TEMPERATURE DEPENDENCE OF HEAT CAPACITY

(Equipment: Ice, liquid Nitrogen, scale, electronic scale, thermos, gloves, paper towels; per station: thermometer, aluminum calorimetry cup set & stopper, hot plate, 2 beakers, metal mass)

OBJECTIVE
To investigate the temperature dependence of heat capacity. Specifically, measure Al’s heat capacity over three temperature ranges and compare your data with the theoretical relationship.

PRE-LAB (to be completed before coming to lab)
Prior to coming to lab, read through this write-up and perform all the exercises labeled ‘Pre-Lab’. During the first 5 minutes of lab, I will check that you have done them. During the course of the lab, you may find that some pre-labs were incorrect – correct them, for I will grade the final versions (provided you’d done respectably before lab).

OVERVIEW
When we considered heat earlier in the semester, we didn’t delve into the nature of heat capacity; we simply approximated it as a constant for a given material. This time around, we see how in a crystalline solid, the heat capacity, 

\[ C_v = \left( \frac{\partial E}{\partial T} \right)_v \]

strongly depends on the population distribution in excited states, which in turn depends on temperature. According to the Debye theory, this dependence is modeled in

\[ C_v = \left( \frac{\partial E}{\partial T} \right)_v = 3Nk \left\{ \frac{3}{x_D^3} \int_0^{x_D} \frac{x^4 e^x dx}{(e^x - 1)^2} \right\} \]

where \( x \equiv \frac{\hbar \omega}{kT} \)

and \( x_D = \frac{\Theta_D}{T} \) as found in equation 7.117 of your text. For aluminum, \( \Theta_D = 380 \) K. Unfortunately, the necessary integral cannot be analytically evaluated, but a computer math package should have little difficulty evaluating it for given temperatures and materials. Qualitatively, the temperature dependence of heat capacity is sketched below.

\[ Y = \frac{C_v}{3Nk} \]

\[ x = \frac{T}{\Theta_D} \]

\( \Theta_D = 380 \) K
**Experiment**

By measuring the average heat capacity of an Al sample over three temperature ranges, you will construct a plot like the one above. Since you’ve already done one calorimetry lab earlier this semester, you should look over that lab to fill-in the procedural details, I’ll just give you a general outline here.

1. **Pre-Lab.** Use the computer math package of your choice to create a theoretical plot like the one above. Print this out.
2. Start a beaker of water boiling.
3. Place a thermos containing powdered dry ice on an electric scale plugged into one computer.
4. Place a thermos containing liquid nitrogen on an electric scale plugged into another computer.
5. Start both computers logging their respective scale’s readings.

**High Temperature**

7. Find the mass of two pieces of aluminum, then put #1 in the boiling water and #2 in the ice chest with ice water.

   \[ m_{m.1} = \quad \quad \quad m_{m.2} = \quad \quad \quad \]

8. Find the mass of the *inner* calorimeter cup. Note: make sure it is aluminum. Fill it about two thirds full with tap water and use the scale again to find the mass of the water.

   \[ m_c = \quad \quad \quad m_w = \quad \quad \quad \]

9. Put the thermometer into the calorimeter cup and record the temperature of the water just before adding the hot piece of metal.

10. One person should lift up the lid of the calorimeter while another person should carefully and quickly remove the metal from the boiling water, dry it, and gently put it in the calorimeter. Quickly replace the lid.

11. *Gently* stir the water for several minutes until you are certain that the temperature has stabilized. Keep the end of the thermometer near the middle of the water. The temperature may start to drop if you wait long enough because the cup is not perfectly insulated, so
you are interested in the *maximum* temperature that the water reaches. Note, you can be fooled if you touch the thermometer to the metal chunk instead of just the water. Record the equilibrium (final) temperature.

\[ T_f = \ldots \]

12. According to your measurements, what is the specific heat of the metal? Show all of your work.

\[
\Delta E_{metal} = -\Delta E_{water} - \Delta E_{cup}
\]

\[
c_{v,Al.high} m_{Al} \Delta T_{Al.high} = -c_{v,water} m_{water} \Delta T_{water} - c_{v,cup} m_{cup} \Delta T_{cup}
\]

\[ c_{v,Al.high} = \ldots \]

13. Roughly speaking, this specific heat corresponds to the average temperature of the metal, which we'll call \( T_{high} \).

\[ T_{high} = \ldots \]

**Medium Temperature**

14. By now, the aluminum chunk in the cooler should be at 0°C, or 273.15 K. Remove it from the cooler, dab it dry with a paper towel, and place it in the thermos with the dry ice.

15. All along, the dry ice has been losing mass, and the scale – computer has been recording that. Now with the aluminum in the dry ice, the dry ice will sublime at an accelerated rate until the
aluminum is cooled to the dry ice’s temperature, 194.67 K. When this happens, you can stop collecting data.

16. Create a new “Corrected Mass” column in the data table and, from the point at which you inserted the aluminum until the end, subtract out the aluminum’s mass.

17. The graph should look similar to the one below with the solid line. The more gradual decreases in dry ice at the beginning and end are due to the imperfect insulation of the thermos. To determine how much dry ice sublimed because of the addition of the aluminum, extend those parts of your graph with dashed lines as shown. Find the mass that sublimed because of the aluminum by measuring the difference between the dashed lines in the middle of the section where the mass is changing rapidly, as indicated by the arrow.

\[ m_{\text{sub}} = \text{______} \]

18. **Pre-Lab:** ignoring heat flow into the thermos, since you have corrected for this. Describe the temperature changes and phase changes that occurred for each substance in this experiment. Indicate whether heat flows in or out of the substance for each change.
19. Given that the latent heat of sublimation for dry ice is \(1.81 \times 10^5\) J/kg and that the dry ice stayed at the same temperature while the aluminum cooled from 273.15 K to 194.67 K, what is the average specific heat of aluminum over this range?

\[ \text{CV Al Medium} = \] 

20. What is the average temperature of the aluminum in this part of the experiment?

\[ \text{T}_{\text{medium}} = \]

**Low Temperature**

21. Now the aluminum is at 194.67 K. It is ready to be transferred to the liquid nitrogen, which is at 77 K. The same basic procedures should be followed as above. Monitoring the mass of liquid nitrogen, you can determine how much mass evaporated due to thermal exchange with the aluminum.

\[ m_{\text{evap}} = \] 

22. Given that the latent heat of vaporization for liquid nitrogen is \(2.01 \times 10^5\) J/kg, what is the specific heat of the aluminum?
23. What is the average temperature of the aluminum in this part of the experiment?

\[ T_{\text{low}} = \ \ \ \ ]

24. You’ve now collected three data points: \((T_{\text{low}}, c_{V, \text{Al low}})\), \((T_{\text{medium}}, c_{V, \text{Al medium}})\), and \((T_{\text{high}}, c_{V, \text{Al high}})\). Put these data points on your theoretical plot and discuss their fit. Note: Experimentally, you measure \(c_v\), heat capacity per mass. To get this into the form of what’s on the Y axis of your theoretical plot, you want

\[ Y = \frac{c_v}{3Nk} = \frac{c_v M}{3Nk} = \frac{c_v M}{3k N} = \frac{c_v m_{\text{Al}}}{3k} \]

where \(m_{\text{Al}}\) is the atomic mass of aluminum (in kg).