Mon. Wed. Fri.,	7.1.1-7.1.3 Ohm's Law & Emf 7.1.3-7.2.2 Emf & Induction 7.2.3-7.2.5 Inductance and Energy of B	
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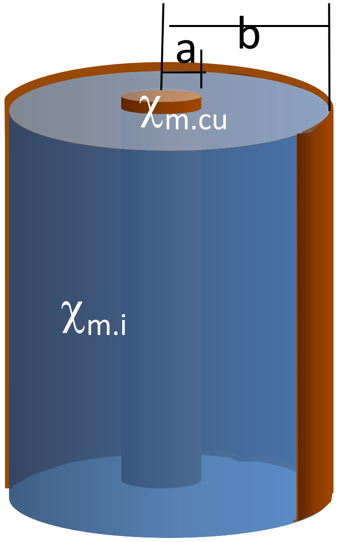
But first: example with linear media

Linear Para/Dia-magnetic

$$I_{free} = \int \vec{J}_{free} \cdot d\vec{a} = \oint \vec{H} \cdot d\vec{l} \qquad \vec{B} = \mu_o\left(\vec{H} + \vec{M}\right) \qquad \vec{M} = \chi_m \vec{H}$$



Example/Exercise: A coaxial cable consists of a copper wire of radius *a* surrounded by a concentric copper sheath of radius *b*. Copper has a magnetic susceptibility $\chi_{m.cu}$. The space between is filled with an insulating material of susceptibility $\chi_{m.i}$. If a current *I* flows up the inner wire (uniformly distributed across the wire's cross-section) and down the outer sheath,

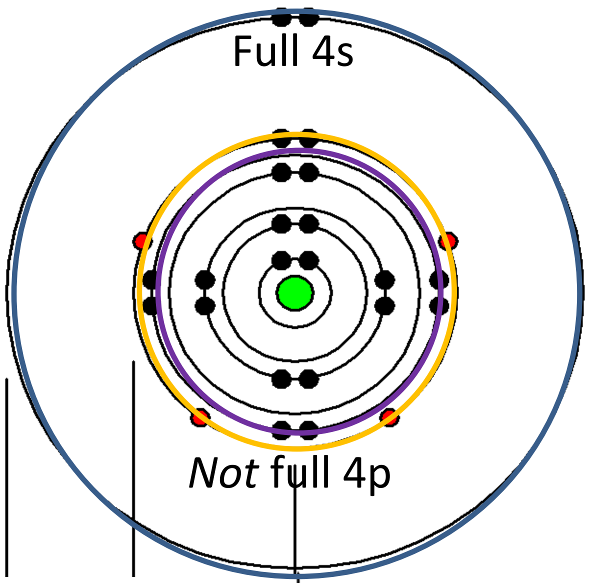
a) find the Auxiliary field everywhere
b) Find the Magnetization everywhere
c) Find the Magnetic field everywhere

I'll do for s<a (inside coper wire), you'll do b<s<a and s<b

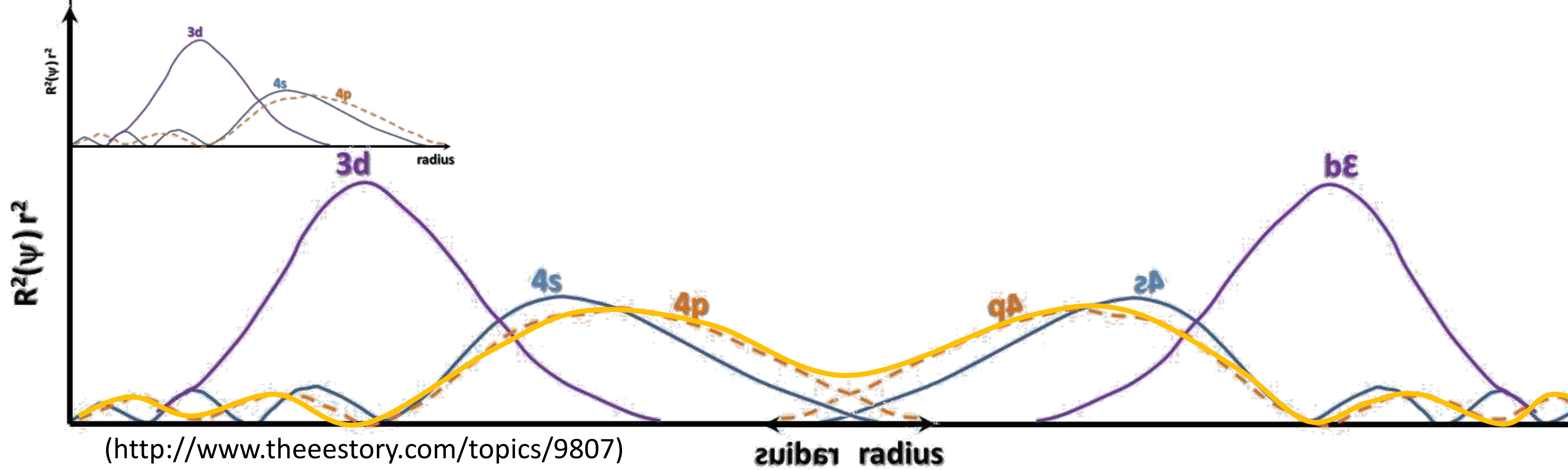
The other magnetism

Ferromagnetism

Fe: $1s^2 2s^2 2p^6 3s^2 3p^6 3d^6 4s^2$



The 3d level is actually higher energy than the on-average larger 4s (which has 4 radial peaks, one closer to the nucleus than the 3d's inner radial peaks), so Iron and its neighbors have filled 4s but only partially filled 3d which is too far in to covalently bond but far enough out to overlap with neighboring irons' 3d's and form a conduction band. So they can share extended wave functions and long wavelength's it's energetically favorable for electrons in this band to be spin-aligned!



(http://www.theeestory.com/topics/9807)

Ohm's Law – charged particles moving in a wire
The classical Drude model

Electron "gas" $v_{thermal} \propto \sqrt{kT}$

Periodic collisions with lattice impurities and vibrations

$$\Delta t_{collisions} = \frac{l_{collisions}}{v_{thermal}}$$

Get's nowhere on average

With Electric Field

$\vec{E}$

q

While the *instantaneous* velocity is hardly change

$$\vec{v} = \vec{v}_{drift} + \vec{v}_{thermal} \approx \vec{v}_{thermal}$$

The random thermal motion averages out while the slow drift forward remains

Between collisions $m\vec{a} = \vec{F} = q\vec{E}$

$$\vec{v}_{ave} = \frac{\vec{v}_{thermal} + \vec{v}_f}{2} = \frac{\vec{v}_{thermal} + \left(\vec{v}_{thermal} + \vec{a}\Delta t_{collision}\right)}{2} = \frac{2\vec{v}_{thermal} + \frac{q}{m}\vec{E}\Delta t_{collision}}{2}$$

$$\left\langle \vec{v}_{ave} \right\rangle = \left\langle \vec{v}_{thermal} \right\rangle + \frac{q}{2m}\vec{E}\Delta t_{collision} = \frac{q}{2m}\vec{E}\frac{l_{collision}}{v_{thermal}} = \vec{v}_{drift}$$

$$\vec{J} = n_{carriers} q \left\langle \vec{v}_{avet} \right\rangle = n_{molecules} f_{carriers/molecule} q \vec{v}_{drift} = \left(n_{molecules} f_{carriers/molecule} \frac{q^2}{2m} \frac{l_{collisions}}{v_{thermal}} \right) \vec{E}$$

Ohm's Law – the current density is *proportional* to the field $\vec{J} = \sigma_{conductivity} \vec{E}$

Ohm's Law – charged particles moving in a wire
The classical Drude model

$$\vec{J} = n_{carriers} q \vec{v}_{drift} = \left(n_{molecules} f_{carriers/molecule} \frac{q^2}{2m} \frac{l_{collisions}}{v_{thermal}} \right) \vec{E}$$

$$\vec{J} = \sigma_{conductivity} \vec{E}$$

Ohm's Law – the current density is *proportional* to the field

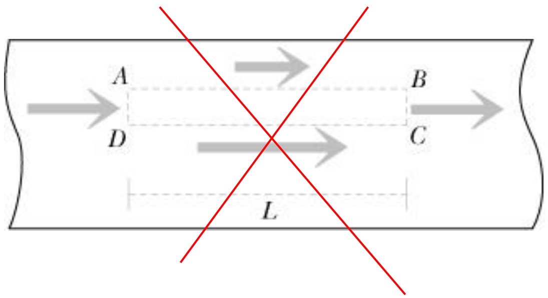
Or, if we integrate over the cross-section perpendicular to the current / field,

$$\int \vec{J} \cdot d\vec{a}_\perp = \int \sigma_{cond} \vec{E} \cdot d\vec{a}_\perp$$

$$I = \left(\int \sigma_{cond} da_\perp \right) E$$

E is uniformly perpendicular to, and constant over this area (otherwise, we'd have a curl)

$$\frac{I}{\left(\int \sigma_{cond} da_\perp \right)} = E$$



Integrate along path current follows.

$$\int \frac{I}{\left(\int \sigma_{cond} da_\perp \right)} dl_\parallel = \int E dl$$

$$I \int \left(\frac{1}{\left(\int \sigma_{cond} da_\perp \right)} \right) dl_\parallel = -\Delta V$$

For steady current, it's constant over the path

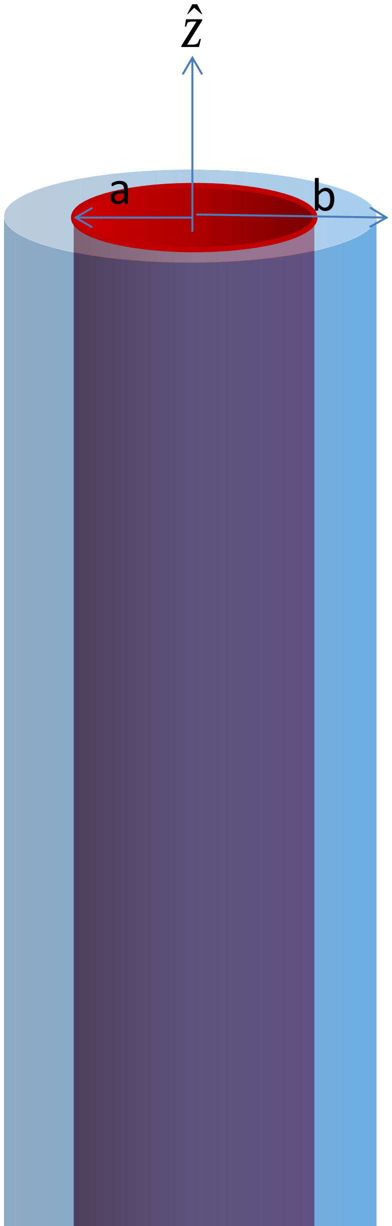
Resistance

$$R \equiv -\frac{dI}{dV} = \int \frac{1}{\left(\int \sigma_{cond} da_\perp \right)} dl_\parallel$$

$$IR = -\Delta V$$

(sign is usually neglected, but means 'current flows down hill.')

Example: Pr. 7.4 Two long, coaxial metal cylinders separated by a material with conductivity $\sigma(s) = k/s$. What is the resistance, *R*?



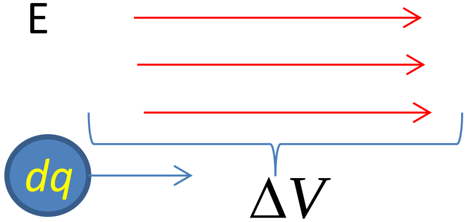
$$R = \int_a^b \frac{1}{\int \sigma(s) da_\perp} dl_\parallel = \int_a^b \frac{1}{\int_0^{2\pi} \int_{-L/2}^{L/2} \frac{k}{s}\left(s d\theta dz\right)} dl_\parallel = \int_a^b \frac{1}{\int_0^{2\pi} k d\theta L} dl_\parallel$$

$$R = \int_a^b \frac{1}{k 2\pi L} dl_\parallel = \frac{b-a}{k 2\pi L}$$

Exercise: Alternatively, imagine charges Q and –Q on the two surfaces, find *E* and a) corresponding *J* and integrate for *I*, then b) corresponding *ΔV*, then take the ratio.

$$R = -\frac{\Delta V = -\int \vec{E} \cdot d\vec{l}}{I = \left(\int \sigma_{cond} da_\perp\right) E}$$

Energy Dissipation

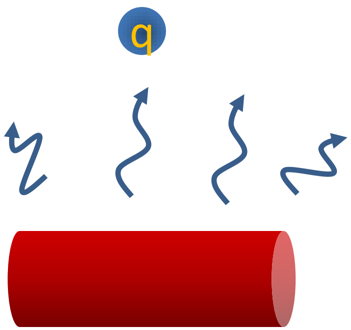


Energy transferred to differential bit of charges when accelerated through a potential difference:

$$(dq)\Delta V = W_{field \rightarrow q} = W_{q \rightarrow wire} = W_{wire \rightarrow environment}$$

But in steady-state, charges moving in a resistive material have no average gain in energy because they repeatedly collide with impurities and vibrations and transfer the energy to the atoms of the wire.

The wire warms up and, unless it melts first (thus stopping the current and the heating), it must shed the energy by heating the environment – conduction and radiation (as in an incandescent lamp's filament.)



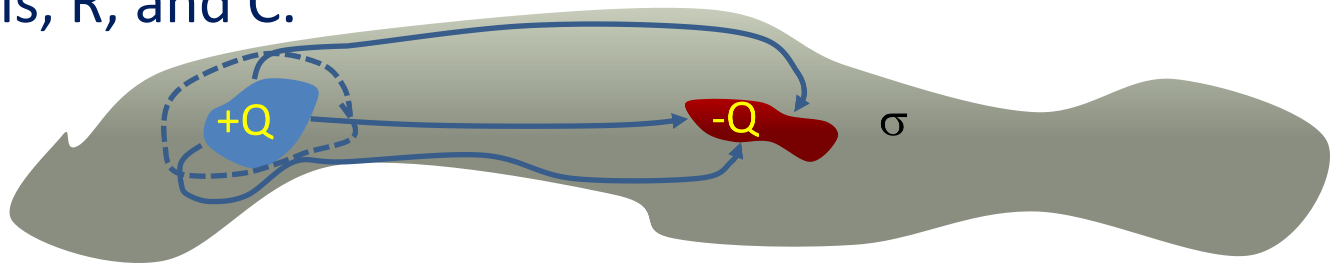
Rate at which energy is transferred :

$$P = \frac{W}{dt} = \frac{dq}{dt}\Delta V = I\Delta V$$

For "ohmic" materials (for which ohm's law applies) $IR = -\Delta V$

so $$|P| = I\Delta V = I^2 R = \frac{(\Delta V)^2}{R}$$

Example: Pr. 7.3 Two metal objects are embedded in a weakly conducting material of constant conductance σ, find the relationship between this, R, and C.



A key point is that current is free to flow *any* direction out of one object to arrive at the other, so when it comes to integrating J over an area, it's over a closed area surrounding one of the objects.

$$R = \frac{-\Delta V}{I} \quad \text{while,} \quad C = \frac{-Q}{\Delta V}$$

So,

$$RC = \frac{-\Delta V / I}{-\Delta V / Q} = \frac{Q}{I} = \frac{\varepsilon_o}{\sigma_{cond}}$$

now,

$$\oint \vec{J} \cdot d\vec{a}_\perp = \oint \sigma_{cond} \vec{E} \cdot d\vec{a}_\perp$$

$$I = \sigma_{cond} \oint \vec{E} \cdot d\vec{a}_\perp$$

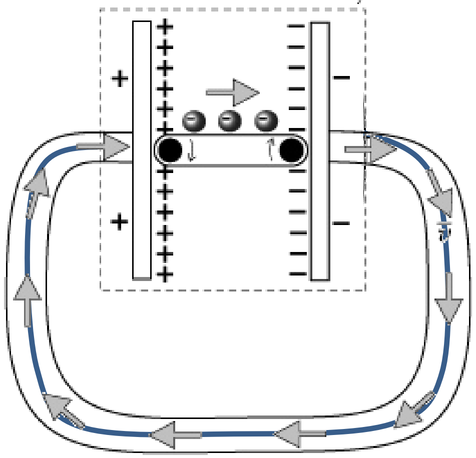
And by Gauss's Law,

$$I = \sigma_{cond} \frac{Q}{\varepsilon_o}$$

So,

Emf (Electro-motive "force")

Some process inside a battery causes charge separation across terminals. The field of those charges drive the current through a circuit.

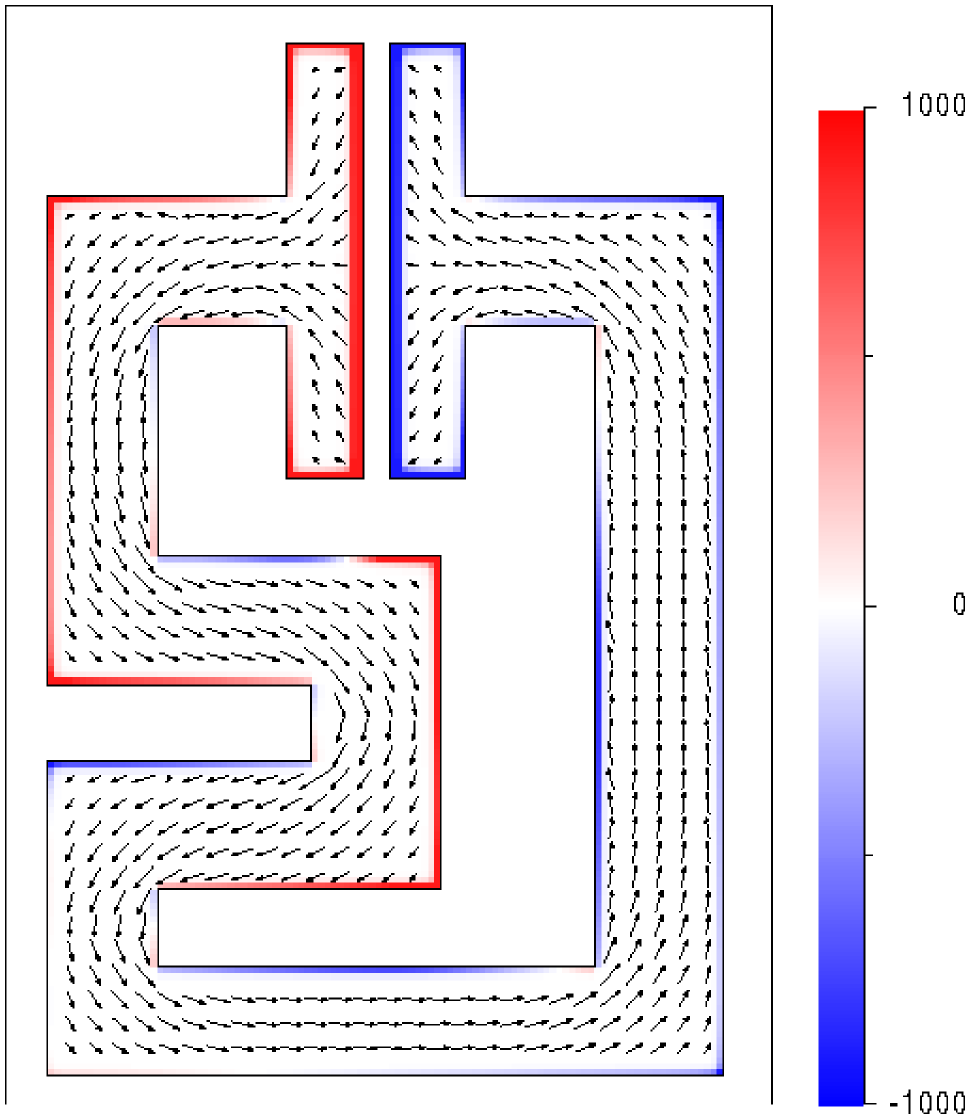


In equilibrium,

$$\vec{F}_{drive\to q} + q\vec{E}_{battery} = \frac{d\vec{p}_q}{dt} = 0 \quad \text{or} \quad -\frac{\vec{F}_{drive\to q}}{q} = \vec{E}_{battery}$$

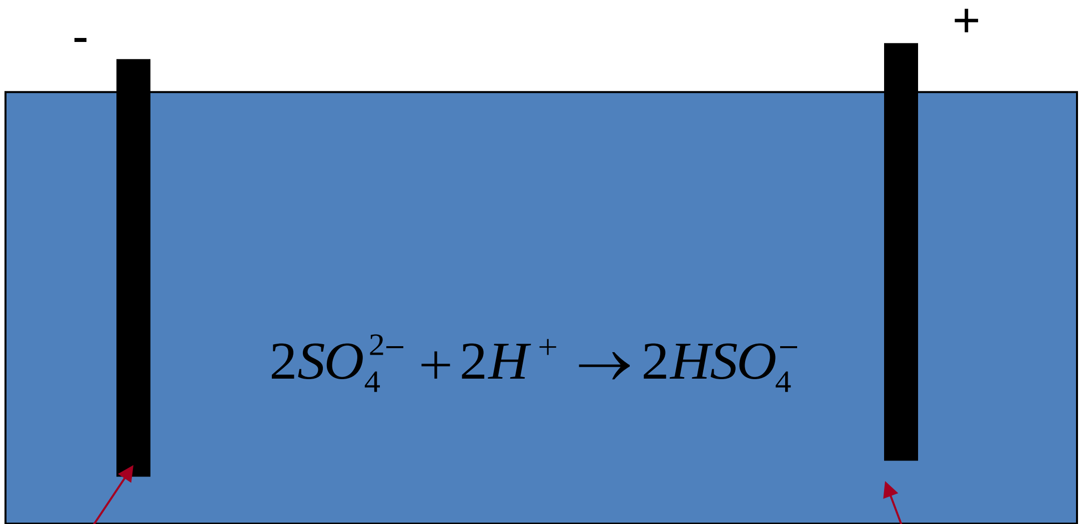
Of course,

$$\Delta V_{battery} = -\int_{+.term}^{-.term} \vec{E}_{battery} \cdot d\vec{l} = -\int_{+.term}^{-.term} \frac{-\vec{F}_{drive\to q}}{q} \cdot d\vec{l} = \frac{W_{drive\to q}}{q} \equiv \mathcal{E}mf$$



1000
0
-1000

Real Chemical Battery



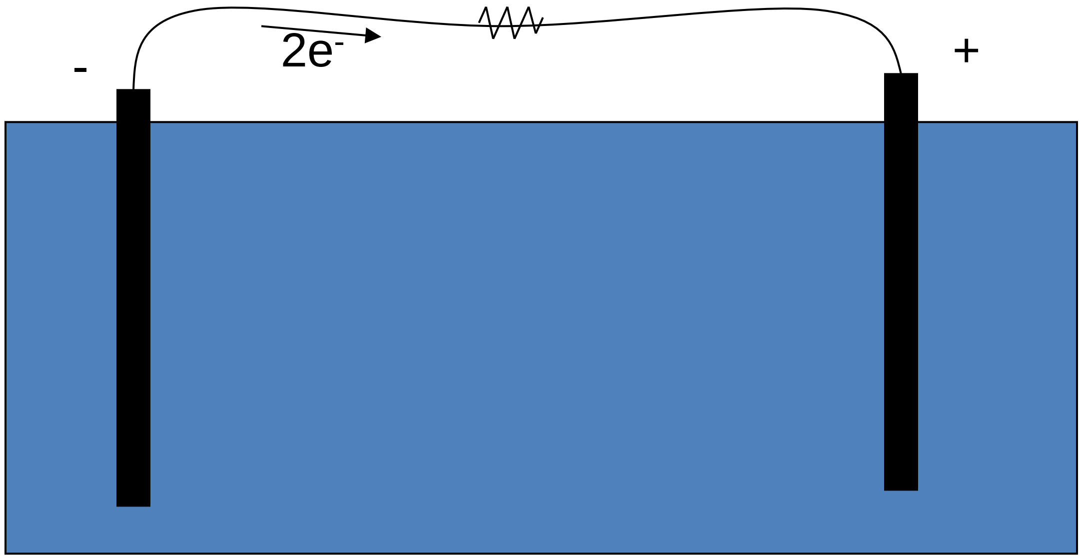
$$Pb + HSO_4^- \rightarrow PbSO_4 + H^+ + 2e^-$$

$$PbO_2 + HSO_4^- + 3H^+ + 2e^- \rightarrow PbSO_4 + 2H_2O$$

Net Effect

$$Pb + PbO_2 + 2\left(2H^+ + SO_4^{2-}\right) \rightarrow 2PbSO_4 + 2H_2O$$

Real Chemical Battery



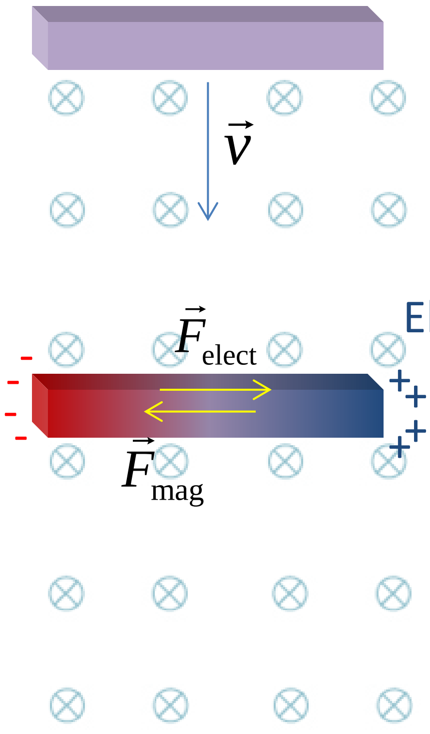
$$Pb + PbO_2 + 2\left(2H^+ + SO_4^{2-}\right) \rightarrow 2PbSO_4 + 2H_2O$$

$$q\Delta V = W_{chem\rightarrow q} \quad \Rightarrow \Delta V = \frac{W_{chem\rightarrow q}}{q} = \text{"}\mathcal{E}mf\text{"}$$

$$W_{chem\rightarrow q} = -\Delta G_{reaction}$$

Difference in Gibbs Free Energy for assembling products and reactants (Phys 344)

Motional *Emf*



Move conducting bar across magnetic field

Mobile electrons move in response to magnetic force

$$\vec{F}_{\text{mag}} = (-e)\vec{v} \times \vec{B}$$

Electron surplus accumulates at one end, deficiency at other

Resulting electric field and force

$$\vec{F}_{\text{elect}} = (-e)\vec{E}$$

grows until

$$\vec{F}_{\text{elect}} + \vec{F}_{\text{mag}} = 0$$

$$(-e)\vec{E} + (-e)\vec{v} \times \vec{B} = 0$$

$$E = vB$$

In terms of Voltage and Emf:

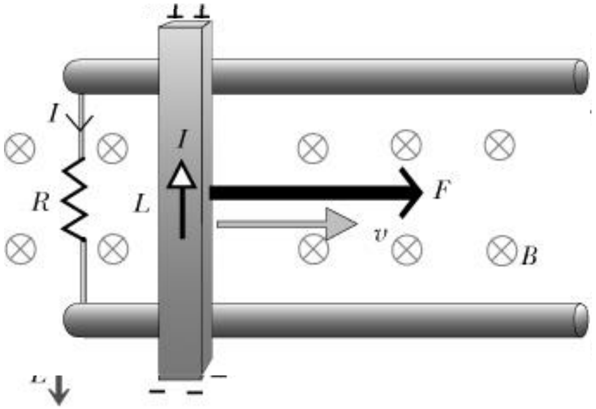
$$\mathcal{E}mf_{mag} = \Delta V$$

$$\mathcal{E}mf_{mag} = \int \left(\frac{\vec{F}_{mag}}{q} \right) \bullet d\vec{\ell} = -\int \vec{E} \bullet d\vec{\ell} = \Delta V$$

$$\mathcal{E}mf_{mag} = \int -vB\hat{y} \bullet d\vec{y} = -\int \vec{E} \bullet d\vec{y} = \Delta V$$

$$\mathcal{E}mf_{mag} = -vBL = -EL = \Delta V$$

Motional *Emf*



$$\mathcal{E}mf_{mag} = -vBL = -EL = \Delta V = -IR$$

so $I = \dfrac{vBL}{R}$

Of course, now that we have established another component of charge motion, a current flowing up, there's *another* component of magnetic force, $\vec{F}_{mag} = \int I d\vec{l} \times \vec{B} = -ILB\hat{x}$

Demo! - "eddy currents"

Example: If the bar has mass *m* and initial speed v_o, what will it be at time t?

$$\frac{d\vec{p}}{dt} = \vec{F}_{mag}$$

$$m\frac{d\vec{v}}{dt} = -\left(\frac{vBL}{R}\right)LB\hat{x}$$

$$\frac{d}{dt}\vec{v} = -\left(\frac{(BL)^2}{mR}\right)\vec{v}$$

$$\vec{v}(t) = \vec{v}_o e^{-t\left(\frac{(BL)^2}{mR}\right)}$$

Show that eventually, all the initial kinetic energy of the bar gets radiated away by the resistor

$$P = I^2 R$$

$$\frac{dE}{dt} = \left(\frac{vBL}{R}\right)^2 R$$

$$\frac{dE}{dt} = \left(\frac{v_o e^{-t\left(\frac{(BL)^2}{mR}\right)} BL}{R}\right)^2 R$$

$$\Delta E = \int_0^t v_o^2 e^{-t\left(2\frac{(BL)^2}{mR}\right)} \frac{(BL)^2}{R} dt$$

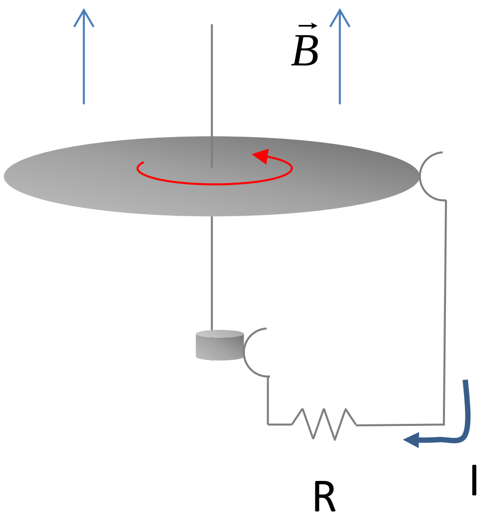
$$\Delta E = \tfrac{1}{2} m\vec{v}_o^{\,2}\left(e^{-2\left(\frac{(BL)^2}{mR}\right)t} - 1\right)$$

$$\Delta E(t = \infty) = -\tfrac{1}{2} m\vec{v}_o^{\,2}$$

Motional *Emf*

Example 7.4 : Faraday Disk A metal disk of radius *a* rotates with an angular frequency ω (counterclockwise viewed from above) about an axis parallel to a uniform magnetic field. A circuit is made by a sliding contact.

What is the current through the resistor *R*?



$$I = \frac{\Delta V}{R}$$

$$\Delta V = \mathcal{E}mf$$

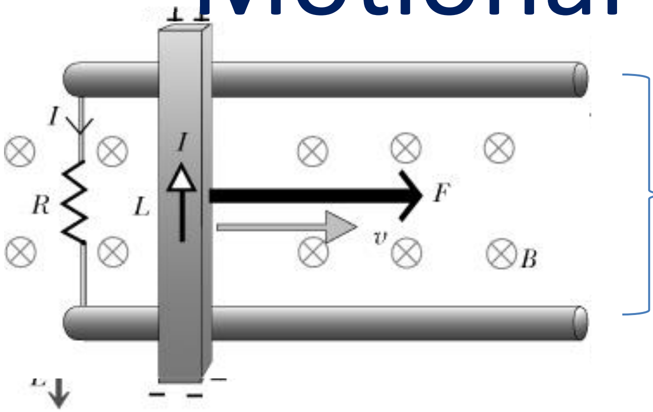
$$\mathcal{E}mf = \int \frac{\vec{F}_{mag}}{q} \cdot d\vec{\ell}$$

$$\vec{F}_{mag \to q} = q\vec{v} \times \vec{B} = q(\omega s)B\,\hat{s}$$

$$\mathcal{E}mf = \int_0^a \omega B\vec{s} \cdot d\vec{s} = \omega B \int_0^a s\,ds = \frac{\omega B a^2}{2}$$

$$I = \frac{\omega B a^2}{2R}$$

Motional *Emf* and Magnetic Flux

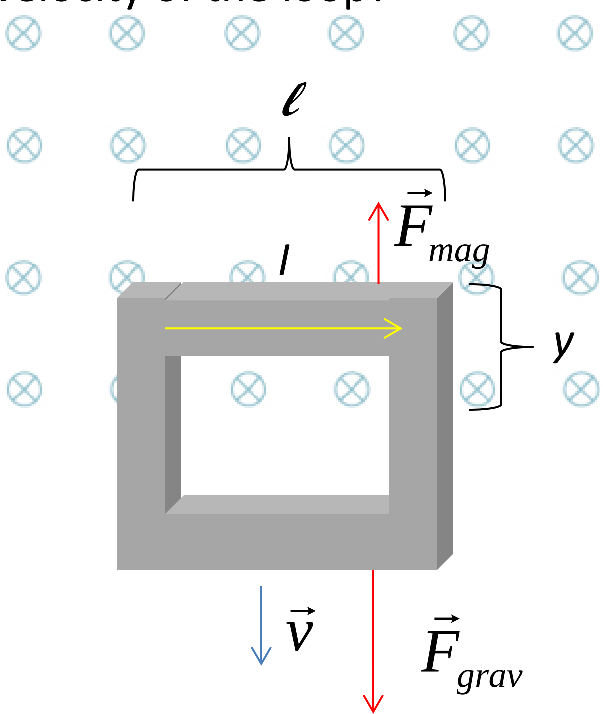


$L = y$

$$\mathcal{E}mf_{mag} = -BvL = -B\underbrace{\frac{dx}{dt}y}_{\frac{da_\perp}{dt}} = -\frac{dBa_\perp}{dt} = -\frac{d\vec{B}\cdot\vec{a}}{dt} = -\frac{d\Phi_B}{dt}$$

Will prove generality soon

Exercise: A square loop is cut out of a thick sheet of aluminum. It is placed so that the top portion is in a uniform, horizontal magnetic field of 1 T <u>into the page</u> (as shown below) and allowed to fall under gravity. The shading indicates the field region. What is the terminal velocity of the loop?



$$\vec{F}_{grav} + \vec{F}_{mag} = \frac{d\vec{p}}{dt}$$

$$m\vec{g} + \int I d\vec{l} \times \vec{B} = m\frac{d\vec{v}}{dt}$$

$$-mg + I\ell B = m\frac{dv}{dt}$$

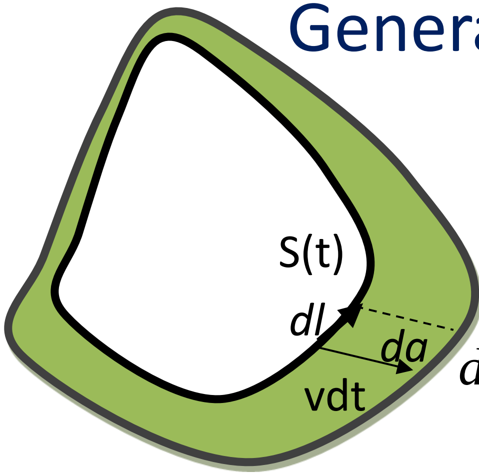
$$I = \frac{|\Delta V|}{R} = \frac{|\mathcal{E}mf|}{R}$$

$$-mg + \frac{B^2\ell^2 v}{R} = m\frac{dv}{dt}$$

$$|\mathcal{E}mf| = \frac{d\Phi}{dt} = B\ell v$$

$$-mg + \frac{B^2\ell^2 v_{ter}}{R} = 0 \qquad v_{ter} = \frac{mgR}{B^2\ell^2}$$

Generalization of Flux Rule



$$d\vec{a} = \vec{v}dt \times d\vec{l}$$

Using vector identity (1)

$$\vec{A}\cdot\left(\vec{B}\times\vec{C}\right) = \left(\vec{A}\times\vec{B}\right)\cdot\vec{C}$$

$$\vec{B}\cdot d\vec{a} = \vec{B}\cdot\left(\vec{v}dt \times d\vec{l}\right) = \left(\vec{B}\times\vec{v}dt\right)\cdot d\vec{l} = -\left(\vec{v}dt\times\vec{B}\right)\cdot d\vec{l}$$

Thus change in magnetic flux through the loop

$$d\Phi_B = -\oint\left(\vec{v}dt\times\vec{B}\right)\cdot d\vec{l}$$

rate of change in magnetic flux through the loop

$$\left.\frac{\partial\Phi_B}{\partial t}\right|_B = -\oint\left(\vec{v}\times\vec{B}\right)\cdot d\vec{l} = -\oint\frac{\vec{F}_{mag}}{q}\cdot d\vec{l}$$

$$\left.\frac{\partial\Phi_B}{\partial t}\right|_B = -Emf_{mag}$$

Warning: our derivation used that the changing, *da/dt*, corresponded to moving charge, *vdl*. Not applicable when that's not the case.
– *thar be "paradoxes"*

(We will later extend this reasoning to discuss stationary charges but changing fields)

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Where we've been

Stationary Charges – producing and interacting via Electric Fields

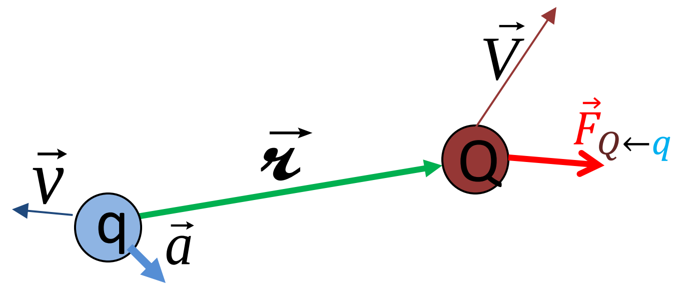
Steady Currents – producing and interacting via Magnetic Fields

Where we're going

Varying currents and charge distributions – producing and interacting with varying Electric and Magnetic Fields

A step closer to

Force between moving charges



$$\vec{F}_{Q\leftarrow q} = Q\left\{\vec{E}_q + \vec{V} \times \vec{B}_q\right\}$$

$$\vec{u} \equiv c\hat{\mathscr{r}} - \vec{v}$$

where

$$\vec{E}_q = \frac{q}{4\pi\varepsilon_o}\frac{\mathscr{r}}{(\vec{\mathscr{r}}\cdot\vec{u})^3}\left[(c^2 - v^2)\vec{u} + \vec{\mathscr{r}} \times (\vec{u} \times \vec{a})\right]$$

and

$$\vec{B}_q = \frac{q}{4\pi\varepsilon_o}\frac{\mathscr{r}}{(\vec{\mathscr{r}}\cdot\vec{u})^3}\frac{1}{c}\left[\hat{\mathscr{r}} \times \left[(c^2 - v^2)\vec{u} + \vec{\mathscr{r}} \times (\vec{u} \times \vec{a})\right]\right]$$

or

$$\vec{F}_{Q\leftarrow q} = \frac{qQ}{4\pi\varepsilon_o}\frac{\mathscr{r}}{(\vec{\mathscr{r}}\cdot\vec{u})^3}\left\{\underbrace{\left[(c^2 - v^2)\vec{u} + \vec{\mathscr{r}} \times (\vec{u} \times \vec{a})\right]}_{\text{Electric}} + \underbrace{\frac{\vec{V}}{c} \times \left[\hat{\mathscr{r}} \times \left[(c^2 - v^2)\vec{u} + \vec{\mathscr{r}} \times (\vec{u} \times \vec{a})\right]\right]}_{\text{Magnetic}}\right\}$$

Electric

Depends on *observer's* perception of *source* charge's velocity and acceleration

Magnetic

Also depends on *observer's* perception of *recipient* charge's velocity