Mon., Tues. Wed.	7.3.1-.3.3 Maxwell's Equations 10.1 - .2.1 Potential Formulation Lunch with UCR Engr – 12:20 – 1:00	HW10

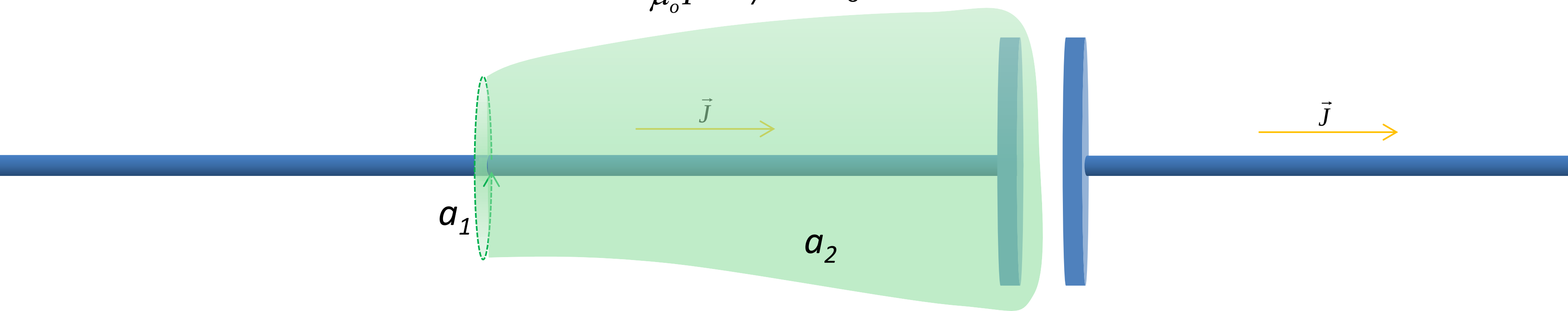
Pizza headcount & Preferences

Correcting Ampere's law

Physical Motivation

$$\oint \vec{B}\cdot d\vec{l} = \mu_o \int \vec{J}\cdot d\vec{a}_1 \quad = \mu_o \int \vec{J}\cdot d\vec{a}_2$$

$$= \mu_o I \quad \neq \quad = 0$$



Mathematical Motivation

It's a mathematical fact that, the divergence of a curl of a vector field is 0

$$\vec{\nabla}\cdot\left(\vec{\nabla}\times\vec{B}\right) = 0$$

We claim $\vec{\nabla}\times\vec{B} \neq \mu_0 \vec{J}$

So,

$$\vec{\nabla}\cdot\left(\vec{\nabla}\times\vec{B}\right) \neq \vec{\nabla}\cdot\left(\mu_0 \vec{J}\right)$$

Continuity Equation:

$$\vec{\nabla}\cdot\vec{J} = -\frac{\partial \rho}{\partial t}$$

Note: in the scenario above this *isn't* zero

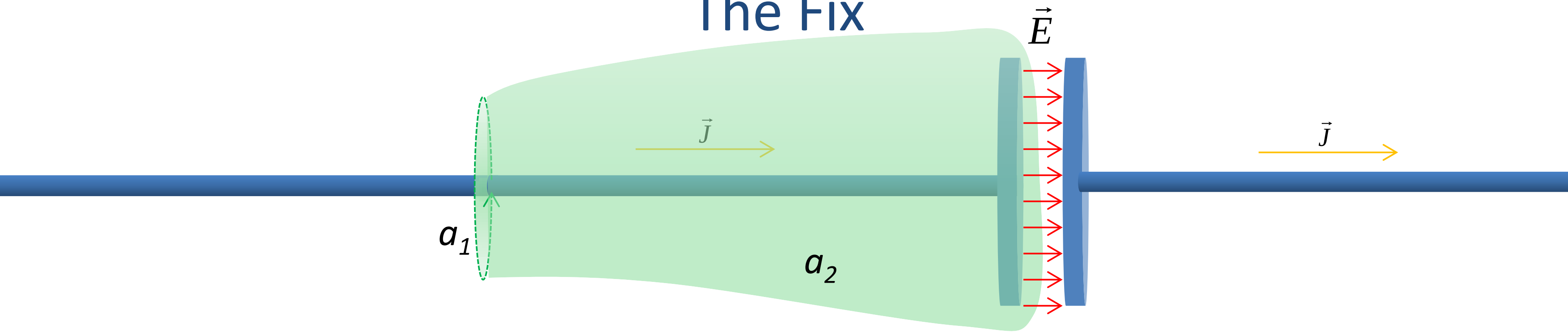
So,

$$\vec{\nabla}\cdot\left(\vec{\nabla}\times\vec{B}\right) \neq -\mu_0 \frac{\partial \rho}{\partial t}$$

At capacitor plate *not 0*

Correcting Ampere's law

The Fix



It's a mathematical **fact** that, the divergence of a curl of a vector field is 0

$$\vec{\nabla}\cdot\left(\vec{\nabla}\times\vec{B}\right)=0$$

So need $\vec{\nabla}\cdot\left(\vec{\nabla}\times\vec{B}\right)+something=0-\mu_0\frac{\partial\rho}{\partial t}$

$$\varepsilon_0\vec{\nabla}\cdot\vec{E}=\rho$$

$$-\mu_o\frac{\partial}{\partial t}\left(\varepsilon_0\vec{\nabla}\cdot\vec{E}\right)=-\mu_o\frac{\partial}{\partial t}\rho$$

In the scenario above, E is changing as the plates charge

$$\vec{\nabla}\cdot\left(\vec{\nabla}\times\vec{B}\right)-\mu_o\frac{\partial}{\partial t}\left(\varepsilon_0\vec{\nabla}\cdot\vec{E}\right)=0-\mu_0\frac{\partial\rho}{\partial t}$$

$$\vec{\nabla}\cdot\left(\vec{\nabla}\times\vec{B}-\mu_o\varepsilon_0\frac{\partial\vec{E}}{\partial t}\right)=-\mu_0\frac{\partial\rho}{\partial t}$$

Or rephrasing in terms of *J* again

$$\vec{\nabla}\cdot\left(\vec{\nabla}\times\vec{B}-\mu_o\varepsilon_0\frac{\partial\vec{E}}{\partial t}\right)=\mu_0\vec{\nabla}\cdot\vec{J}$$

$$\vec{\nabla}\times\vec{B}-\mu_o\varepsilon_0\frac{\partial\vec{E}}{\partial t}=\mu_0\vec{J}$$

Unfortunate historical name: "Displacement Current"

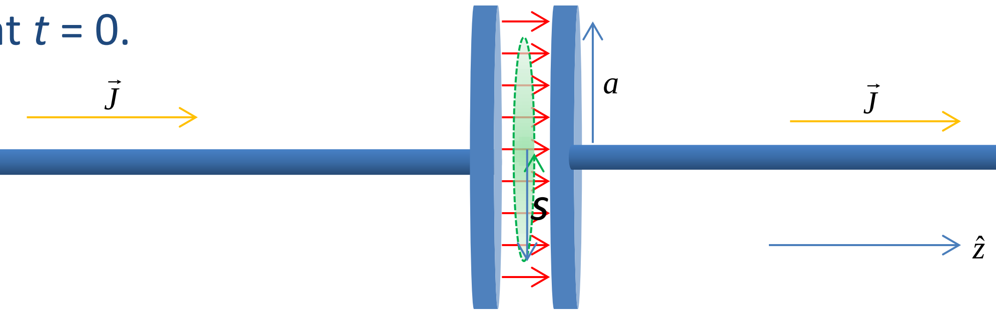
$$\vec{J}_D\equiv\varepsilon_0\frac{\partial\vec{E}}{\partial t}$$

Conceptually, a stand-in for the effect of currents elsewhere

Corrected Maxwell-Ampere's law

$$\vec{\nabla}\times\vec{B}-\mu_o\varepsilon_0\frac{\partial\vec{E}}{\partial t}=\mu_0\vec{J} \qquad \oint\vec{B}\cdot d\vec{\ell}-\mu_0\int\varepsilon_0\frac{\partial\vec{E}}{\partial t}\cdot d\vec{a}=\mu_0\int\vec{J}\cdot d\vec{a}$$

Example: Thin wires connect to the centers of narrow, round capacitor plates. Suppose that the current *I* is constant, the radius of the capacitor is *a*, and the separation of the plates is *w* (<< *a*). Assume that the current flows out over the plates in such a way that the surface charge is uniform at any given time and is zero at *t* = 0.



a) Find the electric field between the plates as a function of time *t.*

Approximating infinite sheets, recall from Gauss's law

$$\vec{E}(t)=\frac{\sigma(t)}{\varepsilon_o}\hat{z} \quad \text{or } \vec{E}(t)=\frac{q(t)/Area}{\varepsilon_o}\hat{z} \quad \text{or since } I_{wire}=\frac{dq(t)}{dt}\Rightarrow q(t)=I_{wire}t \quad \text{and } Area=\pi a^2$$

Constant current and q(0)=0

$$\vec{E}(t)=\left(\frac{I_{wire}}{\varepsilon_o\pi a^2}t\right)\hat{z}$$

b) Using this as an Amperian Loop, find the magnetic field between the capacitor plate.

$$\oint\vec{B}\cdot d\vec{\ell}-\mu_0\int\varepsilon_0\frac{\partial\vec{E}}{\partial t}\cdot d\vec{a}=\mu_0\int\vec{J}\cdot d\vec{a}$$

None across this surface

$$\oint\vec{B}\cdot d\vec{\ell}=\mu_0\int\varepsilon_0\frac{\partial\vec{E}}{\partial t}\cdot d\vec{a}=\mu_o\frac{I_{wire}}{\pi a^2}\pi s^2=\mu_o\frac{I_{wire}}{a^2}s^2$$

Symmetry, as always, tells us B parallels our loop

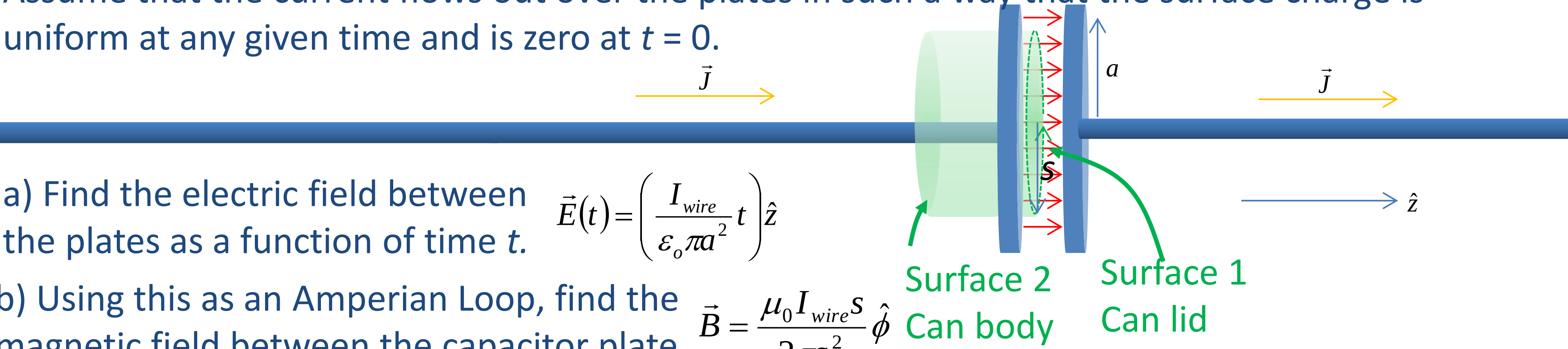
$$B\cdot(2\pi s)=\mu_0\left(I_{wire}\frac{s^2}{a^2}\right) \quad \text{so } \vec{B}=\frac{\mu_0 I_{wire}s}{2\pi a^2}\hat{\phi}$$

Just like field inside the wire!

Corrected Maxwell-Ampere's law

$$\vec{\nabla}\times\vec{B}-\mu_o\varepsilon_0\frac{\partial\vec{E}}{\partial t}=\mu_0\vec{J} \qquad \oint\vec{B}\cdot d\vec{\ell}-\mu_0\int\varepsilon_0\frac{\partial\vec{E}}{\partial t}\cdot d\vec{a}=\mu_0\int\vec{J}\cdot d\vec{a}$$

Example: Thin wires connect to the centers of thin, round capacitor plates. Suppose that the current *I* is constant, the radius of the capacitor is *a*, and the separation of the plates is *w* (<< *a*). Assume that the current flows out over the plates in such a way that the surface charge is uniform at any given time and is zero at *t* = 0.



a) Find the electric field between the plates as a function of time *t*.

$$\vec{E}(t)=\left(\frac{I_{wire}}{\varepsilon_o\pi a^2}t\right)\hat{z}$$

b) Using this as an Amperian Loop, find the magnetic field between the capacitor plate.

$$\vec{B}=\frac{\mu_0 I_{wire}s}{2\pi a^2}\hat{\phi}$$

c) Find the current *along the surface of the capacitor plate.*

Compare Maxwell-Ampere's Law for two, wisely-chosen surfaces bound by our Amperian loop.

$$\mu_0\left(\int_{surface.1}\vec{J}\cdot d\vec{a}+\int_{surface.1}\varepsilon_0\frac{\partial\vec{E}}{\partial t}\cdot d\vec{a}\right)=\oint\vec{B}\cdot d\vec{\ell}=\mu_0\left(\int_{surface.2}\vec{J}\cdot d\vec{a}+\int_{surface.2}\varepsilon_0\frac{\partial\vec{E}}{\partial t}\cdot d\vec{a}\right)$$

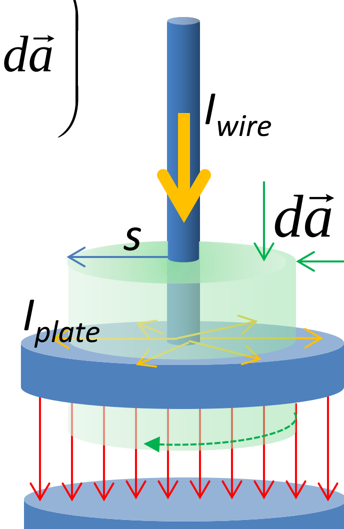
$$\int_{surface.1}\varepsilon_0\frac{\partial\vec{E}}{\partial t}\cdot d\vec{a}=\int_{surface.2}\vec{J}\cdot d\vec{a}=\int_{end.cap}\vec{J}\cdot d\vec{a}+\int_{cylindrical.wall}\vec{J}\cdot d\vec{a}$$

parallel

$$\left(\frac{I_{wire}}{\pi a^2}\right)\pi s^2 \quad = I_{wire}-I_{plate}$$

anti-parallel

$$I_{plate}=I_{wire}\left(1-\left(\tfrac{s}{a}\right)^2\right)$$



Corrected Maxwell-Ampere's law

$$\vec{\nabla}\times\vec{B}-\mu_o\varepsilon_0\frac{\partial\vec{E}}{\partial t}=\mu_0\vec{J} \qquad \oint\vec{B}\cdot d\vec{\ell}-\mu_0\int\varepsilon_0\frac{\partial\vec{E}}{\partial t}\cdot d\vec{a}=\mu_0\int\vec{J}\cdot d\vec{a}$$

Execise: Current $I(t)=I_0\cos(\omega t)$ flows down a long, straight, thin wire and returns along a thin, coaxial conducting tube of radius *a*. From Faraday's Law, the electric field for the region $s < a$ is

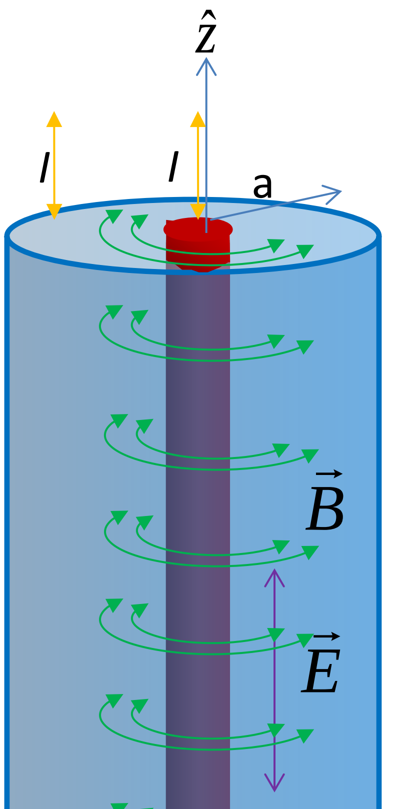
$$\vec{E}(s,t)=\frac{\mu_0 I_0\omega}{2\pi}\ln\left(\frac{a}{s}\right)\sin(\omega t)\hat{z}$$

a) Find an expression for the "displacement current" density.

$$\varepsilon_0\frac{\partial\vec{E}}{\partial t}$$

b) Integrate over a cross-section it pierces to find the "displacement current".

$$\varepsilon_0\int\frac{\partial\vec{E}}{\partial t}\cdot d\vec{a}$$



Integration Note:

$$\int\ln\left(\frac{a}{s}\right)sds=\int\ln\left(\frac{a}{s}\right)\tfrac{1}{2}ds^2=\int\left(-\tfrac{1}{2}\ln\left(\left(\frac{s}{a}\right)^2\right)\right)\tfrac{1}{2}ds^2$$

So it may be convenient to do the change of variables $\zeta\equiv\left(\frac{s}{a}\right)^2$

Maxwell's Laws

Relating Fields and Sources

Relativistically Correct since instantaneous and local

$$\vec{\nabla} \times \vec{B} - \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} = \mu_0 \vec{J}$$ Maxwell – Ampere's Law $$\oint \vec{B} \cdot d\vec{\ell} - \mu_0 \varepsilon_0 \int \frac{\partial \vec{E}}{\partial t} \cdot d\vec{a} = \mu_0 \int \vec{J} \cdot d\vec{a}$$

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$ Gauss's Law $$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\varepsilon_0}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$ Gauss's Law for Magnetism $$\oint \vec{B} \cdot d\vec{a} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$ Faraday's Law $$\oint \vec{E} \cdot d\vec{\ell} = -\left.\frac{\partial \Phi_B}{\partial t}\right|_a = -\int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{a}$$

Not necessarily Relativistically Correct

Helmholtz Theorem: if you know a vector field's curl and divergence (and time derivative), you know everything

Maxwell's Laws

Relating Fields and Sources

Relativistically Correct since instantaneous and local

$$\vec{\nabla}\times\vec{B}-\mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}=\mu_0\vec{J}$$ Maxwell – Ampere's Law $$\oint\vec{B}\cdot d\vec{\ell}-\mu_0\varepsilon_0\int\frac{\partial\vec{E}}{\partial t}\cdot d\vec{a}=\mu_0\int\vec{J}\cdot d\vec{a}$$

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$$\vec{\nabla}\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}$$ Faraday's Law $$\oint\vec{E}\cdot d\vec{\ell}=-\left.\frac{\partial\Phi_B}{\partial t}\right|_a=-\int\frac{\partial\vec{B}}{\partial t}\cdot d\vec{a}$$

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Helmholtz Theorem: if you know a vector field's curl and divergence (and time derivative), you know everything

Example 7.14, Problem 7.34

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Maxwell – Ampere's Law

$$\vec{\nabla}\times\vec{B}-\mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}=\mu_0\vec{J} \qquad \oint\vec{B}\cdot d\vec{\ell}-\mu_0\varepsilon_0\int\frac{\partial\vec{E}}{\partial t}\cdot d\vec{a}=\mu_0\int\vec{J}\cdot d\vec{a}$$

Gauss's Law

$$\vec{\nabla}\cdot\vec{E}=\frac{\rho}{\varepsilon_0} \qquad \oint\vec{E}\cdot d\vec{a}=\frac{Q_{enc}}{\varepsilon_0}$$

Gauss's Law for Magnetism

$$\vec{\nabla}\cdot\vec{B}=0 \qquad \oint\vec{B}\cdot d\vec{a}=0$$

Faraday's Law

$$\vec{\nabla}\times\vec{E}=-\frac{\partial\vec{B}}{\partial t} \qquad \oint\vec{E}\cdot d\vec{\ell}=-\frac{\partial\Phi_B}{\partial t}\bigg|_a=-\int\frac{\partial\vec{B}}{\partial t}\cdot d\vec{a}$$