Quantum Mechanics Homework Week 2

For Wednesday 9/12, review Griffiths' sections 2.1-2.2 and Q7.3 and turn in by 9:30 am:

1. Conceptual: What did Unit Q call a stationary state?
2. Conceptual: Equation 2.15 is related most closely to which of the "rules" from Unit Q?
3. Math: Calculate $\langle x\rangle$, $\langle x_2\rangle$, $\langle p\rangle$, $\langle p_2\rangle$, σ_x, and σ_p, for the nth stationary state of the infinite square well. Check that the uncertainty principle is satisfied. Which state comes closest to the uncertainty limit?
4. Math: A particle in the infinite square well has as its initial wave function: $\Psi(x,0) = A[\psi_1(x) + \psi_4(x)]$
    a. Normalize $\Psi(x,0)$.
    b. Find $\Psi(x,t)$ and $|\Psi(x,t)|^2$. Express the latter as a sinusoidal function of time, as in example 2.1. So simplify the result, use $\omega \equiv \pi^2\hbar / 2ma^2$
    c. Compute $\langle x\rangle$. Notice that it oscillates with time? What is the angular frequency of the oscillation? What is the amplitude?
    d. Compute $\langle p\rangle$. Hint: There is an easy way.
    e. If you measured the energy of this particle, what values might you get, and what is the probability of getting each of them?
    f. Find the expectation value of H. How does it compare with the answer to e?

For Friday 9/14, read Griffiths' section 2.3.1 and Q7.4 and turn in by 9:30 am:

1. Conceptual: Which integral from the back of the book does he use to evaluate the integral before equation 2.59?
2. Fill in: For the equation at the bottom of page 47: write out each term separately. Which term should use integration by parts? Show explicitly and mark which term goes to 0 and why.
3. Easy Math: Find the second excited state of the harmonic oscillator.
    a. Sketch ψ_0, ψ_1, and ψ_2.
    b. Check the orthogonality of ψ_0, ψ_1, and ψ_2, by explicit integration. Hint: If you exploit the even-ness and odd-ness of the functions, there is only one integral left to do.
4. Math: For ψ_2 for the harmonic oscillator:
    a. Compute $\langle x\rangle$, $\langle p\rangle$, $\langle x^2\rangle$, and $\langle p^2\rangle$ by explicit integration. Use the variable $\xi \equiv \sqrt{m\omega/\hbar}x$ and the constant $\alpha \equiv (m\omega/\pi\hbar)^{1/4}$.
    b. Check the uncertainty principle for this state.
    c. Compute $\langle T\rangle$ and $\langle V\rangle$ for these states without integration. Is their sum what you would expect?
5. Math: Find $\langle x\rangle$, $\langle p\rangle$, $\langle x^2\rangle$, $\langle p^2\rangle$, and $\langle T\rangle$, for the nth stationary state of the harmonic oscillator, using the method of example 2.5. Check that the uncertainty principle is satisfied.