Schrodinger Equation: $$i\hbar\frac{\partial\Psi}{\partial t}=-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2}+V\Psi$$

Ψ is the wave function, but what is it?

The wavefunction is used to tell us the probability of possible outcomes for a measurement of location of the particle

$$\Pr(a<x<b,t)=\int_a^b|\Psi(x,t)|^2\,dx$$

Average measured value is the expectation value – note that this is NOT the average of multiple measurements of a single system (because ψ collapses), but is the average of measurements ofr a large number of systems with the same Ψ

To calculate the expectation value:

Discrete data

$$<j>=\sum_{j=0}^{\infty}jP(j)$$

$$<f(j)>=\sum_{j=0}^{\infty}f(j)P(j)$$

continuous data

$$<x>=\int_{-\infty}^{\infty}x\rho(x)dx$$

$$<f(x)>=\int_{-\infty}^{\infty}f(x)\rho(x)dx$$

Using Ψ:

$$<x>=\int_{-\infty}^{+\infty}x|\Psi(x,t)|^2\,dx=\int_{-\infty}^{+\infty}\Psi^*x\Psi dx$$

$$<f(x)>=\int_{-\infty}^{+\infty}f(x)|\Psi(x,t)|^2\,dx=\int_{-\infty}^{+\infty}\Psi^*f(x)\Psi dx$$

Standard deviation about an expectation value is

$$\sigma_x^2\equiv<(\Delta x)^2>=<x^2>-<x>^2$$
$$\sigma_{f(x)}^2\equiv<|f(x)|^2>-<f(x)>^2$$

What is the expectation value for **momentum**? $p(x) = m\frac{dx}{dt}$

As an operator p is $\frac{\hbar}{i}\frac{\partial}{\partial x}$

$$< p >= m\frac{d < x >}{dt} = \int_{-\infty}^{+\infty} \Psi^* (\frac{\hbar}{i}\frac{\partial}{\partial x})\Psi dx$$

Every other important quantity is a function of x and p, so

$$< Q(x, p) >= \int \Psi^* Q(x, \frac{\hbar}{i}\frac{\partial}{\partial x})\Psi dx$$

Momentum and position obey the **uncertainty principle**: $\sigma_x \sigma_p \geq \frac{\hbar}{2}$

Since the particle must be somewhere, the wavefunction is **normalized**.

$$\int_{-\infty}^{+\infty} |\Psi(x,t)|^2 \, dx = 1$$

How do I know the wavefunction will stay normalized?

The time derivative must be zero

$$\frac{d}{dt}\int_{-\infty}^{+\infty} |\Psi(x,t)|^2 \, dx = \int_{-\infty}^{+\infty} \frac{\partial}{\partial t} |\Psi(x,t)|^2 \, dx = \frac{i\hbar}{2m}(\Psi^* \frac{\partial \Psi}{\partial x} - \frac{\partial \Psi^*}{\partial x}\Psi)\Big|_{x=-\infty}^{x=+\infty}$$

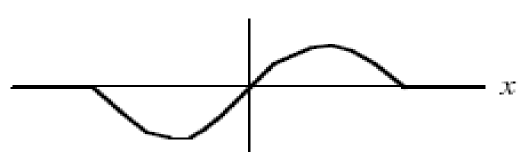
Now we can plug in at the limits Ψ(x=∞) and Ψ(x=-∞)

From our experience with SchroSolver both of these values must be zero for a valid wavefunction, therefore Ψ*(x=∞)=0 and Ψ*(x=-∞)=0

So, the time derivative is zero and the wavefunctiion remains normalized.

Additional questions:

2. I have a particle with wavefunction ψ (shown below).



   a. What is the probability of measuring x>0 at t=0?
   b. I measure the position of the particle at t=0 to be x=5 cm. What is the probability of measuring x>0 cm at t =1 ms?

Solution:

a) Since half the wavefunction is to the right of $x=0$, the probability is ½
b) We need to know how the wavefunction depends on time.

3. Let s be the number of spots shown by a die thrown at random. Calculate $\langle s \rangle$ and σ_s.

Solution:

$\langle s \rangle=(1+2+3+4+5+6)*1/6 = 21/6 = 3$ ½

$\langle s^2 \rangle = (1+4+9+16+25+36) * 1/6 = 91/6 = 15\ 1/6$

σ_s= sqrt(91/6 – (7/2)^2) = sqrt(91/6 – 49/4) = sqrt((182-147)/12)=sqrt(35/12)=1.7078