• **Equipment**
  - Longitudinal wave machine
  - Laptop
  - Simulation of compression /rarefaction passing through gas?
    - [http://www.acs.psu.edu/drussell/Demos/waves/wavemotion.html](http://www.acs.psu.edu/drussell/Demos/waves/wavemotion.html)
  - Ball on spring energy force momentum.py (show with projector as main or only screen)
  - Falstad.com/ripple/ (set to point source, moved to middle)
  - Sound-level intensity meters (deployed around the room)
  - iTunes up on laptop (see if sound input still works; Lenny Kravitz?)

**This Time**

**Chapter 5  Sound Intensity and Its Measurement**

• **Introduction:**
  - **Thus far.** When we first thought of sound in terms of waves, I introduced a few key properties of waves: their wavelengths, their frequencies, their wave speeds, and their amplitudes. Thus far in this class, we’ve mostly focused on the first three properties, both qualitatively and quantitatively. We were able to understand a lot about how sound behaves in terms of these wave properties including, last week, how waves interact with each other (Interference and Beats) and their surroundings (Reflection, Refraction and Diffraction.)
  - **Now.** Of course, something very important happens to a wave’s *amplitude* as it propagates and interacts with its surroundings – it generally gets weaker. Especially for the subject of Architectural Acoustics – how sound interacts with spaces & how spaces can be designed to shape sound, it’s important for us to develop the tools to talk about, measure, and predict the *strength* of a sound.
  - **Demo:** falstad wavetank – see it getting weaker with distance from source.
5.1 Amplitude, Energy, and Intensity

- A sound wave front is characterized by a number of simultaneous disturbances in the air. When it’s suited us, we’ve focused on one or another of them.

- Demo – Longitudinal waves
  - Hanging slinky
    - **Density**: you can see where the rungs of the slinky get bunched up and where they get spread out – higher density and lower density.
    - **Displacement**: hand-in-hand with increases and decreases in density is the displacement of individual rungs from their equilibrium locations.
    - **Tension**: In the slinky, we might describe the force that pulls and pushes the rungs as tension, and clearly as they get bunched or spread that changes.

- Simulation of sound wave: http://www.acs.psu.edu/drussell/Demos/waves/wavemotion.html

- **Density**: Simply for the purpose of visualization, this was the easiest. We could picture the air molecules getting bunched up in places – compressions, and spread out in other places – rarefactions.

- **Displacement**: In wind instruments, it was easiest to focus on the accompanying displacement of air molecules since the closed end of a tube doesn’t allow any – it enforces a node in the displacement.

- **Pressure**: If we want to think about how the sound wave interacts with something, say pushes on your ear drum, we maybe think about accompanying undulations in pressure – where the air particles get pushed together there are going to be more collisions – higher pressure.

- Whatever we choose to focus on, that property undulates. If we watched the density/displacement/pressure at one point along a sound wave, we’d see it vary like

\[
P(t) / \rho(t) / \ddot{\chi}(t) : \text{Since these three properties undulate in synch with each other (or with a static phase shift), we have some choice about which we track or actually measure.}
\]
• **Pressure and measurement.** From a practical perspective, it’s easiest to imagine designing a detector that directly responds to the *pressure* undulations in a wave. A sound sensor responds to this push. When your eardrum gets pushed by the air, it displaces, similarly when a microphone diaphragm gets pushed by the air, it is displaced. So, that’s what a microphone or your ear most directly responds to and so the amplitude of the pressure’s oscillations is a reasonable measure of a sound’s strength.

• **Pressure and Interactions.** If you want to conceptually or quantitatively think about an interaction, like air pushing on the eardrum, then pressure is a great tool.

• **Energy:** Energy is another handy conceptual/mathematical tool. When I first introduced it, I mentioned that it was one of the ways that physicists quantify ‘motion’, either actual motion or the potential for motion.

  • Now, the **fundamental tenet of classical mechanics** (the study of motion and interactions) is that ‘motion is neither created nor destroyed, but transferred via interactions.’ For example, if two hockey pucks were to bounce into each other (at hockey practice I guess), if you kept track of the total amount of energy each had before the collision and the total they had after collision, that *total* would be pretty much the same, maybe one was going faster and now the other is going faster, so some energy was transferred, but the total energy is the same. The same would be true if two air particles bounced into each other.

  • **Conservation of energy and book keeping.** In physics, it’s often very enlightening to trace the ‘flow’ of energy, and knowing that the *total* amount doesn’t change makes that job considerably easier. For example, when we consider how a soundwave’s ‘strength’ changes, it will be easiest to phrase that strength in terms of energy.

  • **Energy refresher.** Back in the second week, when we were considering resonance for a mass hanging from a spring, it was handy to think in terms of work and energy, so I introduced the ideas of kinetic and potential energy.

    • **Kinetic Energy:** \( \frac{1}{2} mv^2 \). A measure of how much something is moving. When the mass sails through the equilibrium position, it’s got a lot of kinetic energy.

    • **Potential Energy:** \( \frac{1}{2} k (dx)^2 \) for a spring. A measure of how much stored up *potential* something has to move. When the mass momentarily comes to a halt when the spring is fully stretched or compressed, it’s got a lot of potential energy.

    • **Total Energy: Kinetic + Potential.** If nothing external interacted with the mass on the spring, then the *total* energy, that of actual motion and that of the potential to have motion, would remain constant, but through an
interaction, energy could be given to or taken from the system.

- **Demo:** mass on spring…py

- **Energy in sound.** These same ideas can be applied to a whole wave of oscillating things – the pieces are moving, so they have kinetic energy, and there’s generally some interaction that helps to push them back and forth, so there’s potential energy. This idea can be applied to a sound wave.

- **Energy and Amplitude.** \[ E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 \] where \( v = \frac{\Delta x}{\Delta t} \), so, the energy is proportional to the displacement squared. And as we’ve already noted, the displacement, density, and pressure all undulate together, so the energy is proportional to the pressure squared. \( E \propto P^2 \).

- **Clicker Example.** I’ve suggested that a microphone responds to pressure changes in a sound wave; let’s say it does so linearly – so when the volume of a sound increases enough that pressure doubles so does the electrical signal that the microphone produces. Now, if the pressure doubles, what happens to the energy of that sound wave?
  - A) doubles
  - B) stays the same
  - C) decreases by \( \frac{1}{2} \)
  - D) quadruples
  - E) decreases by \( \frac{1}{4} \)
  - F) I don’t know, what?

  That means, if you turn up the volume and the sound pressure doubles, the plot the oscilloscope gets twice as tall, then energy **quadruples**.

\[
\frac{E_2}{E_1} = \frac{P_2^2}{P_1^2} = \left( \frac{P_2}{P_1} \right)^2
\]

- **Power:** So, a wave can be said to have energy in it, and as the wave ripples out, you can think of that energy being carried out. A natural question / quantity to track is the rate at which energy is transmitted/carried by the wave. We define Power as the rate of energy transfer:

\[
P_{\text{ow}} = \frac{E}{t} \quad \text{(I’m subscripting with ow to remind us that this is Pow-er, not Pressure or Period)}
\]

- **Units:** The units of energy are Joules, so the units of Power are Joules/Second = Watt.
**Seen Watts before:** That’s the same “Watt” as appears on a lightbulb telling you the rate at which energy is transferred to the bulb from the electrical current running through it and from the bulb to its environment as light.

**Example:** a 40 Watt bulb run for 1 hour translates how much energy from the electrical current to light?

\[ P_{\text{out}} = \frac{E}{t} \implies E = P_{\text{out}} t = 40\text{Watts}\cdot1\text{hour} \]

\[ = 40\text{J/s}\cdot1\text{hour}\cdot\frac{3600\text{s}}{1\text{hr}} = 144,000\text{J} = 144\text{kJ} \]

- (which is about what you’d get from burning 10g of coal, assuming a production-transmission efficiency of around 30%)

**Sound Intensity:** If you think about describing how ‘strong’ a sound is in terms of the energy it delivers, there are two issues – one is that the longer you wait, the more energy you’d measure, so it makes more sense to talk about the rate at which energy is delivered, or the Power = E/t; the other issue is that the larger the detector you have, the more energy it will collect, so if you want to a measurement that speaks to just the strength of the sound, not also the size of your detector, you want to divide out the detector’s area. That gives us the Intensity of the sound.

This is defined as the Sound Intensity, \[ I = \frac{E_{\text{transmitted}}}{A \cdot \Delta t} \] (eq’n 16.8)

- \( E_{\text{transmitted}} \) is the energy transmitted
- \( \Delta t \) is the period of time during which it is detected.
- \( A \) is the area, perpendicular to the sound’s propagation, of the detector.
- Units: \( \text{Energy/(area * time)} \rightarrow J/(m^2 \cdot s) = W/m^2 \)

**Human Range:** Audible sound Intensity spans the range from \(10^{-12} \text{W/m}^2\) to \(10^3 \text{W/m}^2\). This is a huge range.
Example using it: A typical adult ear (not ear-drum mind you) has a surface area of \(2.1 \times 10^{-3} \text{m}^2\). The sound intensity during normal conversation is about \(3.2 \times 10^{-6} \text{W/m}^2\) at the listener’s ear. Assume the sound strikes the surface of the ear perpendicularly. At what rate is energy intercepted by the ear?

- **Quantities**
  - \(A_{\text{ear}} = 2.1 \times 10^{-3} \text{m}^2\)
  - \(I = 3.2 \times 10^{-6} \text{W/m}^2\)
  - \(P = \frac{E}{\Delta t}\)

- **Relations**
  - \(I = \frac{E}{A_{\perp} \cdot \Delta t}\)
  - \(P = \frac{E}{\Delta t}\)

- **Algebra**
  - \(P = \frac{E}{\Delta t}\)
    - \(I = \frac{E}{A_{\perp} \cdot \Delta t} \Rightarrow E = I \cdot A_{\perp}\)
  - \(P = I \cdot A_{\perp}\)

- **Numbers**
  - \(P = I \cdot A_{\perp}\)
    - \(P = 3.2 \times 10^{-6} \text{ W/m}^2 \cdot 2.1 \times 10^{-3} \text{ m}^2\)
    - \(P = 6.72 \times 10^{-9} \text{ W}\)

After 60 seconds of gabbing, how much energy has been transferred to the ear?

- \(P = \frac{E}{\Delta t} \Rightarrow E = P \cdot \Delta t = 6.72 \times 10^{-9} \text{ W} \cdot 60 \text{ s} = 4.03 \times 10^{-7} \text{ J}\)

5.2 Sound Level and the Decibel Scale

- **Motivation for Log scale**
  The range of sound intensities that we can hear – withstand is huge: \(10^{-12} \text{ W/m}^2\) to \(10^3 \text{ W/m}^2\). These are such vastly different values, that it’s hard to even conceive of comparing, say, the intensity of a whisper and the intensity of a rock concert. Often, when dealing with something that measures over such a wide range, rather than talking about the actual values, we talk about the relative magnitudes of them.

- **Powers of 10**: To describe it better, we can speak in terms of factors of 10, orders of magnitude, as some sort of short hand. \(10^{-12}\) to \(10^3\) is 15 orders of magnitude.

- **Comparison against Threshold of Hearing**: We can take the threshold of Audibility and compare everything to that.
### Example:
An empty library, with Intensity of $10^{-8} \text{W/m}^2$, is
\[
\frac{I_L}{I_o} = \frac{10^{-8} \text{W/m}^2}{10^{-12} \text{W/m}^2} = 10^4 \text{times more intense than just audible.}
\]
A short hand is just quoting the exponent: 4. The units we put to this is the bel. A library is 4 bels louder than just audible.

### Example:
A jet engine, \[
\frac{I_j}{I_o} = \frac{10^3 \text{W/m}^2}{10^{-12} \text{W/m}^2} = 10^{15} \text{ times louder than just audible, or is 15 ‘bels’ above the audible limit.}
\]

### Log:
In these two examples, I’ve chosen very simple ratios – easy to see how many of factors of 10. So we just read off the “4” or “15”. Conveniently, there is a button on your calculator which does the same thing, and can do it for even uglier ratios. That’s the Log.
\[
\log \left( \frac{I_j}{I_o} \right) = \log \left( \frac{10^3 \text{W/m}^2}{10^{-12} \text{W/m}^2} \right) = \log (0^{15}) = 15
\]

This measure is called the **Sound Intensity Level**, or SIL and it is measured with the Sound Intensity Meter, which we’ll use in lab on Thursday.

\[
\Delta \text{SIL} = 10 \text{dB} \cdot \log \left( \frac{I}{I_o} \right).
\]

The unit is the bel, B, but since people are sensitive to changes in loudness around a tenth of that, or 1 dB (decibel), sound intensity levels are more commonly quoted in dB’s.

Just as 1 meter is 10 dm or 100 cm. 1 B is 10 dB. So we say that a jet engine is 150dB, a library is 40 dB.

### Music Dynamic Range
Where does this touch home? In music, the rough correspondence of the dynamics are

<table>
<thead>
<tr>
<th>Dynamics</th>
<th>Levels</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pppp</td>
<td>40 dB</td>
</tr>
<tr>
<td>ppp</td>
<td>50 dB</td>
</tr>
<tr>
<td>pp</td>
<td>60’s</td>
</tr>
<tr>
<td>p</td>
<td>70’s</td>
</tr>
<tr>
<td>mp</td>
<td>85 dB</td>
</tr>
<tr>
<td>mf</td>
<td>95 dB</td>
</tr>
<tr>
<td>f</td>
<td>100 dB</td>
</tr>
</tbody>
</table>

Though, as the text cautions, pure ‘loudness’ isn’t all that determines our sense of how violently something’s being played.

**Demo:** **SIL’s** There are Sound Intensity Level meters deployed all around the room, let’s see if we can get familiar with some of these levels. If there’s one near you, turn it on (if it isn’t already on) to the LO setting, and we’ll first all be as quiet as we can and see the lowest level it reads.

- **Quiet.** \( \text{SIL} = 62\text{dB} \)
- **Lecture.** \( \text{SIL} = 64\text{dB} \)
- **Talking to neighbors** \( \text{SIL} = 70\text{dB} \)
- **Rockin’ out** \( \text{SIL} = 75\text{dB} \)

### Got this far Tues.
- So we started out simply wanting to pick a measure of a sound wave's amplitude. We thought it would be simply a measure of push, maybe pressure. But scientists chose energy, the field guys chose to divide by area and time to give us intensity, and in an attempt to come up with a unit for the layman to relate to, we’ve arrived at the
SIL, measured in dB something many times removed from our picture of a sound wave.

- **Example** When I’m talking on the phone to my grandmother, who’s a bit hard of hearing, I have to talk a bit louder than usual. If, say, the sound power or intensity to talk to Grandma about 40 fold over my normal phone volume, what is the change in my Sound Intensity Level?
  - **Quantities**
    - $\frac{I_{\text{grandma}}}{I_{\text{norm}}} = 40$
    - $\Delta \text{SIL}$
  - **Relations**
    - $\Delta \text{SIL} = 10 \text{dB} \cdot \log \left( \frac{I_{\text{grandma}}}{I_{\text{norm}}} \right)$
  - **Algebra**
    - $\Delta \text{SIL} = 10 \text{dB} \cdot \log \left( \frac{I_{\text{grandma}}}{I_{\text{norm}}} \right)$
  - **Numbers**
    - $\Delta \text{SIL} = 10 \text{dB} \cdot \log 40 \approx 16 \text{dB}$

- **Example like 14** The bellow of a territorial bull hippopotamus has been measured at 115 dB above the threshold of hearing. What is the ratio if its intensity to that of the threshold of hearing?
  - **Quantities**
    - $\beta = 115 \text{ dB}$ (above threshold of hearing)
    - $I_o = 1.0 \times 10^{-12} \text{ W/m}^2$ (Intensity at threshold of hearing)
    - $I = ?$
  - **Relations**
    - $\beta = 10 \text{dB} \cdot \log \left( \frac{I}{I_o} \right)$
  - **Algebra**
    - $\beta = 10 \text{dB} \cdot \log \left( \frac{I}{I_o} \right)$
    - $\frac{\beta}{10 \text{dB}} = \log \left( \frac{I}{I_o} \right)$
    - $10^{\frac{\beta}{10 \text{dB}}} = \frac{I}{I_o}$
  - **Numbers**
\[
\frac{I}{I_o} = 10^{\frac{\text{dB}}{10}}
\]

- \[
\frac{I}{I_o} = 10^{\frac{115\text{dB}}{10}} = 3.16 \times 10^{11}
\]

- What is the ratio of their amplitudes?
  - Sound intensity is proportional to the square of its amplitude,
  \[
  \frac{I}{I_o} = \left(\frac{A}{A_o}\right)^2 \Rightarrow \frac{A}{A_o} = \sqrt{\frac{I}{I_o}} = 5.62 \times 10^5
  \]

5.3 The Inverse-Square law

- Spherically uniform radiation
  - We are all quite familiar with the fact that our perceptions of a sound’s loudness depends strongly on how far we are from its source.
  - The explanation and mathematical description are simplest if we make imagine a source that radiates sound evenly in all directions – this would be the case for a speaker diaphragm that was a sphere that pulsed larger and smaller.
  - The push of the diaphragm is transferred to the air molecules just around it, they in turn push straight out on the neighbors just beyond…In this fashion, the push is transmitted radially outward. The front looks like an expanding spherical surface of molecules pushing molecules. However, there is only so much ‘oomph’ to get passed around. The rate of energy transfer through the air remains the same, but that energy gets shared by more and more atoms as it moves to bigger and bigger spherical shells.
  - So the intensity, power per area would go like:
    \[
    I = \frac{P}{A_{\text{sphere}}} = \frac{P}{4\pi r^2}
    \]
    - It drops off like \(1/r^2\).
  - Since how loud something sounds depends on how hard your eardrum gets pushed, thus the sound intensity, the loudness drops off the further you are from the sound source.

- Imagined Demo: A similar thing would happen if I were standing in the middle of a big crowd, I tore a sheet of paper into equal parts and gave each to each of my neighbors & asked each of them to distribute their pieces evenly among their outward neighbors, and so on. It started out that I had all the paper, when it reaches the end, each one of, say, 100 people, would have a \(1/100\)th of the piece of paper. The ‘intensity’ of paper per person drops off the further out it goes.

- Sound Intensity Level Decay
  - We take this to our expression for Sound Intensity Level:
    \[
    \Delta \text{SIL} = 10\text{dB} \cdot \log \left( \frac{I_2}{I_1} \right)
    \]
\[ I = \frac{P}{A_{\text{sphere}}} = \frac{P}{4\pi r^2}, \]

\[ I_1 = \frac{A_{\text{sphere},1}}{P} = \frac{A_{\text{sphere},2}}{P} = \frac{4\pi r_2^2}{r_2^2} = \frac{4\pi r_1^2}{r_1^2} = \left(\frac{r_1}{r_2}\right)^2 \]

\[ \Delta I = 10dB \cdot \log \left( \left(\frac{r_1}{r_2}\right)^2 \right) = 20dB \cdot \log \left( \frac{r_1}{r_2} \right) \]

**Example using it** A rocket in a fireworks display explodes high in the air. The sound spreads out uniformly in all directions. How much lower is the Sound Intensity Level 190 m away than 120 m away?

- **Quantities**
  - \( r_1 = 120 \text{ m} \)
  - \( r_2 = 190 \text{ m} \)

- **Relations**
  - \( \Delta I = 20dB \cdot \log \left( \frac{r_1}{r_2} \right) = 20dB \cdot \log \left( \frac{120}{190} \right) = -4 \)