This is a closed book/notes exam. Calculator and equation sheet are allowed; however, the equation sheet may have no words and must be turned-in with the exam.

1) If a mass on a spring bobbed up and down 10 times in 5 seconds, what would be the period of its oscillation?

\[
T = \frac{t}{\text{# of oscillations}} = \frac{5\text{s}}{10} = 0.5\text{s}
\]

2) A 25-coil spring has a spring constant of 350 N/m. What would be the spring constant of four such springs hung in parallel (side-by-side)?

\[
k_{\text{parallel}} = 4k = 4 \cdot 350 \text{N/m} = 1400 \text{N/m}
\]

3) Say you took a spring with a 100 N/m spring constant and hung a 30 kg mass from it. By how much would it stretch?

\[
\Delta x = \frac{F}{k} = \frac{mg}{k} = \frac{30\text{kg} \cdot 9.8 \text{m/s}^2}{100 \text{N/m}} = 2.9\text{m}
\]

4) The plot below represents the position vs. time of a mass on a spring. Decide which of the two labeled points best completes the following phrases.

![Position vs. time graph]

a. The mass is momentarily stationary at ___A___
b. The mass is moving the fastest at _______B____
c. The mass experiences the largest net force at __A__
d. The mass experiences no net force at _________B____

5) A grandfather clock’s pendulum has a period of 2 seconds. How long should the pendulum be?

\[
T = 2\pi \sqrt{\frac{L}{g}} \Rightarrow L = \left(\frac{T}{2\pi}\right)^2 g = \left(\frac{2s}{2\pi}\right)^2 9.8\text{m/s}^2 = 0.99\text{m}
\]

6) How is the period of a pendulum’s swing affected if the mass of its bob is doubled?

It isn’t affected at all. Fundamentally, this is for the same reason that the free fall acceleration of an object doesn’t depend on its mass.
7) In which condition is a vibration dampened out the soonest: when it is under-damped, over-damped, or critically damped?

Critically damped. Under-damped lets it vibrate for a long time, over-damped drags out the return to equilibrium too long. Critically damped gets the mass back to equilibrium expediently and keeps it there.

8) Throughout history, the standard musical scale has slid higher and higher. We currently use a scale with the note A₄ at 440 Hz. Stringed instruments accommodated by simply tightening their strings to higher tensions (modern violins require internal braces so the high tension strings won’t break them.) By what factor must the tension in a string increase to shift its A from 420 Hz to 440 Hz?

\[
\frac{f_2}{f_1} = \frac{1}{\lambda} \sqrt{\frac{F_2}{m/L}} \Rightarrow \frac{F_2}{F_1} = \left( \frac{f_2}{f_1} \right)^2 = \left( \frac{440 \text{ Hz}}{420 \text{ Hz}} \right)^2 = 1.098
\]

9) Say a fire cracker goes off in the sky, 30 m from you. If the sound intensity is 100 W/m² where you sit, how much is it for a person 60 m from the fire cracker?

\[
I = \frac{P}{4\pi r^2} \Rightarrow \frac{I_2}{I_1} = \frac{P}{4\pi r_2^2} = \frac{r_1^2}{r_2^2} \Rightarrow I_2 = I_1 \left( \frac{r_1}{r_2} \right)^2 = 100 \text{ W/m}^2 \left( \frac{30 \text{ m}}{60 \text{ m}} \right)^2 = 25 \text{ W/m}^2
\]

10) Measured at the same distance from the source, a whisper’s intensity is around \(1.0 \times 10^{-10} \text{ W/m}^2\) and normal conversation is \(3.2 \times 10^{-6} \text{ W/m}^2\). What is the intensity level (in dB’s) of a normal conversation relative to a whisper?

\[
\beta = 10 \log_{10} \left( \frac{I_{\text{conversation}}}{I_{\text{whisper}}} \right) = 10 \log_{10} \left( \frac{3.2 \times 10^{-6} \text{ W/m}^2}{1.0 \times 10^{-10} \text{ W/m}^2} \right) = 45 \text{ dB}
\]
11) Waiting at a train crossing, you hear the whistle of a 35 mph (15.6 m/s) train approach, and then recede. Taking the motion to be nearly directly at, and then away from you, put in ratio the frequency heard when the train was receding vs. that heard when approaching: \( \frac{f_{\text{recede}}}{f_{\text{approach}}} \).

Take the speed of sound to be 343 m/s.

\[
f_{\text{recede}} = f_o \left( \frac{1}{1 + \frac{v_{\text{source}}}{v_{\text{wave}}}} \right), \quad f_{\text{approach}} = f_o \left( \frac{1}{1 - \frac{v_{\text{source}}}{v_{\text{wave}}}} \right)
\]

\[
f_o \left( \frac{1}{1 + \frac{v_{\text{source}}}{v_{\text{wave}}}} \right) = \frac{1}{1 - \frac{v_{\text{source}}}{v_{\text{wave}}}} = \frac{v_{\text{wave}} - v_{\text{source}}}{v_{\text{wave}} + v_{\text{source}}} = \frac{343\text{ m/s} - 15.6\text{ m/s}}{343\text{ m/s} + 15.6\text{ m/s}} = 0.91
\]

12) 20,000 Hz is around the highest frequency that humans can hear. What diameter speaker would produce this high pitch with a broad diffraction angle of 80°? Take the speed of sound to be 343 m/s.

\[
v = f\lambda \quad \sin \theta = 1.22 \frac{\lambda}{D} \Rightarrow D = \frac{1.22 \cdot \lambda}{\sin \theta} = \frac{1.22 \cdot v}{\sin \theta} = \frac{1.22 \cdot 343\text{ m/s}}{20.000\text{Hz} \cdot \sin 80^\circ} = 0.021\text{m}
\]

13) The two plots below represent a wave traveling down the torsion-wave machine (the one with the long black beams). Write the Y position as a function of x and t. In other words, write the basic form of the equation and fill in values for all the constants.

\[
Y(x,t) = 0.5m \cdot \sin \left[ 2\pi \left( \frac{t}{0.04s} - \frac{x}{0.8m} \right) \right]
\]

14) Tuning a guitar, say you play A\(_4\), 440 Hz, ... use electronic tuner and adjust the tension of your guitar string until you hear a beat every 1.5 seconds. You call that good enough. What are the two possible frequencies you’ve tuned the string to?

\[
f_b = |f_1 - f_2| \Rightarrow f_1 = f_2 \pm f_b = 440\text{Hz} \pm \frac{1}{1.5s} = 440.6\text{Hz}, 439.3\text{Hz}
\]
15) A rope of length L is clamped at both ends. In terms of L, what are the 3 longest wavelengths of vibrations that it can support?

\[ 2L, L, \frac{2}{3}L \]

16) A 3-m long string is anchored at both ends. The frequency of the vibration shown below is 400 Hz. What is the wave speed?

\[ \lambda = \frac{1}{2}L = \frac{1}{2} \cdot 3m \]

\[ f = 400Hz \]

\[ v = f\lambda = 400Hz \cdot \frac{1}{2} \cdot 3m = 600m/s \]

17) When Alan DeWeerd substituted in class he spoke like Mickey Mouse after inhaling He, and Barry White after inhaling Xe (monatomic gas, mass = \(2.18 \times 10^{-25} \text{kg}\)). Assuming that the air in his throat is body temperature, 36°C, what was the speed of sound in his throat full of Xe?

\[ v = \sqrt{\frac{\gamma kT}{m}} = \sqrt{\frac{\frac{5}{3}1.38 \times 10^{-23} \gamma /k (273 + 36)K}{2.18 \times 10^{-25} kg}} = 181m/s \]

18) What fundamental frequency would be played by a 2-m long organ pipe with one open end? Take the speed of sound to be 343 m/s.

\[ v = f\lambda \Rightarrow f = \frac{v}{\lambda} = \frac{343m/s}{4 \cdot 2m} = 43Hz \]

19) Say you have a G₃ string on a guitar. Played ‘open string’ (no fingers on the fret board) its fundamental frequency is 196 Hz. What frequency would it play if you put you finger \(\frac{1}{4}\) of the way down the neck (allowed \(\frac{3}{4}\) of the string to vibrate)?

\[ L = \frac{\lambda_1}{2} = \frac{\lambda_2}{4} \Rightarrow \lambda_1 = \lambda_2 \cdot \frac{4}{3} \]
\[ v = f_1 \lambda_1 = f_2 \lambda_2 = f_2 \lambda_1 \frac{3}{4} \]

\[ f_1 = f_2 \frac{3}{4} \Rightarrow f_2 = f_1 \frac{4}{3} = \frac{196 \text{Hz}}{3} = 65.33 \text{Hz} \]