Fri., 11/1	**7.1.3-7.2.2** *Emf & Induction*	
Mon. 11/4 Wed., 11/6 Fri., 11/8	**Exam 2 (Ch 3 & 5)** **7.2.3-7.2.5** *Inductance and Energy of B* **7.3.1-.3.3** Maxwell's Equations	

Equipment:

- Crank generator
- Cow magnet and copper pipe
- Eddy current demo: magnet and swinging fins – with and without fingers
- Homemade electric motor
- Two of our old induction coils

Last Time

Ohm's Law

$$\vec{J} = \sigma_{conductivity}\vec{E}$$

$$I = \frac{V}{R}$$

EMF

$$\varepsilon \equiv \oint \vec{f} \cdot d\vec{\ell}$$

Motional emf

Example 7.4 – Faraday's Disk

A metal disk of radius *a* rotates with an angular frequency ω (counterclockwise viewed from above) about an axis parallel to a uniform magnetic field. A circuit is made by a sliding contact. What is the current through the resistor *R*?

Note: This problem can<u>not</u> be solved using $\varepsilon = -d\Phi/dt$.

Find the *emf* by calculating the line integral of the force per charge from the center to the contact point. The speed of a point at a distance *s* from the center is $v = \omega s$, so the force per charge is $\vec{f}_{mag} = \vec{v} \times \vec{B} = \omega s B\,\hat{s}$. The *emf* is

$$\varepsilon = \int \vec{f}_{mag} \cdot d\vec{\ell} = \int_0^a f_{mag} ds = \omega B \int_0^a s\, ds = \frac{\omega B a^2}{2}.$$

The current found using Ohm's law is

$$I = \frac{\varepsilon}{R} = \frac{\omega B a^2}{2R}.$$

By the RHR, it flows from the center to the outer edge of the disk.

Phrasing in terms of Magnetic Flux

Let's return to the original result for the emf. It can be rephrased a bit. Here, we are changing the area of a loop through which the field is flowing.

$$emf = BvL = B\frac{dA}{dt} = \frac{d(BA)}{dt} = \frac{d\Phi_B}{dt}$$

If we impose the Right Hand Rule for sign conventions, we'd have

$$emf = -\frac{d\Phi_B}{dt}$$

Problem 7.11

A square loop is cut out of a thick sheet of aluminum. It is placed so that the top portion is in a uniform, horizontal magnetic field of 1 T <u>into the page</u> (as shown below) and allowed to fall under gravity. The shading indicates the field region. What is the terminal velocity of the loop? How long does it take to reach 90% of the terminal velocity?

Use *y* for the distance from the bottom of the field region. The magnetic flux is $\Phi = B\ell y$, so the size of the *emf* is

$$|\varepsilon| = \frac{d\Phi}{dt} = B\ell v.$$

By Ohm's law, the size of the current is $I = \varepsilon/R = B\ell v/R$. By the RHR, the current flows in the direction of $\vec{v}\times\vec{B}$ (for the top segment), which is to the right.

The magnetic field is perpendicular to the current in the loop so the force is

$$|F| = I\left|\int d\vec{\ell}\times\vec{B}\right| = I\ell B = \frac{B^2\ell^2 v}{R},$$

where ℓ is the length of a side. By the RHR, the direction of $d\vec{\ell}\times\vec{B}$ and the force on the loop is to the upward. This is a 1-D problem. Using downward as positive, Newton's second law is

$$mg - \frac{B^2\ell^2}{R}v = ma = m\frac{dv}{dt}.$$

Terminal "velocity" is reached when the acceleration is zero, so

$$mg - \frac{B^2\ell^2}{R}v = 0 \quad \Rightarrow \quad v_t = \frac{mgR}{B^2\ell^2}.$$

The equation of motion can be written as

$$g - \frac{B^2\ell^2}{mR}v = \left(1 - \frac{v}{v_t}\right)g = \frac{dv}{dt}.$$

This can be integrated to get (starts from rest):

$$\frac{dv}{(v_t - v)} = \frac{g}{v_t}dt,$$

$$\int_0^{v(t)} \frac{dv}{(v_t - v)} = \frac{g}{v_t}\int_0^t dt,$$

$$\left[-\ln(v_t - v)\right]_0^v = -\ln\left(\frac{v_t - v}{v_t}\right) = \frac{gt}{v_t},$$

$$\frac{v_t - v}{v_t} = e^{-gt/v_t} \quad \Rightarrow \quad v = v_t\left(1 - e^{-gt/v_t}\right).$$

At 90% of terminal velocity,

$$\frac{v}{v_t} = 0.9 = \left(1 - e^{-gt/v_t}\right) \quad \Rightarrow \quad e^{-gt/v_t} = 0.1,$$

$$-gt/v_t = \ln(1/10) \quad \Rightarrow \quad t = \frac{v_t}{g}\ln(10).$$

Suppose the cross sectional area is *A*. The mass is $m = 4\eta(A\ell)$, where $\eta = 2.7\times10^3\,\mathrm{kg/m^3}$ is the mass density of aluminum. The resistance of the loop is $R = 4(\ell/A\sigma) = 4\ell\rho/A$, where $\rho = 2.8\times10^{-8}\,\Omega m$ is the resistivity of aluminum. The terminal velocity is

$$v_t = \frac{mgR}{B^2\ell^2} = \frac{(4\eta A\ell)g(4\ell\rho/A)}{B^2\ell^2} = \frac{16g\eta\rho}{B^2},$$

so the time to reach 90% of terminal velocity is

$$t = \frac{v_t}{g}\ln(10) = \frac{16\eta\rho}{B^2}\ln(10) = \frac{16\left(2.7\times10^3\,\mathrm{kg/m^3}\right)\left(2.8\times10^{-8}\,\Omega m\right)}{(1\ \mathrm{T})^2} = 2.8\times10^{-3}\,\mathrm{s} = 2.8\ \mathrm{ms}.$$

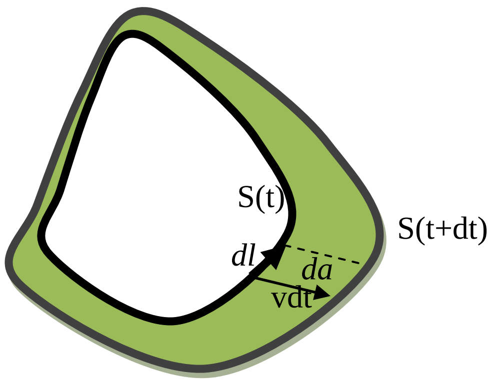
The units work because ohm = V/A and T = N/(Am).

Phrasing in terms of Magnetic Flux

$$emf_{motion} = -\left.\frac{\partial\Phi_B}{\partial t}\right|_B$$

Proof of Generality

Say you have a fairly elastic and mobile wire loop in the presence of a non-uniform magnetic field (steady in time, but varying from one location to another.) Say you flex and move this wire. Here's a picture representing the *old* wire configuration and the new one.



The green region represents the *change in area.* That can be described in terms of each little point on the loop having its own velocity such that it gets to the new location in time dt.

Note that the area swept out by moving our little line segment dl from the inner curve position to the outer curve position is $d\vec{a} = \vec{v}dt \times d\vec{l}$

So, the little bit of flux gained by moving line segment *dl* out is

$$\vec{B} \cdot d\vec{a} = \vec{B} \cdot \left(\vec{v}dt \times d\vec{l}\right) = \left(\vec{B} \times \vec{v}dt\right) \cdot d\vec{l} = -\left(\vec{v}dt \times \vec{B}\right) \cdot d\vec{l}$$

The last step makes use of Vector Identity (1): $\vec{A} \cdot \left(\vec{B} \times \vec{C}\right) = \left(\vec{A} \times \vec{B}\right) \cdot \vec{C}$ and the fact that flipping the order of a cross-product flips signs.

Then the total gain in flux from expanding the loop is gotten by summing over the whole loop.

$$d\Phi_B = -\oint \left(\vec{v}dt \times \vec{B}\right) \cdot d\vec{l}$$

Finally, divide by the *dt*, the time over which we're expanding the loop and we get the rate of change of flux.

$$\frac{d\Phi_B}{dt} = -\oint \left(\vec{v} \times \vec{B}\right) \cdot d\vec{l} = -\oint f_{mag} \cdot d\vec{l} = -Emf_{motion}$$

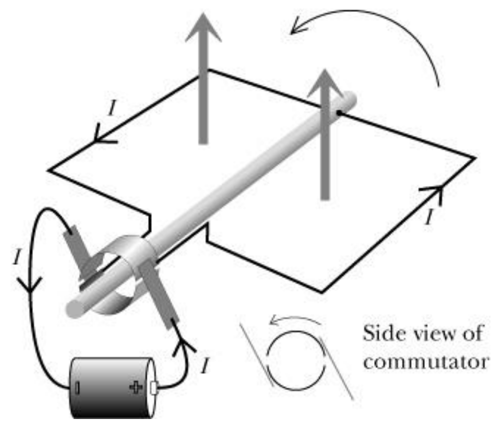
$$\frac{d\Phi_B}{dt} = -Emf_{motion}$$

In slightly better mathematical notation, since we're holding something constant (the field) over time, this is the *partial* derivative

$$emf_{motion} = -\left.\frac{\partial \Phi_B}{\partial t}\right|_B$$

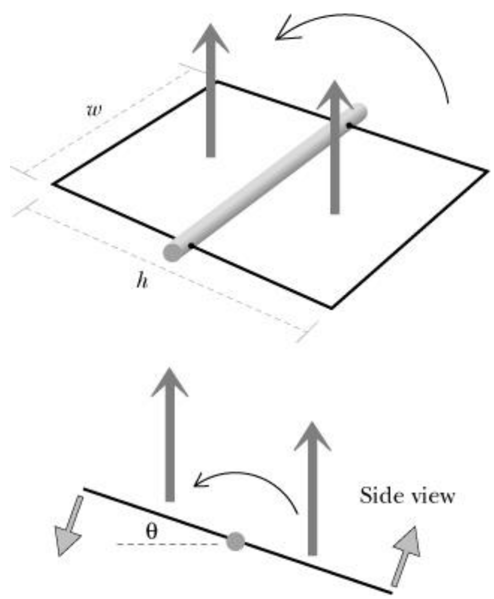
Example 7.10 Motors:

The commutator switches the direction of the current as the loop spins so that it is always moving in the same direction on each side of the axis.



Generators: Suppose a loop rotates at an angular speed ω.

As the loop spins, there is a motional emf on each side with length *w*, but in opposite directions. That leads to a conventional current around the loop.



The emf is largest when the angle θ is 90°, because the wires are moving the fastest in the direction perpendicular to the magnetic field. The size of the emf on the left wire depends on the component of the velocity perpendicular to the magnetic field:

$$\text{emf}_{\text{left}} = -\frac{d\Phi}{dt} = -\frac{d\left(\vec{B}\cdot\vec{A}\right)}{dt} = -\frac{d\left(BA\cos\theta\right)}{dt} = BA(\sin\theta)\frac{d\theta}{dt} = BA(\sin\theta)\omega\ .$$

Faraday's Law

So, we actually *derived* $\varepsilon = -\frac{d\Phi}{dt}$ for *motional* emf – when the charges are moving relative to the magnetic field, but what about if the field is moving relative to the charges? On the one hand, isn't motion just a relative thing? When *we* see a wire moving in a field doesn't *it* see the field moving? Either way you look at it, a current should get flowing around the wire. So shouldn't there again be some kind of force per charge, some kind of *emf*? On the other hand, the way we've defined the magnetic interaction (the one that depends upon the motion of the sensing charge) we can't call that interaction 'magnetic.'

So, as we've reasoned

$$\varepsilon = -\frac{d\Phi}{dt}$$

Holds not just if the area is changing, but if the field is changing too. That observation is called Faraday's Law. To be *particularly* specific about it,

$$\varepsilon_F = -\left.\frac{d\Phi}{dt}\right|_a$$

That is, the *Faraday* effect is that there's an emf generated when the flux changes due to the magnetic field changing (the area remaining constant.) Either (or both) way you cut it, there's an emf.

$$\varepsilon = -\frac{d\Phi}{dt}$$

$$\varepsilon_{motion} + \varepsilon_F = -\left.\frac{\partial\Phi}{\partial t}\right|_B - \left.\frac{\partial\Phi}{\partial t}\right|_a$$

Faraday's contribution, $\varepsilon_F = -\left.\frac{\partial\Phi}{\partial t}\right|_a$, can be rephrased if we dig into our definitions of emf and flux:

$$\varepsilon_F = -\left.\frac{\partial\Phi}{\partial t}\right|_a$$

$$\oint \frac{\vec{F}}{q}\cdot d\vec{l} = -\frac{\partial}{\partial t}\int \vec{B}\cdot d\vec{a}\Big|_a$$

$$\int\left(\vec{\nabla}\times\frac{\vec{F}}{q}\right)\cdot d\vec{a} = -\int\frac{\partial\vec{B}}{\partial t}\cdot d\vec{a}$$

Where we used Stoke's theorem on the left and the fact that we're holding the area constant on the right.

Of course, if the two integrals are the same, then the two integrands must be the same, so

$$\vec{\nabla} \times \frac{\vec{F}}{q} = -\frac{\partial \vec{B}}{\partial t}$$

What is this force? Well, process of elimination leaves Electric as the only candidate – it isn't magnetic since we defined magnetism as the thing that depends on the sensing charge's velocity (and we're imagining keeping those steady), and there are no other players *than* the charges – so this must be the charge-charge force that *doesn't* depend on sensor velocity; that is, electric.

$$\vec{\nabla} \times \frac{q\vec{E}}{q} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

I will speak a little more carefully than Griffiths does at this point,

A changing magnetic field is accompanied by *an electric field.*

Misnomer Warning: I do not say "induces" because that word implies "causes" and many physicists (even Griffiths later in this section) mistakenly make that connection. This law here is *not* a causal one; it is a *correlation* – there's no way to deduce form it whether E's curl causes B's time variation or the other way around, or neither. The correct answer is "neither." Both are caused by time-varying current densities; we'll

$$\varepsilon_F = -\left.\frac{\partial \Phi}{\partial t}\right|_a$$

see the proof of that in Ch. 10. $\displaystyle \oint \frac{\vec{F}}{q} \cdot d\vec{l} = -\frac{\partial}{\partial t} \int \vec{B} \cdot d\vec{a} \Big|_a$

$$\int \left(\vec{\nabla} \times \frac{\vec{F}}{q} \right) \cdot d\vec{a} = -\int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{a}$$

At any rate, this is a very powerful deductive tool, and we can phrase it a few different ways:

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

(note: direction of E's circulation is opposite to the direction of change in flux, i.e., use a *left* hand rule to grab the change in flux and see the direction of E.)

$$\oint \vec{E} \cdot d\vec{l} = -\int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{a}$$

(note: similarity to Ampere's Law)

$$\varepsilon_F = -\left.\frac{\partial \Phi}{\partial t}\right|_a$$

In any of these equivalent forms, it's called Faraday's Law.

In both cases (varying area or field), the size of the emf is equal to the rate of change of the magnetic flux. Some situations can be seen as one or the other effect depending on the reference frame – but there is not always one frame that works for all parts of the circuit, so that more generally only works locally.

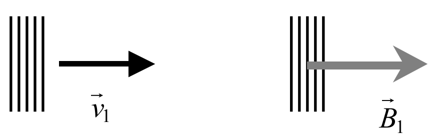
Several ways to change the Magnetic Flux:

Exercise – Come up with ways to change the magnetic flux through a coil using either a second coil or a permanent magnet

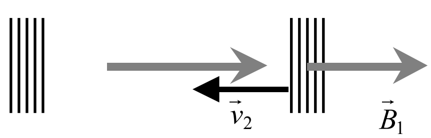
All of the following will result in an induced emf in the coil 2 on the right.

1. Change the current in coil 1

2. Move coil 1 (with current through it)



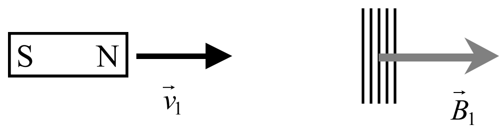
3. Move coil 2 (with current through coil 1)



4. Rotate coil 1

Rotate coil 2

5. Move the magnet relative to the coil (includes moving coil toward magnet)



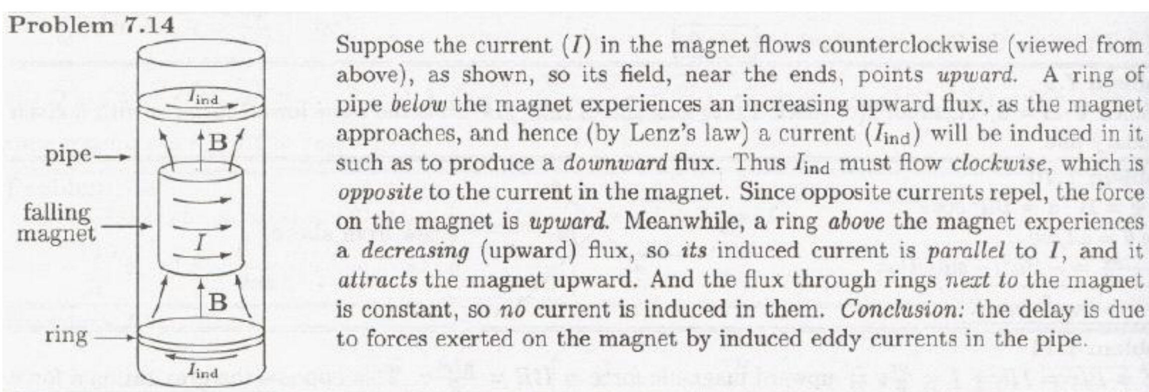
6. Rotate the magnet

7. Rotate the coil

Problem 7.14

Explain why a cylindrical magnet takes much longer to drop through a vertical copper pipe than an unmagnetized piece of iron does.

(Ignore the part about the "current in the magnet")



Problem 7.14

Suppose the current (I) in the magnet flows counterclockwise (viewed from above), as shown, so its field, near the ends, points *upward*. A ring of pipe *below* the magnet experiences an increasing upward flux, as the magnet approaches, and hence (by Lenz's law) a current (I_{ind}) will be induced in it such as to produce a *downward* flux. Thus I_{ind} must flow *clockwise*, which is *opposite* to the current in the magnet. Since opposite currents repel, the force on the magnet is *upward*. Meanwhile, a ring *above* the magnet experiences a *decreasing* (upward) flux, so *its* induced current is *parallel* to I, and it *attracts* the magnet upward. And the flux through rings *next to* the magnet is constant, so *no* current is induced in them. *Conclusion:* the delay is due to forces exerted on the magnet by induced eddy currents in the pipe.

Demo: drop a magnet down a copper tube (not ferromagnetic) –very slow compared to free fall!

Each cross section of the pipe can be considered a loop. There will be induced currents around the pipe. These in turn produce magnetic fields, so it's like having two magnets interact. You will explain the slowing in terms of <u>forces</u> in Prob. 22.1 (c).

Lenz's Law

We usually just used Faraday's law to find the magnitude of the *emf* and don't worry about the minus sign. Lenz's law can be used to determine the direction of the induced current. It states that, "Nature abhors a change in magnetic flux." In other words, the induced current will produce a magnetic field that will somewhat oppose the change in flux.

Apply this to the figures above.

Examples

Pr. 7.12

Example 7.7 (an Ampere's Law type approach)

Pr. 7.15 $\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} \quad \Phi_B = \int \vec{B} \cdot d\vec{a} = \int \mu_o I(t) n da$

Problem 7.16 – Coaxial Cable with Change Current

An (slowly) alternating current $I(t) = I_0 \cos(\omega t)$ flows down a long, straight, thin wire and returns along a thin, coaxial conducting tube of radius *a*.

a. In what direction does the induced electrical field point?

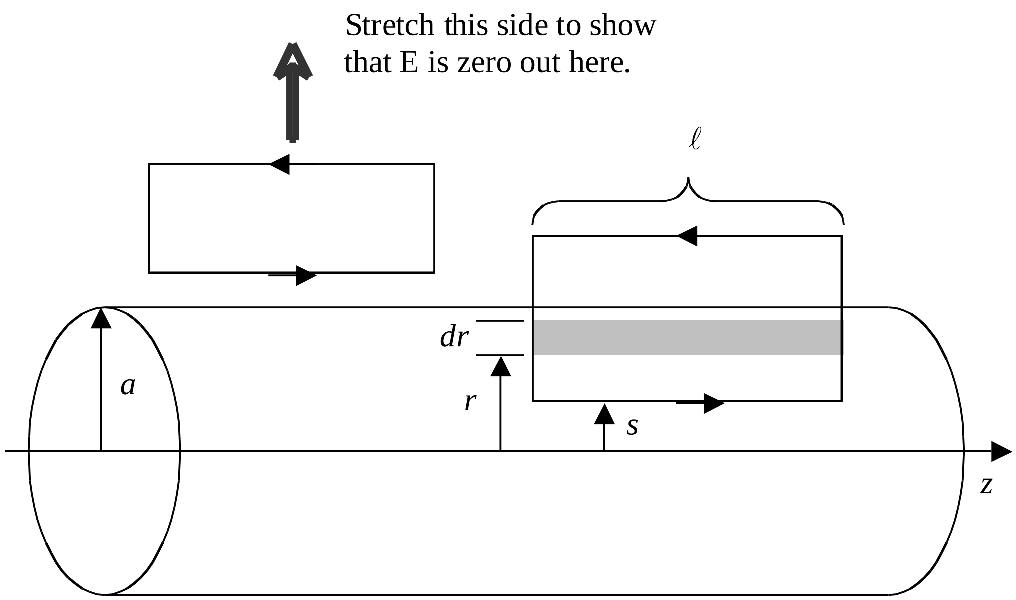
Let the current on the central wire be in the +*z* direction. In the quasistatic approximation (current changes slowly), the magnetic field is circumferential. A changing magnetic field in this direction is analogous to the current for a solenoid, which produces a longitudinal (in *z* direction) magnetic field. Therefore, the direction of the induced electric field is longitudinal.

b. Assuming that the field goes to zero as $s \to \infty$, find the induced electric field $\vec{E}(s,t)$.

The magnetic field in the quasistatic approximation is (use Ampere's law)

$$\vec{B} = \begin{cases} \frac{\mu_0 I}{2\pi s} \hat{\phi} & s < a, \\ 0 & s > a. \end{cases}$$

By symmetry, we also know that the induced electric field only *s* (and *t*). Use the same shape of "amperian loop" as for a solenoid (see the diagram below).



We can argue that the induced electric field is the same at all distances outside the coaxial cable, so it must be zero (use the loop on the left).

For a loop with one side inside the cable (on the right), the line integral of the electric field around the loop is $\oint \vec{E}\cdot d\vec{\ell} = E\ell$, because only the bottom side is non-zero.

Consider a thin strip between distances r and $r + dr$ from the long wire that is enclosed by the loop. The magnetic flux through this segment is

$$d\Phi = \left(\frac{\mu_0 I}{2\pi r}\right)(\ell\, dr).$$

The magnetic field comes out of the page, so the flux is positive by the RHR. The total flux through the loop is

$$\Phi = \frac{\mu_0 I\ell}{2\pi}\int_s^a \frac{dr}{r} = \frac{\mu_0 I\ell}{2\pi}\left[\ln r\right]_s^a = \frac{\mu_0 I\ell}{2\pi}\ln\left(\frac{a}{s}\right).$$

Putting in the function for the current and applying Faraday's law, $\oint \vec{E}\cdot d\vec{\ell} = -d\Phi/dt$, gives

$$\vec{E}(s,t) = \begin{cases} \dfrac{\mu_0 I_0 \omega}{2\pi}\ln\left(\dfrac{a}{s}\right)\sin(\omega t)\hat{z} & s < a, \\ 0 & s > a. \end{cases}$$

Pr. 7.17 $IR = emf = -\dfrac{d\Phi_B}{dt}$ (use Lenz's Law for direction)

"After equation 7.17 ("Faraday's Law") Griffiths mentions that this is actually determined by both magnetic and electric field. The second of which is 'caused' by mag field why does the third experiment work out the same as the first two?"
Casey P,

Also I noticed an interesting author in footnote [8]... ;) Casey P,

Yes, I don't suppose you have a copy of said discussion mentioned... Casey McGrath

"I put this in the questions last time, but since we didn't quite get to it, I still don't really understand all of the mechanics leading up to how he derived equation 7.13, which is pretty crucial to the rest of these sections." Casey McGrath

Also, maybe I'm just missing something, but how did he make the jump from 7.14 to 7.15? Was that just an observational argument as stated in the prior sentence, or did he derive that using some other equation? Casey McGrath

I may be able to help. Flux is area hx*B through it. Induced EMF should change in field through over change in time. Also -Bhv = - Bh (dx/dt). That gives 7.13.

For 7.14-7.15 Flux is also the B field through an area, so it makes sense that change in flux would make sense for change in flux to turn into dB(dot)da.

I don't know if that's what you needed but I hope it helps.Anton

"I don't understand why Griffith does not consider Experiment 1 to be an example of Faraday's law."Davies

"Griffiths was mentioning how Faraday's Law breaks down when we talk about induction. I'm not sure I understand his discussion of quasistatic magnetic fields soonafter."
Rachael Hach

"This might not be a big plot point but I was curious as to why exactly in example 7.9 the use of the quasistatic approximation would cause E to blow up."Ben Kid

I think it has to do with the definition of the speed of light, in which B depends on the current as it was earlier. The quasistatic approximation assumes that at any distance the information of the system travels instantly, but with relativity this is not true.Davies

"How is the emf different than the change in voltage? Is there a reason why we redefine it as the emf? Also, can we do some sketches of emf vs. time tomorrow (Figure 7.23)?"Spencer

My understanding so far, emf is more of a measure of a result than the cause. So a change in voltage would cause an emf, but the changes in magnetic fields also cause emf, but not voltage. I'm not sure though, this is just a thought.Freeman,

"I'm not sure if i understand example 7.8. Its kinda confusing how they got some of the equations they used."Connor W,