Mon Wed Thurs Fri	3.4.1-.4.2 Multipole Expansion 3.4.3-.4.4 Multipole Expansion Review	 HW4

Hooks and Crook: Interesting ways of finding V and E

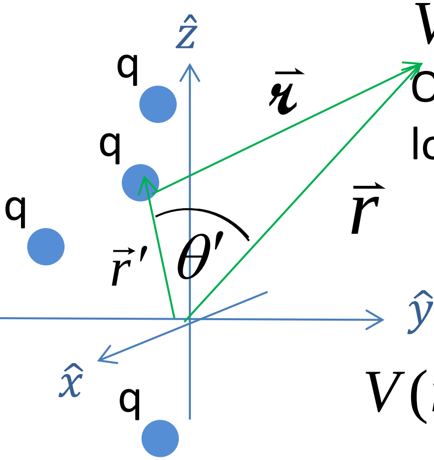
Images: replace a problem with a simpler equivalent one (based on corollary of the first uniqueness theorem)

Relaxation: a computational method based on the potential at a point being the average of the values at the same distance (more about Next Time).

→ **Multipole Expansion:** a method for getting approximate answers for *V* far from a charge distribution (section 3.4)

Multi-pole Expansion

Discrete charge distribution



Observation location

$$V(r) = \frac{1}{4\pi\varepsilon_o}\sum_{i}^{\text{charges}} \frac{q_i}{\mathscr{r}_i} = \frac{1}{4\pi\varepsilon_o}\sum_{i}^{\text{charges}} \frac{q_i}{\sqrt{r^2 + r'^2 - 2rr'\cos\theta_{r\to r'}}}$$

$$\frac{1}{\mathscr{r}_i} = \frac{1}{r}\sum_{n=0}^{\infty}\left(\frac{r_i'}{r}\right)^n P_n(\cos\theta_i')$$

$P_n(\cos\theta')$

nth Legendre polynomial

$$P_0 = 1$$
$$P_1(u) = u$$
$$P_2(u) = (3u^2 - 1)/2$$

$$V(r) = \frac{1}{4\pi\varepsilon_o}\sum_{\text{i}}^{\text{charges}}\left(\frac{q_i}{r}\sum_{n=0}^{\infty}\left(\left(\frac{r_i'}{r}\right)^n P_n(\cos\theta_i')\right)\right)$$

For each charge… sum terms in expansion

Flip order of summation

$$V(r) = \frac{1}{4\pi\varepsilon_o}\sum_{n=0}^{\infty}\left(r^{-(n+1)}\sum_{i}^{\text{charges}}\left(r_i'^n{}_i P_n(\cos\theta_i')q_i\right)\right)$$

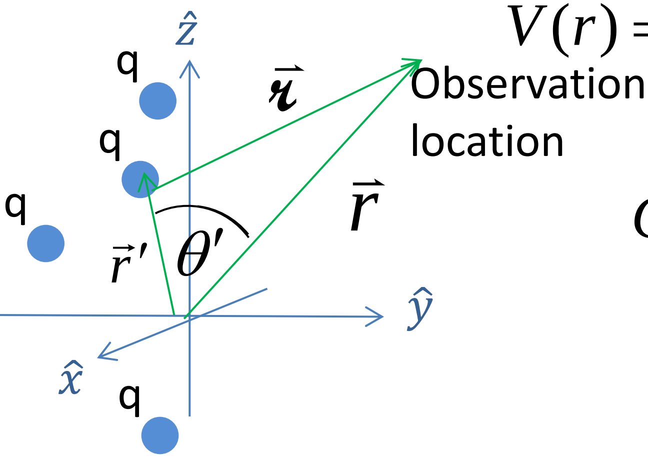
For each term in expansion… sum contribution of charges

$$V(r) = \frac{1}{4\pi\varepsilon_o}\frac{\sum_i^{\text{charges}} q_i}{r} + \frac{1}{4\pi\varepsilon_o}\frac{\sum_i^{\text{charges}} r_i' q_i \cos\theta_i'}{r^2} + \dots$$

monopole dipole

Multi-pole Expansion

Discrete charge distribution



$$V(r) = \frac{1}{4\pi\varepsilon_o}\frac{\sum_i^{\text{charges}} q_i}{r} + \frac{1}{4\pi\varepsilon_o}\frac{\sum_i^{\text{charges}} r_i' q_i \cos\theta_i'}{r^2} + \ldots$$

monopole — dipole

$$Q_{net} = \sum_i^{\text{charges}} q_i$$

Observe that

$$\vec{r}_i\,' \cdot \vec{r} = r_i' r \cos\theta_i'$$

or

$$\vec{r}_i\,' \cdot \hat{r} = \vec{r}_i\,' \cdot \frac{\vec{r}}{r} = r_i'\frac{r}{r}\cos\theta' = r_i'\cos\theta_i'$$

so

$$\sum_i^{\text{charges}} r_i' q_i \cos\theta_i' = \sum_i^{\text{charges}} q_i \vec{r}_i\,' \cdot \hat{r} = \left(\sum_i^{\text{charges}} q_i \vec{r}_i\,'\right)\cdot\hat{r}$$

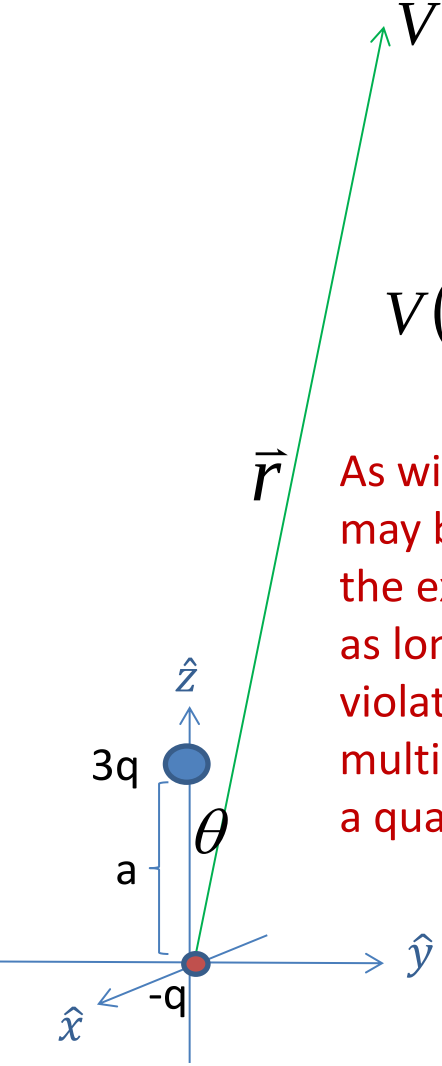
$$= \vec{p}\cdot\hat{r}$$

Electric Dipole Moment

$$V(r) = \frac{1}{4\pi\varepsilon_o}\frac{Q_{net}}{r} + \frac{1}{4\pi\varepsilon_o}\frac{\vec{p}\cdot\hat{r}}{r^2} + \ldots$$

monopole — dipole

Example: Find the first two terms in the mulitpole expansion for the figure shown below.



$$V(r)_{mono} = \tfrac{1}{4\pi\varepsilon_o}\frac{Q}{r}$$

$$Q = 2q$$

$$V(r)_{mono} = \tfrac{1}{4\pi\varepsilon_o}\frac{2q}{r}$$

$$V(r)_{dipole} = \tfrac{1}{4\pi\varepsilon_o}\frac{\vec{p}\cdot\hat{r}}{r^2}$$

$$\vec{p} \equiv \int \vec{r}\,'\rho(\vec{r}\,')d\tau'$$

$$\vec{p} = \vec{r}_1'q_1 + \vec{r}_2'q_2$$

$$\vec{p} = 0(-q) + a3q\hat{z}$$

$$\vec{p} = 3aq\hat{z}$$

As with a Taylor series; though the sum of all terms may be insensitive to where your reference point, the exact contribution of each term *is* sensitive. But as long as you don't change your choice of origin to violate the condition (r>>r') for using a multipole approximation, you won't get a quantitatively *significantly* different solution

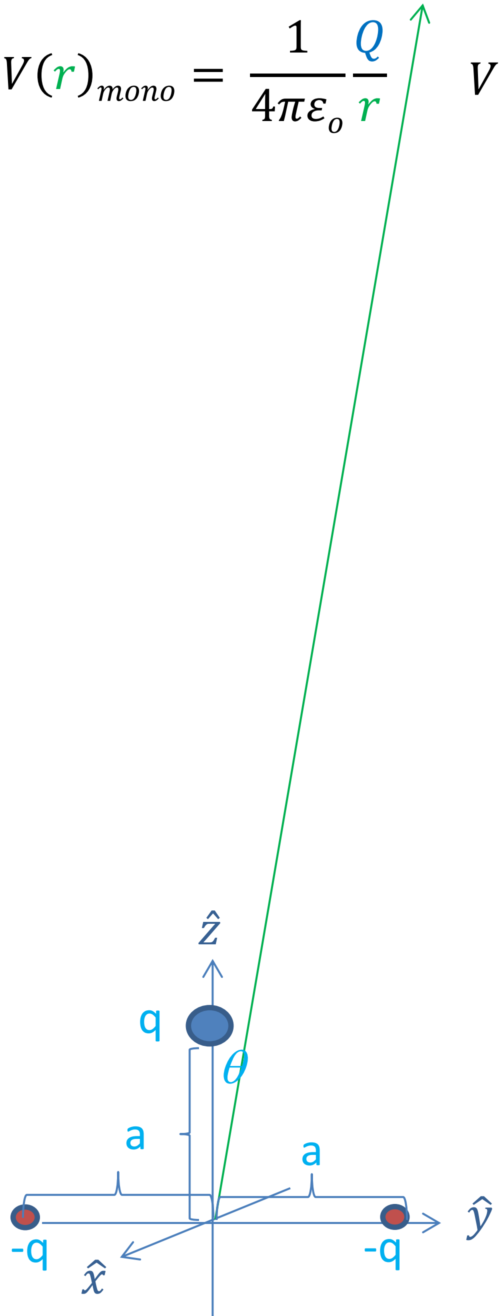
$$V(r)_{dipole} = \tfrac{1}{4\pi\varepsilon_o}\frac{3qa\hat{z}\cdot\hat{r}}{r^2}$$

$$V(r)_{dipole} = \tfrac{1}{4\pi\varepsilon_o}\frac{3qa\cos\theta}{r^2}$$

Q: What if we move the origin up, half-way between the charges?

Exercise: Find the first two terms in the mulitpole expansion for the figure shown below.

$$V(r)_{mono} = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r} \qquad V(r)_{dipole} = \frac{1}{4\pi\varepsilon_o}\frac{\vec{p}\cdot\hat{r}}{r^2} \qquad \vec{p} = \int \vec{r}'\rho(\vec{r}')d\tau' = \vec{r}_1'q_1 + \vec{r}_2'q_2$$



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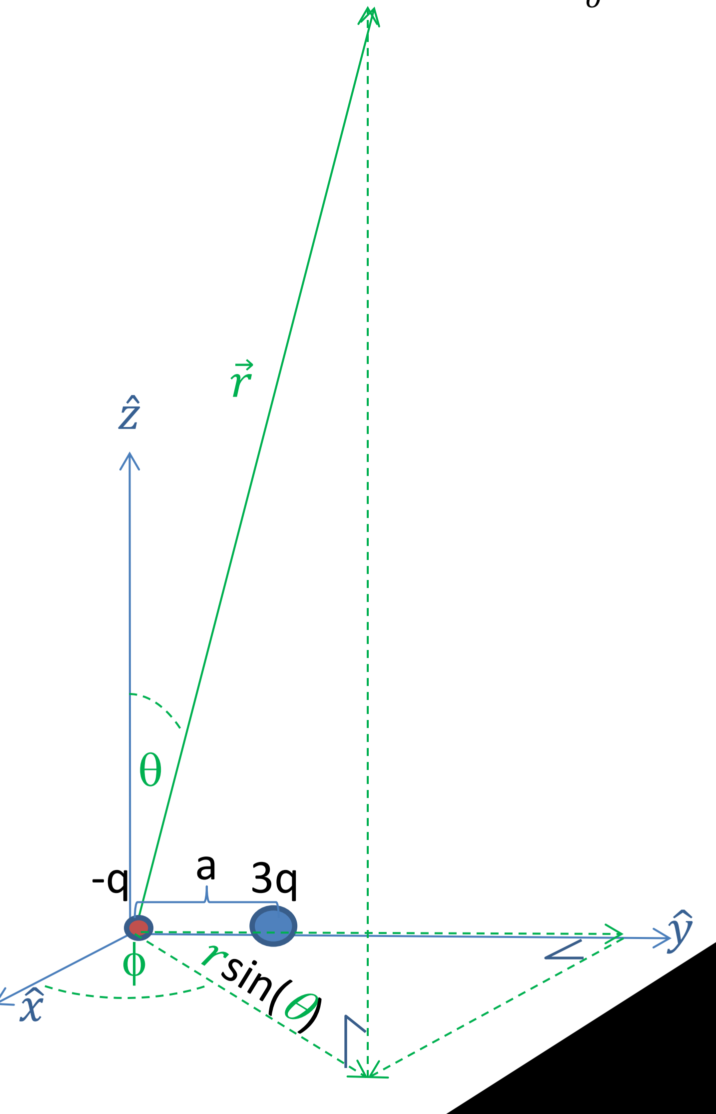
Exercise: Find the first two terms in the mulitpole expansion for the figure shown below.

$$V(r)_{mono} = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r} \qquad V(r)_{dipole} = \frac{1}{4\pi\varepsilon_o}\frac{\vec{p}\cdot\hat{r}}{r^2} \qquad \vec{p} = \int \vec{r}'\rho(\vec{r}')d\tau' = \vec{r}_1'q_1 + \vec{r}_2'q_2$$



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