Mon.	10.3 Point Charges	HW11

Continuous Source Distribution

$$\vec{E}(\vec{r},t)=\frac{1}{4\pi\varepsilon_o}\int\left(\frac{\dot{\rho}(\vec{r}\,',t_r)\hat{\mathscr{r}}}{c\mathscr{r}}+\frac{\rho(\vec{r}\,',t_r)\hat{\mathscr{r}}}{\mathscr{r}^2}-\frac{\dot{\vec{J}}(\vec{r}\,',t_r)}{c^2\mathscr{r}}\right)d\tau'$$

$$\vec{B}(\vec{r},t)=\frac{\mu_o}{4\pi}\int\left(\frac{\dot{\vec{J}}(\vec{r}\,',t_r)}{c\mathscr{r}}+\frac{\vec{J}(\vec{r}\,',t_r)}{\mathscr{r}^2}\right)\times\hat{\mathscr{r}}d\tau'$$

$$V(\vec{r},t)=-\tfrac{1}{4\pi\varepsilon_o}\int\frac{\rho(\vec{r}\,',t_r)}{\mathscr{r}}d\tau'$$

$$\vec{A}(\vec{r},t)=-\tfrac{\mu_o}{4\pi}\int\frac{\vec{J}(\vec{r}\,',t_r)}{\mathscr{r}}d\tau'$$

Point Source

[redacted]

Continuous Source Distribution

$$\vec{E}(\vec{r},t)=\frac{1}{4\pi\varepsilon_o}\int\left(\frac{\dot{\rho}(\vec{r}',t_r)\hat{\boldsymbol{\imath}}}{c\boldsymbol{\imath}}+\frac{\rho(\vec{r}',t_r)\hat{\boldsymbol{\imath}}}{\boldsymbol{\imath}^2}-\frac{\dot{\vec{J}}(\vec{r}',t_r)}{c^2\boldsymbol{\imath}}\right)d\tau'$$

$$\vec{B}(\vec{r},t)=\frac{\mu_o}{4\pi}\int\left(\frac{\dot{\vec{J}}(\vec{r}',t_r)}{c\boldsymbol{\imath}}+\frac{\vec{J}(\vec{r}',t_r)}{\boldsymbol{\imath}^2}\right)\times\hat{\boldsymbol{\imath}}\,d\tau'$$

$$V(\vec{r},t)=-\tfrac{\mu_o}{4\pi}\int\frac{\rho(\vec{r}',t_r)}{\boldsymbol{\imath}}d\tau'$$

$$\vec{A}(\vec{r},t)=-\tfrac{\mu_o}{4\pi}\int\frac{\vec{J}(\vec{r}',t_r)}{\boldsymbol{\imath}}d\tau'$$

Point Source

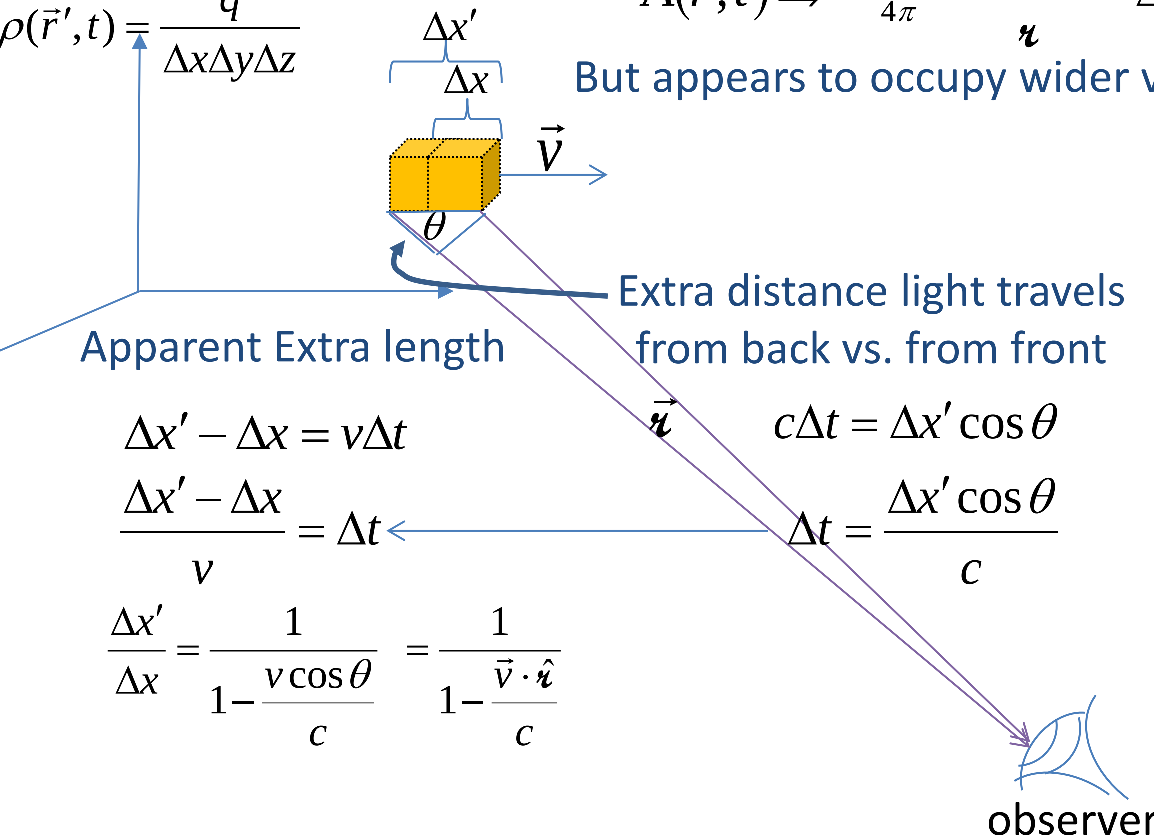
$$V(\vec{r},t)=-\tfrac{1}{4\pi\varepsilon_o}\frac{qc}{\boldsymbol{\imath}c-\vec{v}\cdot\vec{\boldsymbol{\imath}}}$$

Differentially small volume of charge

$$\vec{A}(\vec{r},t)\Rightarrow-\tfrac{\mu_o}{4\pi}\frac{\rho(\vec{r}',t_r)\vec{v}}{\boldsymbol{\imath}}\Delta\tau\quad=-\tfrac{\mu_o}{4\pi}\frac{\left(\frac{q}{\Delta x\Delta y\Delta z}\right)\vec{v}}{\boldsymbol{\imath}}(\Delta x'\Delta y\Delta z)$$

$$\rho(\vec{r}',t)=\frac{q}{\Delta x\Delta y\Delta z}$$

But appears to occupy wider volume



$$\vec{A}(\vec{r},t)=-\tfrac{\mu_o}{4\pi}\frac{q\vec{v}}{\boldsymbol{\imath}}\left(\frac{\Delta x'}{\Delta x}\right)$$

$$\Delta x'-\Delta x=v\Delta t$$

$$\frac{\Delta x'-\Delta x}{v}=\Delta t$$

$$c\Delta t=\Delta x'\cos\theta$$

$$\Delta t=\frac{\Delta x'\cos\theta}{c}$$

$$\frac{\Delta x'}{\Delta x}=\frac{1}{1-\frac{v\cos\theta}{c}}=\frac{1}{1-\frac{\vec{v}\cdot\hat{\boldsymbol{\imath}}}{c}}$$

$$\vec{A}(\vec{r},t)=-\tfrac{\mu_o}{4\pi}\frac{qc\vec{v}}{\boldsymbol{\imath}c-\vec{v}\cdot\vec{\boldsymbol{\imath}}}$$

$$\vec{A}(\vec{r},t)=-\tfrac{\mu_o}{4\pi}\tfrac{c^2}{c^2}\frac{qc\vec{v}}{\boldsymbol{\imath}c-\vec{v}\cdot\vec{\boldsymbol{\imath}}}$$

$$\vec{A}(\vec{r},t)=-\tfrac{1}{4\pi\varepsilon_o}\tfrac{1}{c^2}\frac{qc\vec{v}}{\boldsymbol{\imath}c-\vec{v}\cdot\vec{\boldsymbol{\imath}}}$$

$$\vec{A}(\vec{r},t)=\frac{\vec{v}}{c^2}V(\vec{r},t)$$

Point Source

$$V(\vec{r},t) = -\frac{1}{4\pi\varepsilon_o}\frac{qc}{\mathscr{r}c - \vec{v}\cdot\vec{\mathscr{r}}} \qquad \vec{A}(\vec{r},t) = -\frac{\mu_o}{4\pi}\frac{qc\vec{v}}{\mathscr{r}c - \vec{v}\cdot\vec{\mathscr{r}}} = \frac{\vec{v}}{c^2}V(\vec{r},t)$$

How about Fields

$$\vec{E} = -\vec{\nabla}V - \frac{\partial}{\partial t}\vec{A} \qquad \vec{B} = \vec{\nabla}\times\vec{A}$$

$$\vec{\nabla}_r V(\vec{r},t) = \frac{qc}{4\pi\varepsilon_o}\vec{\nabla}_r\frac{1}{\mathscr{r}c - \vec{v}\cdot\vec{\mathscr{r}}}$$

$$= \frac{qc}{4\pi\varepsilon_o}\frac{-1}{(\mathscr{r}c - \vec{v}\cdot\vec{\mathscr{r}})^2}\vec{\nabla}_r(\mathscr{r}c - \vec{v}\cdot\vec{\mathscr{r}})$$

Notational note: to reinforce that our r now points to a *moving* source, Griffiths replaces "r'", that's stationary in time, with "w", that tracks the moving source.

$$\vec{\nabla}_r(\mathscr{r}c - \vec{v}\cdot\vec{\mathscr{r}}) = \vec{\nabla}_r\mathscr{r}c - \vec{\nabla}_r(\vec{v}\cdot\vec{\mathscr{r}}) = \hat{\mathscr{r}}c - \vec{\nabla}_r(\vec{v}\cdot\vec{\mathscr{r}})$$

Product Rule 4

$$\vec{\nabla}_r(\vec{v}\cdot\vec{\mathscr{r}}) = \vec{v}\times(\vec{\nabla}_r\times\vec{\mathscr{r}}) + \vec{\mathscr{r}}\times(\vec{\nabla}_r\times\vec{v}) + (\vec{\mathscr{r}}\cdot\vec{\nabla}_r)\vec{v} + (\vec{v}\cdot\vec{\nabla}_r)\vec{\mathscr{r}}$$

$$\vec{\nabla}_r\times\vec{\mathscr{r}} = \vec{\nabla}_r\times(\vec{r} - \vec{w}(t_r))$$

$$\vec{\mathscr{r}} = \vec{r} - \vec{r}\,' \Rightarrow \vec{r} - \vec{w}(t_r)$$

Focus on just one component

$$(\vec{\nabla}_r\times\vec{w}(t_r))_x = \frac{\partial w_z(t_r)}{\partial y} - \frac{\partial w_y(t_r)}{\partial z} \qquad = 0 - \vec{\nabla}\times\vec{w}(t_r)$$

$$\frac{\partial w_z}{\partial t_r}\frac{\partial t_r}{\partial y} - \frac{\partial w_y}{\partial t_r}\frac{\partial t_r}{\partial z} = v_z\frac{\partial t_r}{\partial y} - v_y\frac{\partial t_r}{\partial z}$$

$$\vec{\nabla}\times\vec{w}(t_r) = \left(v_z\frac{\partial t_r}{\partial y} - v_y\frac{\partial t_r}{\partial z}\right)\hat{x} + \left(v_x\frac{\partial t_r}{\partial z} - v_z\frac{\partial t_x}{\partial z}\right)\hat{y} + \left(v_y\frac{\partial t_r}{\partial x} - v_x\frac{\partial t_r}{\partial y}\right)\hat{z} = \nabla t_r\times\vec{v} = -\vec{v}\times\nabla t_r$$

So far: $\vec{\nabla}_r\times\vec{\mathscr{r}} = (\vec{v}\times\nabla t_r)$

Point Source

$$V(\vec{r},t) = -\frac{1}{4\pi\varepsilon_o}\frac{qc}{\mathscr{r}c - \vec{v}\cdot\vec{\mathscr{r}}}$$

$$\vec{A}(\vec{r},t) = -\frac{\mu_o}{4\pi}\frac{qc\vec{v}}{\mathscr{r}c - \vec{v}\cdot\vec{\mathscr{r}}} = \frac{\vec{v}}{c^2}V(\vec{r},t)$$

How about Fields

$$\vec{E} = -\vec{\nabla}V - \frac{\partial}{\partial t}\vec{A} \qquad \vec{B} = \vec{\nabla}\times\vec{A}$$

$$\vec{\nabla}_r V(\vec{r},t) = \frac{qc}{4\pi\varepsilon_o}\vec{\nabla}_r\frac{1}{\mathscr{r}c - \vec{v}\cdot\vec{\mathscr{r}}}$$

$$= \frac{qc}{4\pi\varepsilon_o}\frac{-1}{(\mathscr{r}c - \vec{v}\cdot\vec{\mathscr{r}})^2}\vec{\nabla}_r(\mathscr{r}c - \vec{v}\cdot\vec{\mathscr{r}})$$

$$\vec{\nabla}_r(\mathscr{r}c - \vec{v}\cdot\vec{\mathscr{r}}) = \vec{\nabla}_r\mathscr{r}c - \vec{\nabla}_r(\vec{v}\cdot\vec{\mathscr{r}}) = \hat{\mathscr{r}}c - \vec{\nabla}_r(\vec{v}\cdot\vec{\mathscr{r}})$$

Product Rule 4

$$\vec{\nabla}_r(\vec{v}\cdot\vec{\mathscr{r}}) = \vec{v}\times(\vec{\nabla}_r\times\vec{\mathscr{r}}) + \vec{\mathscr{r}}\times(\vec{\nabla}_r\times\vec{v}) + (\vec{\mathscr{r}}\cdot\vec{\nabla}_r)\vec{v} + (\vec{v}\cdot\vec{\nabla}_r)\vec{\mathscr{r}}$$

$$\vec{\nabla}_r\times\vec{\mathscr{r}} = \vec{\nabla}_r\times(\vec{r} - \vec{w}(t_r)) = -\vec{\nabla}\times\vec{w}(t_r)$$

$$\vec{\nabla}_r\times\vec{\mathscr{r}} = (\vec{v}\times\nabla t_r)$$

$$\nabla t_r = \nabla\left(t - \frac{|\mathscr{r}|}{c}\right) = -\frac{1}{c}\nabla|\mathscr{r}|$$

$$= -\frac{1}{c}\nabla\sqrt{\mathscr{r}^2} = -\frac{1}{c}\nabla\sqrt{\vec{\mathscr{r}}\cdot\vec{\mathscr{r}}}$$

So far:

$$\nabla t_r = -\frac{1}{\mathscr{r}c}\left[\vec{v}(\vec{\mathscr{r}}\cdot\nabla t_r) - \nabla t_r(\vec{\mathscr{r}}\cdot\vec{v}) + (\vec{\mathscr{r}}\cdot\nabla)\vec{\mathscr{r}}\right] = -\frac{1}{c}\frac{1}{2}\frac{1}{\sqrt{\vec{\mathscr{r}}\cdot\vec{\mathscr{r}}}}\nabla(\vec{\mathscr{r}}\cdot\vec{\mathscr{r}})$$

Product Rule 4   quoting

$$2\left[\vec{v}(\vec{\mathscr{r}}\cdot\nabla t_r) - \nabla t_r(\vec{\mathscr{r}}\cdot\vec{v}) + (\vec{\mathscr{r}}\cdot\nabla)\vec{\mathscr{r}}\right] = \nabla(\vec{\mathscr{r}}\cdot\vec{\mathscr{r}}) = 2\left[\vec{\mathscr{r}}\times(\nabla\times\vec{\mathscr{r}}) + (\vec{\mathscr{r}}\cdot\nabla)\vec{\mathscr{r}}\right]$$

$$\vec{\mathscr{r}}\times(\nabla\times\vec{\mathscr{r}}) = \vec{\mathscr{r}}\times(\vec{v}\times\nabla t_r)$$

Product Rule 2

$$= \vec{v}(\vec{\mathscr{r}}\cdot\nabla t_r) - \nabla t_r(\vec{\mathscr{r}}\cdot\vec{v})$$

Point Source

$$V(\vec{r},t) = -\frac{1}{4\pi\varepsilon_o}\frac{qc}{\mathscr{r}c - \vec{v}\cdot\vec{\mathscr{r}}}$$

$$\vec{A}(\vec{r},t) = -\frac{\mu_o}{4\pi}\frac{qc\vec{v}}{\mathscr{r}c - \vec{v}\cdot\vec{\mathscr{r}}} = \frac{\vec{v}}{c^2}V(\vec{r},t)$$

How about Fields

$$\vec{E} = -\vec{\nabla}V - \frac{\partial}{\partial t}\vec{A} \qquad \vec{B} = \vec{\nabla}\times\vec{A}$$

$$\vec{\nabla}_r V(\vec{r},t) = \frac{qc}{4\pi\varepsilon_o}\vec{\nabla}_r\frac{1}{\mathscr{r}c - \vec{v}\cdot\vec{\mathscr{r}}}$$

$$= \frac{qc}{4\pi\varepsilon_o}\frac{-1}{\left(\mathscr{r}c - \vec{v}\cdot\vec{\mathscr{r}}\right)^2}\vec{\nabla}_r\left(\mathscr{r}c - \vec{v}\cdot\vec{\mathscr{r}}\right)$$

$$\vec{\nabla}_r\left(\mathscr{r}c - \vec{v}\cdot\vec{\mathscr{r}}\right) = \vec{\nabla}_r\mathscr{r}c - \vec{\nabla}_r\left(\vec{v}\cdot\vec{\mathscr{r}}\right) = \hat{\mathscr{r}}c - \vec{\nabla}_r\left(\vec{v}\cdot\vec{\mathscr{r}}\right)$$

Product Rule 4

$$\vec{\nabla}_r\left(\vec{v}\cdot\vec{\mathscr{r}}\right) = \vec{v}\times\left(\vec{\nabla}_r\times\vec{\mathscr{r}}\right) + \vec{\mathscr{r}}\times\left(\vec{\nabla}_r\times\vec{v}\right) + \left(\vec{\mathscr{r}}\cdot\vec{\nabla}_r\right)\vec{v} + \left(\vec{v}\cdot\vec{\nabla}_r\right)\vec{\mathscr{r}}$$

$$\vec{\nabla}_r\times\vec{\mathscr{r}} = \vec{\nabla}_r\times\left(\vec{r} - \vec{w}(t_r)\right) = -\vec{\nabla}\times\vec{w}(t_r)$$

$$\vec{\nabla}_r\times\vec{\mathscr{r}} = \left(\vec{v}\times\nabla t_r\right)$$

$$\nabla t_r = -\frac{1}{\mathscr{r}c}\left[\vec{v}\left(\vec{\mathscr{r}}\cdot\nabla t_r\right) - \nabla t_r\left(\vec{\mathscr{r}}\cdot\vec{v}\right) + \left(\vec{\mathscr{r}}\cdot\nabla\right)\vec{\mathscr{r}}\right]$$

$$\left(\vec{\mathscr{r}}\cdot\nabla\right)\vec{\mathscr{r}} = \left(\mathscr{r}_x\frac{\partial}{\partial x} + \mathscr{r}_y\frac{\partial}{\partial y} + \mathscr{r}_z\frac{\partial}{\partial z}\right)\left(\vec{r}\text{-}\vec{w}(t_r)\right)$$

$$\left(\vec{\mathscr{r}}\cdot\nabla\right)\vec{\mathscr{r}} = \left(\mathscr{r}_x\frac{\partial\vec{r}}{\partial x} + \mathscr{r}_y\frac{\partial\vec{r}}{\partial y} + \mathscr{r}_z\frac{\partial\vec{r}}{\partial z}\right) - \left(\mathscr{r}_x\frac{\partial t_r}{\partial x}\frac{\partial\vec{w}(t_r)}{\partial t_r} + \mathscr{r}_y\frac{\partial t_r}{\partial y}\frac{\partial\vec{w}(t_r)}{\partial t_r} + \mathscr{r}_z\frac{\partial t_r}{\partial z}\frac{\partial\vec{w}(t_r)}{\partial t_r}\right)$$

$$\left(\vec{\mathscr{r}}\cdot\nabla\right)\vec{\mathscr{r}} = \left(\mathscr{r}_x\hat{x} + \mathscr{r}_y\hat{y} + \mathscr{r}_z\hat{z}\right) - \frac{\partial\vec{w}(t_r)}{\partial t_r}\left(\mathscr{r}_x\frac{\partial t_r}{\partial x} + \mathscr{r}_y\frac{\partial t_r}{\partial y} + \mathscr{r}_z\frac{\partial t_r}{\partial z}\right)$$

$$\left(\vec{\mathscr{r}}\cdot\nabla\right)\vec{\mathscr{r}} = \vec{\mathscr{r}} - \vec{v}\left(\vec{\mathscr{r}}\cdot\nabla t_r\right)$$

So

$$\nabla t_r = -\frac{1}{\mathscr{r}c}\left[\vec{v}\left(\vec{\mathscr{r}}\cdot\nabla t_r\right) - \nabla t_r\left(\vec{\mathscr{r}}\cdot\vec{v}\right) + \vec{\mathscr{r}} - \vec{v}\left(\vec{\mathscr{r}}\cdot\nabla t_r\right)\right] = -\frac{1}{\mathscr{r}c}\left[-\nabla t_r\left(\vec{\mathscr{r}}\cdot\vec{v}\right) + \vec{\mathscr{r}}\right] = \frac{-\vec{\mathscr{r}}}{\mathscr{r}c - \vec{\mathscr{r}}\cdot\vec{v}}$$

Point Source

$$V(\vec{r},t) = -\frac{1}{4\pi\varepsilon_o}\frac{qc}{\mathscr{r}c - \vec{v}\cdot\vec{\mathscr{r}}}$$

$$\vec{A}(\vec{r},t) = -\frac{\mu_o}{4\pi}\frac{qc\vec{v}}{\mathscr{r}c - \vec{v}\cdot\vec{\mathscr{r}}} = \frac{\vec{v}}{c^2}V(\vec{r},t)$$

How about Fields

$$\vec{E} = -\vec{\nabla}V - \frac{\partial}{\partial t}\vec{A}$$

$$\vec{B} = \vec{\nabla}\times\vec{A}$$

More of the same…

$$\vec{E}(r,t) = \frac{q}{4\pi\varepsilon_o}\frac{\mathscr{r}}{(\vec{\mathscr{r}}\cdot\vec{u})^3}\left[\left(c^2 - v^2\right)\vec{u} + \vec{\mathscr{r}}\times(\vec{u}\times\vec{a})\right]$$

$$\vec{u} \equiv c\hat{\mathscr{r}} - \vec{v}$$

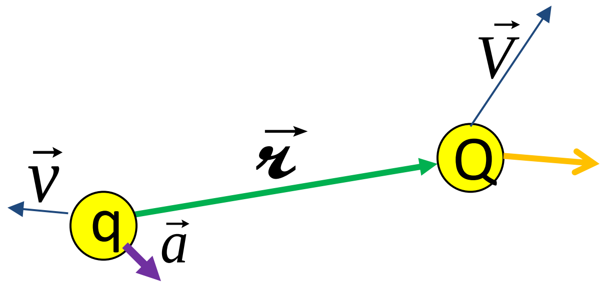
$$\vec{B} = \nabla\times\left(\frac{\vec{v}}{c^2}V(\vec{r},t)\right) = \frac{1}{c^2}\left(V(\vec{r},t)\left(\nabla\times\vec{v}\right) + \vec{v}\times\nabla V(\vec{r},t)\right)$$

$$\vec{B} = \frac{1}{c}\frac{q}{4\pi\varepsilon_o}\frac{1}{(\vec{\mathscr{r}}\cdot\vec{u})^3}\vec{\mathscr{r}}\times\left[\left(c^2 - v^2\right)\vec{u} + \vec{\mathscr{r}}\times(\vec{u}\times\vec{a})\right]$$

$$\vec{B} = \frac{1}{c}\hat{\mathscr{r}}\times\vec{E}$$

Force between moving charges

(Eq’n 10.74)



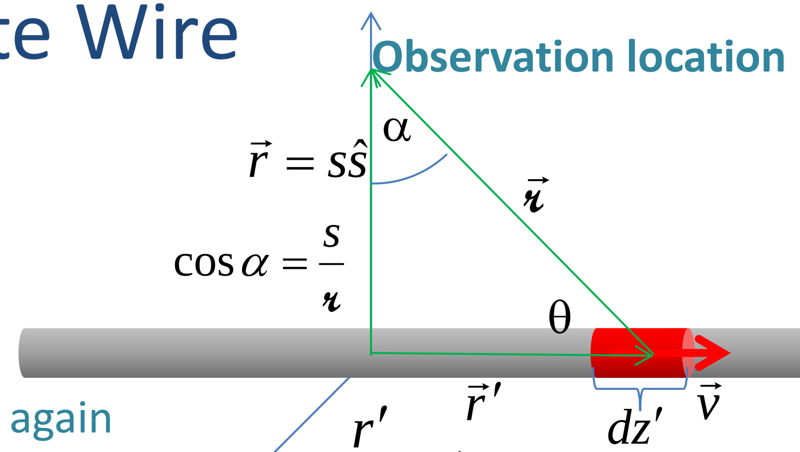
$$\vec{u} \equiv c\hat{\mathscr{r}} - \vec{v}$$

$$\vec{F}_{Q\leftarrow q} = \frac{qQ}{4\pi\varepsilon_o}\frac{\mathscr{r}}{\left(\vec{\mathscr{r}}\cdot\vec{u}\right)^3}\left\{\left[\left(c^2-v^2\right)\vec{u}+\vec{\mathscr{r}}\times\left(\vec{u}\times\vec{a}\right)\right]+\frac{\vec{V}}{c}\times\left[\hat{\mathscr{r}}\times\left[\left(c^2-v^2\right)\vec{u}+\vec{\mathscr{r}}\times\left(\vec{u}\times\vec{a}\right)\right]\right]\right\}$$

“The entire theory of classical electrodynamics is contained in that equation…but you see why I preferred to start out with Coulomb’s law.” - Griffiths

Example Pr 10.22: Field of Infinite Wire

(Like Book's approach to Ex. 5.5)



Point charge:

$$\vec{E}(\vec{r}) = \frac{q\left(1-\left(\frac{v}{c}\right)^2\right)\hat{R}}{4\pi\varepsilon_o\left(1-\left(\frac{v}{c}\right)^2\sin^2\theta\right)^{3/2}R^2}$$

Line charge:

$$\vec{E}(\vec{r}) = \int_{-\infty}^{\infty}\frac{\lambda dz'\left(1-\left(\frac{v}{c}\right)^2\right)\hat{\mathscr{r}}}{4\pi\varepsilon_o\left(1-\left(\frac{v}{c}\right)^2\sin^2\theta\right)^{3/2}\mathscr{r}^2}$$

Back to r since again using as integration variable, without an end anchored to a moving charge.

$$\frac{r'}{s} = \tan\alpha$$

$$\frac{dz'}{s} = d\left(\frac{z'}{s}\right) = d(\tan\alpha)$$

$$\frac{dz'}{s} = \frac{1}{\cos^2\alpha}d\alpha$$

$$\vec{E}(\vec{r}) = \frac{\lambda\left(1-\left(\frac{v}{c}\right)^2\right)}{4\pi\varepsilon_o}\left(\int_{-\infty}^{\infty}\frac{dz' s\hat{s}}{\left(1-\left(\frac{v}{c}\right)^2\sin^2\theta\right)^{3/2}\mathscr{r}^3} + \int_{-\infty}^{\infty}\frac{dz'(-z')\hat{z}}{\left(1-\left(\frac{v}{c}\right)^2\sin^2\theta\right)^{3/2}\mathscr{r}^3}\right)$$

Rephrase in terms of ratios of distances to prepare to rewrite in terms of trig functions

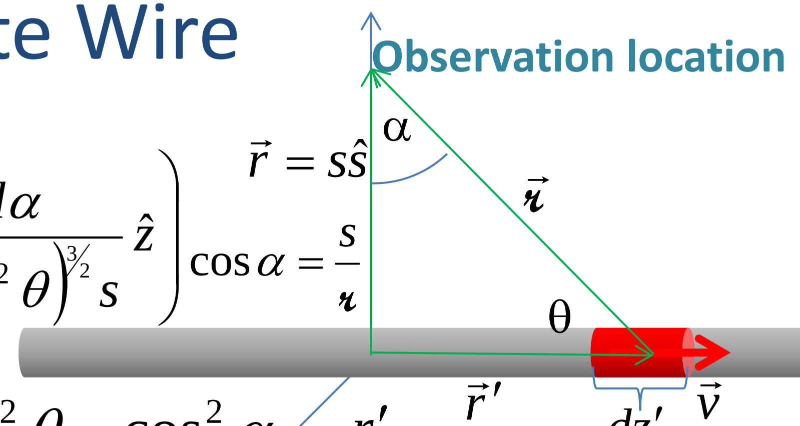
$$\vec{E}(\vec{r}) = \frac{\lambda\left(1-\left(\frac{v}{c}\right)^2\right)}{4\pi\varepsilon_o}\left(\int_{-\infty}^{\infty}\frac{1}{\left(1-\left(\frac{v}{c}\right)^2\sin^2\theta\right)^{3/2}}\frac{1}{s}\frac{s}{\mathscr{r}}\frac{s}{\mathscr{r}}\frac{s}{\mathscr{r}}\frac{dz'}{s}\hat{s} - \int_{-\infty}^{\infty}\frac{\hat{z}}{\left(1-\left(\frac{v}{c}\right)^2\sin^2\theta\right)^{3/2}}\frac{1}{s}\frac{s}{\mathscr{r}}\frac{s}{\mathscr{r}}\frac{z'}{\mathscr{r}}\frac{dz'}{s}\right)$$

$$\vec{E}(\vec{r}) = \frac{\lambda\left(1-\left(\frac{v}{c}\right)^2\right)}{4\pi\varepsilon_o}\left(\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{\cos(\alpha)d\alpha}{\left(1-\left(\frac{v}{c}\right)^2\sin^2\theta\right)^{3/2}s}\hat{s} - \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{\sin(\alpha)d\alpha}{\left(1-\left(\frac{v}{c}\right)^2\sin^2\theta\right)^{3/2}s}\hat{z}\right)$$

Example Pr 10.22: Field of Infinite Wire

(Like Book's approach to Ex. 5.5)

$$\vec{E}(\vec{r}) = \frac{\lambda\left(1-\left(\frac{v}{c}\right)^2\right)}{4\pi\varepsilon_o}\left(\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos(\alpha)d\alpha}{\left(1-\left(\frac{v}{c}\right)^2\sin^2\theta\right)^{3/2} s}\hat{s} - \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin(\alpha)d\alpha}{\left(1-\left(\frac{v}{c}\right)^2\sin^2\theta\right)^{3/2} s}\hat{z}\right)$$



Observe that θ and α are complementary angles, so $\sin^2\theta = \cos^2\alpha$

$$\frac{r'}{s} = \tan\alpha$$

$$\vec{E}(\vec{r}) = \frac{\lambda\left(1-\left(\frac{v}{c}\right)^2\right)}{4\pi\varepsilon_o s}\left(\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos(\alpha)d\alpha}{\left(1-\left(\frac{v}{c}\right)^2\cos^2\alpha\right)^{3/2}}\hat{s} - \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin(\alpha)d\alpha}{\left(1-\left(\frac{v}{c}\right)^2\cos^2\alpha\right)^{3/2}}\hat{z}\right)$$

$$\frac{dz'}{s} = d\left(\frac{z'}{s}\right) = d(\tan\alpha)$$

$$\frac{dz'}{s} = \frac{1}{\cos^2\alpha}d\alpha$$

Observing that the second integral is odd about 0, so will sum to 0, I focus on just the first integral

$$\vec{E}(\vec{r}) = \frac{\lambda\left(1-\left(\frac{v}{c}\right)^2\right)}{4\pi\varepsilon_o s}\left(\left.\frac{\sin(\alpha)}{\left(1-\left(\frac{v}{c}\right)^2\right)\left(1-\frac{1}{2}\left(\frac{v}{c}\right)^2(1+\cos(2\alpha))\right)^{1/2}}\right|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\right)\hat{s}$$

$$\vec{E}(\vec{r}) = \frac{\lambda}{2\pi\varepsilon_o s}\hat{s}$$

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