# R for the Learned 

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June 13, 2012

## 1 Loading R Packages

While R itself is powerful, the use of packages makes it even more so. By using code generated by others and not having to reinvent the wheel, the user can save a lot of time.

Packages must first be installed from a CRAN or other source. Although periodic updating is suggested, once the R workspace is saved reinstallation is not necessary.

```
> ##install.packages can be used in place of the R-gui
> #install.packages("xtable")
> ## load a few helpful packageslibrary(lattice)
> library(Rcmdr)
> library(Hmisc)
> library(xtable)
> library(ggplot2)
> library(survival)
```


## 2 Data Entry

Data may be entered into $R$ in a number of ways. Three commonly used methods will be discussed.

### 2.1 Manual Entry

Perhaps the easiest way to enter small datasets is to enter each variable indvidually and then combine them into a data frame. Using the data from BPS5 problem 4.9, this might look like:

```
> sex = c(rep("Female",12),rep("Male",7))
> mass = c(36.1, 54.6, 48.5, 42.0, 50.6, 42.0, 40.3, 33.1, 42.4,
+ 34.5, 51.1, 41.2, 51.9, 46.9, 62, 62.9, 47.4, 48.7, 51.9)
> rate = c(995, 1425, 1396, 1418, 1502, 1256, 1189, 913, 1124, 1052,
+ 1347, 1204, 1867, 1439, 1792, 1666, 1362, 1614, 1460)
> gender = c(rep (1,12),rep (2,7))
> bps5.4.9 = data.frame(sex, mass, rate, gender)
```

We can now check to see if the data frame has been created by entering

```
> ls()
```

```
[1] "bps5.4.9" "gender" "mass" "rate" "sex"
```

Note that the listing also shows the individual variables that were used to create the data frame. These can be deleted by using rm().

```
> rm("sex", "mass", "rate", "gender")
> ls()
```

[1] "bps5.4.9"
The attributes of the data frame and some summary statistics can be computed using the attributes and summary functions.

```
> attributes(bps5.4.9)
```

\$names
[1] "sex" "mass" "rate" "gender"
\$row.names
[1] $1 \begin{array}{lllllllllllllllllll} & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 & 18 & 19\end{array}$
\$class
[1] "data.frame"
> summary(bps5.4.9)

| sex | mass |  | rate |  |
| :--- | :--- | :--- | :--- | :--- |
| Female:12 | Min. $: 33.10$ | Min. $: 913$ | Min. $: 1.000$ |  |
| Male $: 7$ | 1st Qu.:41.60 | 1st Qu.:1196 | 1st Qu. $: 1.000$ |  |
|  | Median $: 47.40$ | Median $: 1396$ | Median $: 1.000$ |  |
|  | Mean $: 46.74$ | Mean $: 1370$ | Mean $: 1.368$ |  |
|  | 3rd Qu.:51.50 | 3rd Qu.:1481 | 3rd Qu. $: 2.000$ |  |
|  | Max. $: 62.90$ | Max. $: 1867$ | Max. $: 2.000$ |  |

Notice that while sex was treated as a categorical variable, gender was treated as if it was cardinal. R is smart in that it recognizes the difference between cardinal and categorical (which it calls "factor") variables. To make gender a factor variable we can enter

```
> bps5.4.9$gender = factor(bps5.4.9$gender,levels=c(1,2),
+ labels=c("F","M"))
```

Using summary we can see that gender is treated as a factor, or categorical, variable.

```
> summary(bps5.4.9)
\begin{tabular}{|c|c|c|c|}
\hline sex & mass & rate & gender \\
\hline Female:12 & Min. :33.10 & Min. : 913 & F: 12 \\
\hline Male : 7 & 1st Qu.: 41.60 & 1st Qu.:1196 & M: 7 \\
\hline & Median : 47.40 & Median :1396 & \\
\hline & Mean : 46.74 & Mean :1370 & \\
\hline & 3rd Qu.:51.50 & 3rd Qu.:1481 & \\
\hline & Max. : 62.90 & Max. :1867 & \\
\hline
\end{tabular}
```


### 2.2 Using Rcmdr

The package Rcmdr allows us to import data created in a number of packages. While the Windows version of R will import Excel (.XLS) files, the Mac version of R does not. However, both versions will import SPSS transport files.

To use Rcmdr we first need to load the package. This can be accomplished using menus or by using the library function. Assuming that Rcmdr is installed we enter
> library(Rcmdr)
If everything is working correctly, the Rcmdr GUI interface should start. After selecting Data - Import Data - from Excel, Access, or dBase data set, R will ask us for a name for our data set. Enter something descriptive but easy to type (e.g. HtWt). Remeber that R is case sensitive.

Next, you will have to select the Excel file that contains your data. R will then ask which sheet in the Excel file you wish to import. Once you have selected a sheet, R will complete the import and the data set/frame will be created.

Rcmdr will indicate that the data frame has been created and selected by showing Data set: HtWt above the script window. You can now view the data by clicking on View data set.

Noting that the Group variable (which is really a sex variable) is coded as a numeric (1 or 2), we should probably recode it as a factor variable. Rcmrd makes this easy. Click on Data - Manage variables in active data set - Convert numeric variables to factors. Select the variable we wish to change - in this case Group. We will supply level names and use the same variable for the factor recoding. Click on OK. We are going to overwrite Group so click on Yes. In this case a 1 is a Male and a 2 is a Female. Once the level names have been entered, click on OK.

Clicking on View data set we see that the Group variable is now coded as Female and Male. R now recognizes Group as a factor/categorical variable.

Data that is stored in SPSS portable or save formats can be imported in a similar manner. The files that come with BPS5e are actually in the portable format so you can use the menus to create a new data frame.

### 2.3 Reading Comma Seperated Value (CSV) Files

$R$ has a utility for reading comma seperated value (CSV) ascii files. These files can reside on the host machine or on a server. If the files are in standard CSV format,
either of

```
> HtWt = read.csv("c:/stat/ncssdata/htwt.csv")
> htwt = read.csv(
+ "http://newton.uor.edu/facultyfolder/jim_bentley/downloads/math111/htwt.csv")
```

will create a data frame that contains the NCSS Sample data set's height and weight data. Note the use of forward slashes instead of backslashes.

The group variable will be imported as a numeric. To help R function efficiently, it will need to be converted to a factor variable using one of the methods from above.

### 2.4 Saving and Loading Data Frames

Regardless of how they were created, data frames may be saved in $R$ as part of the $R$ workspace. The workspace contains all of the variables, data frames, and functions that you have defined. A workspace is a snapshot of your work to the point of the save.

To save a workspace click on File - Save Workspace. Select the folder to which you wish to save the file and a file name and then click on Save. Your workspace is now safely tucked away on your drive. This file can later be Loaded or you can open it by double clicking on the file.

History files store the commands that you used during your R session. These can be saved ans loaded in a manner similar to that of workspaces. These files are are text files and can be edited using Wordpad or something similar.

## 3 Graphics

$R$ contains a number of predefined data frames. Some of these will be used in the examples that are presented below.
$R$ supports a number of different approaches to generating graphics. We will look at standard R graphics, the lattice package, and graphics using the ggplot2 package.

### 3.1 Standard R Graphics

To use the standard graphics within R we do not need to load any additional packages. A simple scatterplot of the data from BPS5e problem 4.9 (Figure 1) can be created by entering

```
> plot(bps5.4.9$mass,bps5.4.9$rate,
+ xlab="Lean Body Mass (kilograms)",
+ ylab="Metabolic Rate (calories)")
```

A boxplot of the rate variable (Figure 2) can be generated using

```
> boxplot(bps5.4.9$rate, ylab="Metabolic Rate (calories)")
```



Figure 1: Plot of metabolic rate as a function of lean body mass for the data from BPS5 problem 4.9.


Figure 2: Boxplot of metabolic rate for the data from BPS5 problem 4.9.

A histogram of metabolic rate for the data from BPS5 problem 4.9 (Figure 3) can be generated using
> hist(bps5.4.9\$rate, xlab="Metabolic Rate (calories)")
The corresponding stemplot for the rate data is given by entering
> print(stem(bps5.4.9\$rate))
The decimal point is 2 digit(s) to the right of the $\mid$
8 | 1
10 | 0529
12 | 0656
14 | 023460
16 | 179
18 | 7


Figure 3: Histogram of the metabolic rates (colories) from BPS5e problem 4.9.

NULL
Since this generates a stemplot with too few stems, we may wish to expand the stems a bit. The following function call provides more stems-10 to be exact.
> print(stem(bps5.4.9\$rate, 2))
The decimal point is 2 digit(s) to the right of the $\mid$
9 | 1
10 | 05
11 | 29
12 | 06
13 | 56
14 | 02346
15 | 0
16 | 17
17 | 9
18 | 7

NULL
Of course, it is possible to have too many stems as is shown in the following example.
> print (stem(bps5.4.9\$rate,5))
The decimal point is 2 digit(s) to the right of the $\mid$
9 | 1
9 |
10 | 0

```
10 | 5
11 | 2
11 | 9
12 | 0
12 | 6
13
13 | 56
14 | 0234
14 | 6
15 | 0
15 |
16 | 1
16 | 7
17 |
17 | 9
18 |
18 | 7
```

NULL

### 3.2 Lattice Graphics

Use of the lattice package requires that the package be loaded. Entering

```
> library(lattice)
```

accomplishes this.
A simple scatterplot of the data from BPS5e problem 4.9 (Figure 4) can be created by entering

```
> latticeplot = xyplot(rate~mass, data=bps5.4.9,
+ xlab="Lean Body Mass (kilograms)",
+ ylab="Metabolic Rate (calories)")
> print(latticeplot)
```

Comparison of sexes can be made by using conditioning (Figure 5).

```
> latticeplot = xyplot(rate~mass/sex, data=bps5.4.9,
+ xlab="Lean Body Mass (kilograms)",
+ ylab="Metabolic Rate (calories)")
> print(latticeplot)
```

or by the using different symbols for the two groups in overlayed plots (Figure 6).

```
> latticeplot = xyplot(rate~mass, group=sex,
+ pch=c(1,3), data=bps5.4.9,
+ xlab="Lean Body Mass (kilograms)",
+ ylab="Metabolic Rate (calories)")
> print(latticeplot)
```



Figure 4: Plot of metabolic rate as a function of lean body mass for the data from BPS5 problem 4.9.


Figure 5: Plot of metabolic rate as a function of lean body mass while controlling for sex for the data from BPS5 problem 4.9.

A boxplot of the rate variable (Figure 7) can be generated using

```
> latticeplot = bwplot(~rate, data=bps5.4.9,
+ xlab="Metabolic Rate (calories)")
> print(latticeplot)
```

A boxplot of the rate variable comparing sexes (Figure 8) can be generated using

```
> latticeplot = bwplot(sex^rate, data=bps5.4.9,
+ ylab="Sex", xlab="Metabolic Rate (calories)")
> print(latticeplot)
```

The lattice package includes a few sample data frames. One of these is the singer data frame that contains information on various characteristics of some group of singers.

We can create a histogram of the heights of the singers (Figure 9) using


Figure 6: Plot of metabolic rate as a function of lean body mass while controlling for sex for the data from BPS5 problem 4.9.


Figure 7: Boxplot of metabolic rate for the data from BPS5 problem 4.9.

```
> latticeplot = histogram(~height, data=singer)
> print(latticeplot)
```

We can gain additional information by controlling for voice part when creating a histogram of the heights of the singers (Figure 10) using

```
> latticeplot = histogram(~height|voice.part, data=singer)
> print(latticeplot)
```

Similarly, we can look at the distribution of the heights of the singers using density plots. Again, we can gain additional information by controlling for voice part (Figure 11).

```
> latticeplot = densityplot(`height|voice.part,data=singer)
> print(latticeplot)
```

One of the nice things about $R$ is that its use of objects means that it is smart about data types. R knows the difference between cardinal (numerical) and categorical


Figure 8: Boxplot of metabolic rate controlling for sex for the data from BPS5 problem 4.9.


Figure 9: Histogram of the heights of singers in the singer data frame.
(factor) data. The histogram function from the lattice package will revert to a bargraph when asked to plot a factor variable. Figure 12 shows how this works for the voice.part variable.
> latticeplot = histogram(~voice.part,data=singer)
> print(latticeplot)
Figure 13 is the plot that made the whole idea of trellised graphics famous. The barley data that is presented had been analyzed for years by both the investigators and students. It was not until trellised graphics came along that it was recognized that one of the sites appears to have had its year data swapped.

```
> latticeplot = dotplot(variety ~ yield | site, data = barley,
+ groups = year, pch=c(1,3),
+ key = simpleKey(levels(barley$year),
+ space = "right"),
+ xlab = "Barley Yield (bushels/acre) ",
```



Figure 10: Histogram of the heights of singers in the singer data frame controlling for voice part.


Figure 11: Density plot of the heights of singers in the singer data frame controlling for voice part.

```
+ aspect=0.5, layout = c(1,6), ylab=NULL)
> print(latticeplot)
```


### 3.3 GGPLOT2 Graphics

Use of the GGPLOT2 package requires that the package be loaded. Entering

```
> library(ggplot2)
```

accomplishes this. The structure of ggplot is quite different from standard R and lattice graphics. To generate a boxplot of metabolic rate that allows a comparison by sex (Figure 14) one enters the following commands.

```
> bw = ggplot(bps5.4.9,aes(sex,rate))
> bw = bw + ylab("Metabolic Rate (calories)") + xlab("Sex")
> bw = bw + geom_boxplot() + coord_flip()
> print(bw)
```



Figure 12: Bargraph generated using histogram on the factor variable voice.part from the singer data frame.


Figure 13: Density plot of the barley data showing the reversal of year for one of the study sites.

A histogram of metabolic rate (Figure 15) is made by entering the following code.

```
> plt = ggplot(bps5.4.9, aes(x=rate))
> plt = plt + xlab("Metabolic Rate (calories)")
> plt = plt + geom_histogram(binwidth=200)
> print(plt)
```

The sentax for a bar chart is similar to that of a histogram. Figure 16 shows a bar chart of the sex variable from the BPS5e problem 4.9 data.

```
> plt = ggplot(bps5.4.9, aes(x=sex))
> plt = plt + geom_bar()
> plt = plt + xlab("Sex")
> print(plt)
```



Figure 14: Boxplots of metabolic rate by sex for the data from BPS5 problem 4.9.


Figure 15: Histogram of metabolic rate for the data from BPS5 problem 4.9.

GGPLOT2 also provides scatterplots that can be enhanced with things like LOESS smooths (Figure 17).

```
> plt = ggplot(bps5.4.9, aes(mass, rate, shape=sex, linetype=sex))
> plt = plt + xlab("Mass (kilograms)") + ylab("Metabolic Rate (calories)")
> plt = plt + geom_point(size=3)
> plt = plt + stat_smooth(span=0.8, colour="black", lwd=0.25)
> print(plt)
```

As with the lattice package, it is possible to create separate plots for each of the sexes by using (Figure 18).

```
> plt = ggplot(bps5.4.9, aes(mass, rate)) + facet_grid(sex~.)
> plt = plt + xlab("Mass (kilograms)") + ylab("Metabolic Rate (calories)")
> plt = plt + geom_point(size=3)
> plt = plt + stat_smooth(span=0.8, colour="black", lwd=0.25)
> plt = plt + theme_bw()
> print(plt)
```



Figure 16: Bar chart of sex for the data from BPS5 problem 4.9.

## 4 Simple Univariate Descriptives

Summary statistics for the htwt data can be obtained via the summary function.

```
> summary(htwt)
```

| Height |  | Weight |  | Group |  |
| :--- | :--- | :--- | :--- | ---: | :---: |
| Min. $\quad: 51.0$ | Min. $\quad: 82.0$ | Min. $: 1.00$ |  |  |  |
| 1st Qu. $: 56.0$ | 1st Qu. $: 108.2$ | 1st Qu.: 1.00 |  |  |  |
| Median $: 59.5$ | Median $: 123.5$ | Median $: 2.00$ |  |  |  |
| Mean $: 62.1$ | Mean $: 139.6$ | Mean | $: 1.55$ |  |  |
| 3rd Qu. $: 68.0$ | 3rd Qu. $: 166.8$ | 3rd Qu. $: 2.00$ |  |  |  |
| Max. $\quad: 79.0$ | Max. $: 228.0$ | Max. $: 2.00$ |  |  |  |

> summary(subset(htwt,Group=="Male"))

| Height | Weight | Group |
| :---: | :---: | :---: |
| Min. : NA | Min. : NA | Min. |
| st Qu.: NA | 1st Qu.: NA | 1st Qu.: NA |
| dian : NA | Median : NA | Median : NA |
| Mean : NaN | Mean : NaN | Mean |
| rd Qu.: NA | 3rd Qu.: NA | 3rd Qu.: |
| Max. : NA | Max. : NA | Max |

> summary(subset(htwt,Group=="Female"))

| Height | Weight | Group |
| :---: | :---: | :---: |
| Min. : NA | Min. : NA | Min. |
| 1st Qu.: NA | 1st Qu.: NA | 1st Qu. |
| Median : NA | Median : NA | Median |
| Mean : NaN | Mean : NaN | Mean |
| u.: NA | 3rd Qu.: NA | 3rd Qu. |
| Max. : NA | Max. : NA | Max |



Figure 17: LOESS fits with approximate $95 \%$ confidence bounds for all data by sex.


Figure 18: LOESS fits with approximate $95 \%$ confidence bounds for all data by sex.

Note that subsets of the data can be summarized using the Group option.
Specific values may be obtained by using specialized functions. The sample mean is computed using the mean function. The same value can be found through the use of the sum function.

```
> mean(htwt$Weight)
```

[1] 139.6
> sum(htwt\$Weight)
[1] 2792

```
> length(htwt$Weight)
```

[1] 20
> sum(htwt\$Weight)/length(htwt\$Weight)
[1] 139.6
> colMeans(htwt[,1:2])
Height Weight
62.1139 .6

We now compute the variance by summing the squared deviations from the mean and dividing by $n-1$. Computing the mean once and assigning it to xbar and then calling xbar is more efficient than using mean(htwt\$Weight) in the sum.

```
> xbar = mean(htwt$Weight)
> sum((htwt$Weight-xbar)^2)
```

[1] 35330.8
$>\operatorname{sum}((h t w t \$ W e i g h t-x b a r) \wedge 2) /($ length(htwt\$Weight)-1)
[1] 1859.516
Or, we can use the var fucntion to compute the variance.
> var(htwt\$Weight)
[1] 1859.516
> apply(htwt[,1:2],2,var)
Height Weight
71.252631859 .51579

The standard deviation is the square root of the variance. Thus, it is simple to compute the standard deviation for the Weight data.

```
> sqrt(var(htwt$Weight))
```

[1] 43.1221

```
> sqrt(apply(htwt[,1:2],2,var))
```

```
    Height Weight
8.441127 43.122103
```

When outliers or skewness are present, the above measures of centrality and spread become suspect. At these times we often turn to the median and the IQR. R makes it easy to compute these values.

We can compute the median and quartiles by sorting and then counting. The sort function makes this easy.

```
> sort(htwt$Weight)
```

[1] 8288787101103110112119119122125151155157159190191195199 [20] 228

However, for large data sets this may be problematic. Using the R functions median and quantile are more efficient.

```
> median(htwt$Weight)
[1] 123.5
> quantile(htwt$Weight)
    0% 25% 50% 75% 100%
82.00 108.25 123.50 166.75 228.00
> apply(htwt[,1:2],2,median)
Height Weight
    59.5 123.5
```

Rcmdr has the function numSummary which can be called from the Rcmdr menu -Statistics - Summaries - Numerical Summaries. It can also be called from the command prompt. numSummary computes all of the above statistics with a single call.

```
> numSummary(htwt[,"Weight"], statistics=c("mean", "sd", "quantiles"))
    mean sd 0% 25% 50% 75% 100% n
139.6 43.1221 82 108.25 123.5 166.75 228 20
```

While it is possible to use mean, var, etc. and get results by group, using numSummary with groups= is easier.

```
> numSummary(htwt[,c("Height","Weight")], groups=htwt$Group,
+ statistics=c("mean", "sd", "quantiles"))
Variable: Height
    mean sd 0% 25% 50% 75% 100% n
1 65.00000 8.972179 52 59 64 71 79 9
2 59.72727 7.564270}51 51 54 58 64 76 11
Variable: Weight
    mean sd 0% 25% 50% 75% 100% n
1 155 48.99235 87 119.0 159 191 228 9
2 127 34.99714 82 106.5 119 153 199 11
```


## 5 Tables

Tables can be created both from the command line and from Rcmdr. We will take a look at the hospitals data set.

### 5.1 Manual Tables

The hospitals data frame contains three variables and 2900 observations.

```
> hospitals =
+ read.csv("http://newton.uor.edu/facultyfolder/jim_bentley/downloads/math111/hospi
> names(hospitals)
[1] "hospital" "condition" "survival"
> hospitals[c(1:3,2900),]
    hospital condition survival
1 A Good Survived
2 A Good Survived
3 A Good Survived
2900 B Poor Died
```

To get simple frequencies of each of the variables we can enter

```
> table(hospitals[,"hospital"])
```

```
    A B
2100 800
> table(hospitals[,"condition"])
```

```
Good Poor
1200 1700
> table(hospitals[,"survival"])
    Died Survived
        79 2821
```

Two way tables are created by providing two columns of data. Examples might be survival by hospital or survival by condition.

```
> table(hospitals[,c("hospital","survival")])
    survival
hospital Died Survived
    A 63 2037
    B 16 784
> table(hospitals[,c("survival","condition")])
        condition
survival Good Poor
    Died 14 65
    Survived 1186 1635
```

Notice that the order of the columns determines the rows and columns respectively.
The table function will also generate three-way tables.

```
> table(hospitals[,c("survival","hospital","condition")])
```

, , condition $=$ Good

|  | hospital |  |
| :---: | ---: | ---: |
| survival | A | B |
| Died | 6 | 8 |
| Survived | 594 | 592 |

```
, , condition = Poor
```

        hospital
    survival A B
Died 578
Survived 1443192

The table function assumes that the columns are entered as rows, columns, and tables respectively.

While the table function is good for getting counts, it does not generate row, column, or table percentages. Rcmdr does this through the use of the xtabs, colPercents, and rowPercents functions which are accessible through its menus - Statistics - Contingency Tables. These functions can also be called from the command line.

We first generate a counts table using xtabs.

```
> .Table = xtabs(~survival+hospital+condition, data=hospitals)
> .Table
```

, , condition = Good
hospital

| survival | A | B |
| :--- | ---: | ---: |
| Died | 6 | 8 |
| Survived | 594 | 592 |

```
, , condition = Poor
```

        hospital
    survival A B
Died 578
Survived 1443192

To get column percents we us colPercents on the saved table.

```
> colPercents(.Table)
, , condition = Good
        hospital
survival A B
    Died 1 1.3
    Survived 99 98.7
    Total 100 100.0
    Count 600 600.0
```

, , condition $=$ Poor
hospital
survival A B
Died $3.8 \quad 4$
Survived 96.296
Total 100.0100
Count 1500.0200

Row percents can be generated in a similar manner by using rowPercents.

```
> rowPercents(.Table)
```

, , condition = Good
hospital

```
survival A B Total Count
    Died 42.9 57.1 100 14
    Survived 50.1 49.9 100 1186
, , condition = Poor
        hospital
survival A B Total Count
    Died 87.7 12.3 100 65
    Survived 88.3 11.7 100 1635
```

Finally, we can compute table percentages for two-way tables by using totPercents. We clean up by removing the table with rm.

```
> .Table = xtabs(~survival+hospital, data=hospitals)
> totPercents(.Table)
```

|  | A | B | Total |
| :--- | ---: | ---: | ---: |
| Died | 2.2 | 0.6 | 2.7 |
| Survived | 70.2 | 27.0 | 97.3 |
| Total | 72.4 | 27.6 | 100.0 |

> rm(.Table)

## 6 Testing The Population Mean

### 6.1 The One Sample Test

A simple test for the population mean of the Weight variable in the htwt data can be obtained via the t.test function. To compute the one sample t-test of $H_{0}: \mu=145$ we enter:

```
> t.test(htwt$Weight, mu=145, alternative='two.sided',
+ conf.level=.95)
    One Sample t-test
data: htwt$Weight
t = -0.56, df = 19, p-value = 0.582
alternative hypothesis: true mean is not equal to 145
95 percent confidence interval:
    119.4182 159.7818
sample estimates:
mean of x
    139.6
```

An equivalent test of $H_{0}: \mu=145$ may be carried out using a linear model via the 1 m function.

```
> summary(lm((Weight-145)~1, data=htwt))
Call:
lm(formula = (Weight - 145) ~ 1, data = htwt)
Residuals:
    Min 1Q Median 3Q Max
-57.60 -31.35 -16.10 27.15 88.40
Coefficients:
            Estimate Std. Error t value Pr (> |t|)
(Intercept) -5.400 9.642 -0.56 0.582
```

Residual standard error: 43.12 on 19 degrees of freedom
Notice that adding the coefficient from the model to the hypothesized mean gives the sample mean. That is $145+(-5.4)=139.6$. Note, too that the p-values computed by t .test and lm are the same $(p=0.582)$.

### 6.2 The Two Sample Test

A simple test to compare the male and female population means of theWeight variable in the htwt data can also be obtained via the t.test function. To compute the two sample t-test of $H_{0}: \mu_{m}=\mu_{f}$ we enter:

```
> t.test(Weight~Group, alternative='two.sided', conf.level=.95,
+ var.equal=TRUE, data=htwt)
    Two Sample t-test
data: Weight by Group
t = 1.4903, df = 18, p-value = 0.1534
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
    -11.4713 67.4713
sample estimates:
mean in group 1 mean in group 2
    155 127
```

An equivalent test of $H_{0}: \beta_{1}=0=\mu_{m}-\mu_{f}$ may be carried out using a linear model via the lm function.

```
> summary(lm(Weight~Group, data=htwt))
```

Call:
$\operatorname{lm}(f o r m u l a=$ Weight $\sim$ Group, data $=$ htwt $)$
Residuals:


Signif. codes: $0{ }^{\prime * * * '} 0.001{ }^{\prime * * '} 0.01 '^{\prime \prime} 0.05$ '.' 0.1 ' ' 1
Residual standard error: 41.8 on 18 degrees of freedom
Multiple R-squared: 0.1098, Adjusted R-squared: 0.06039
F-statistic: 2.221 on 1 and 18 DF, p-value: 0.1534
Notice that intercept term (155) is the sample mean for the males. The sample mean for the females is the model evaluated for a female $(155+(-28)=127)$. As in the one sample problem the p-values computed by t.test and lm are the same ( $p=0.153$ ).

### 6.3 Correcting for Height

It is fairly clear from graphing Weight as a function of Height that when modeling a person's weight we should correct for height. While this cannot be accomplished using a t-test, a linear model makes the correction fairly easy.

To test for $H_{0}: \beta_{1}=0$ when controlling for Height using the model

$$
\text { Weight }=\beta_{0}+\beta_{1} \text { Female }+\beta_{2} \text { Height }+\epsilon
$$

we compute
> summary(lm(Weight $\sim 1+$ Group+Height, data=htwt))
Call:
lm(formula $=$ Weight ~ 1 + Group + Height, data = htwt)
Residuals:


Height $5.0108 \quad 0.2103 \quad 23.826 \quad 1.68 \mathrm{e}-14$ ***
---
Signif. codes: $0{ }^{\prime * * * '} 0.001{ }^{\prime * * '} 0.01 '^{\prime \prime} 0.05 '^{\prime} 0.1$ ' 1
Residual standard error: 7.334 on 17 degrees of freedom
Multiple R-squared: 0.9741, Adjusted R-squared: 0.9711
F-statistic: 319.9 on 2 and 17 DF, p-value: $3.239 \mathrm{e}-14$
Notice that as before there does note appear to be a difference between females and males $(p=0.655)$. However, it is clear that Height is predictive of Weight ( $p<0.001$ ).

### 6.4 Interaction Terms

At this point we may be convinced that no differences exist in the weights of our two groups. Clearly the means for this sample are not significantly different. A little more insight may be gained by including an interaction term.

We now fit the model

```
            Weight = \beta
> lm.htwt = lm(Weight~1+Group*Height, data=htwt)
> summary(lm.htwt)
Call:
lm(formula = Weight ~ 1 + Group * Height, data = htwt)
Residuals:
\begin{tabular}{rrrr} 
Min & 1Q Median & 3Q & Max \\
-9.968 & -3.413 & -1.104 & 2.697 \\
13.163
\end{tabular}
Coefficients:
\begin{tabular}{lrrrrr} 
& Estimate & Std. Error t value \(\operatorname{Pr}(>|\mathrm{t}|)\) \\
(Intercept) & -252.7467 & 37.1333 & -6.806 & \(4.22 \mathrm{e}-06\) & \(* * *\) \\
Group & 54.4858 & 23.2997 & 2.338 & 0.0327 & \(*\) \\
Height & 6.3360 & 0.5766 & 10.989 & \(7.28 \mathrm{e}-09 \quad * * *\) \\
Group: Height & -0.9013 & 0.3713 & -2.427 & \(0.0274 *\)
\end{tabular}
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 6.463 on 16 degrees of freedom
Multiple R-squared: 0.9811, Adjusted R-squared: 0.9775
F-statistic: 276.6 on 3 and 16 DF, p-value: 5.425e-14
```

It is now clear that not only is height predictive of weight ( $p<0.0001$ ), more importantly, females and males put weight on differently. Since the interaction term is
significant ( $p=0.0274$ ) this indicates that their slopes are different with the women putting on about one pound less per inch than the men.

Diagnostic plots can be generated by using the plot function on the lm object, lm.htwt. Figure 19 shows the four diagnostic plots that are the default. The analysis of variance table may also be generated.

```
> # Set up the page to take all four images
> par(mfrow=c(2,2))
> plot(lm.htwt)
> anova(lm.htwt)
Analysis of Variance Table
Response: Weight
\begin{tabular}{lrrrrcc} 
& Df & Sum Sq Mean Sq & F value & \(\operatorname{Pr}(>F)\) & \\
Group & 1 & 3880.8 & 3880.8 & 92.9116 & \(4.570 \mathrm{e}-08\) & \(* * *\) \\
Height & 1 & 30535.6 & 30535.6 & 731.0636 & \(8.778 \mathrm{e}-15\) & \(* * *\) \\
Group:Height & 1 & 246.1 & 246.1 & 5.8921 & \(0.02738 \quad *\) \\
Residuals & 16 & 668.3 & 41.8 & & &
\end{tabular}
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The question of mean differences is thus shown to be the wrong question. The investigator should have been looking to see if men and women put on an equivalent number of pounds for each inch difference in height. This is something that is not apparent when looking at t-tests.

## 7 Fitting Logistic Models Using GLM

The examples that follow are based upon data from the Titanic disaster. Importing of the data into R can be carried out using the following code.

```
> titanic =
+ read.csv("http://newton.uor.edu/facultyfolder/jim_bentley/downloads/math111/tit
> titanic$AGE=factor(titanic$AGE,labels=c('Child','Adult'))
> titanic$CLASS=factor(titanic$CLASS,labels=c('0','1','2','3'))
> titanic$SEX=factor(titanic$SEX, labels=c('Female','Male'))
> titanic$SURVIVED=factor(titanic$SURVIVED,labels=c('No','Yes'))
```

The models fitted here give results that are equivalent to those obtained by using SAS or NCSS.

### 7.1 CLASS

A model to test for the difference in odds of survival as determined by class may be fitted using the glm function with binomial error and logit link.

```
> titanic.logistic.class=glm(SURVIVED~}\mp@subsup{}{~}{CLASS,
+ family=binomial(logit),data=titanic)
> summary(titanic.logistic.class)
Call:
glm(formula = SURVIVED ~ CLASS, family = binomial(logit), data = titanic)
Deviance Residuals:
\begin{tabular}{rrrrr} 
Min & 1Q & Median & 3Q & Max \\
-1.3999 & -0.7623 & -0.7401 & 0.9702 & 1.6906
\end{tabular}
Coefficients:
Estimate Std. Error z value Pr(> | | )
(Intercept) -1.15516 0.07876 -14.667 < 2e-16 ***
CLASS1 1.66434 0.13902 11.972 < 2e-16 ***
CLASS2 0.80785 0.14375 5.620 1.91e-08 ***
CLASS3 0.06785 0.11711 0.579 0.562
---
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
(Dispersion parameter for binomial family taken to be 1)
    Null deviance: 2769.5 on 2200 degrees of freedom
Residual deviance: 2588.6 on 2197 degrees of freedom
AIC: 2596.6
Number of Fisher Scoring iterations: 4
Note that the (log) odds of survival do not differ for classes 0 (viewed as baseline)
and 3. However, classes }1\mathrm{ and 2 differ from 0 (and thus 3) as well as from each other.
This can most easily be seen using the odds ratios.
```

```
> coefs=summary(titanic.logistic.class)$coef
```

> coefs=summary(titanic.logistic.class)\$coef
> est=exp(coefs[,1])
> est=exp(coefs[,1])
> upper.ci=exp(coefs[,1]+1.96*coefs[,2])
> upper.ci=exp(coefs[,1]+1.96*coefs[,2])
> lower.ci<-exp(coefs[,1]-1.96*coefs[,2])
> lower.ci<-exp(coefs[,1]-1.96*coefs[,2])
> cbind(est,lower.ci,upper.ci)
> cbind(est,lower.ci,upper.ci)
est lower.ci upper.ci
est lower.ci upper.ci
(Intercept) 0.3150074 0.2699482 0.3675878
(Intercept) 0.3150074 0.2699482 0.3675878
CLASS1 5.2822069 4.0223687 6.9366366
CLASS1 5.2822069 4.0223687 6.9366366
CLASS2 2.2430799 1.6923031 2.9731124
CLASS2 2.2430799 1.6923031 2.9731124
CLASS3 1.0702008 0.8507054 1.3463295
CLASS3 1.0702008 0.8507054 1.3463295
> rm(coefs)

```
> rm(coefs)
```

While the odds for class 3 relative to class 0 are essentially 1:1, class 1 has a 5.28:1 odds of survival and class 2 has a $2.24: 1$ odds of survival relative to class 0 .

### 7.2 AGE and SEX

A model to test for the difference in odds of survival as determined by age and sex may be fitted using the glm function with binomial error and logit link.

```
> titanic.logistic.agesex=glm(SURVIVED~AGE*SEX,
+ family=binomial(logit),data=titanic)
> summary(titanic.logistic.agesex)
Call:
glm(formula = SURVIVED ~ AGE * SEX, family = binomial(logit),
    data = titanic)
```

Deviance Residuals:

| Min | 1Q | Median | 3Q | Max |
| ---: | ---: | ---: | ---: | ---: |
| -1.6497 | -0.6732 | -0.6732 | 0.7699 | 1.7865 |

Coefficients:

|  | Estimate | Std. Error | z value | $\operatorname{Pr}(>\|z\|)$ |  |
| :--- | ---: | ---: | ---: | ---: | :--- |
| (Intercept) | 0.4990 | 0.3075 | 1.623 | 0.1046 |  |
| AGEAdult | 0.5654 | 0.3269 | 1.729 | 0.0837 | . |
| SEXMale | -0.6870 | 0.3970 | -1.731 | 0.0835 | . |
| AGEAdult:SEXMale | -1.7465 | 0.4167 | -4.191 | $2.77 e-05$ | *** |

Signif. codes: $0{ }^{\prime * * * ' ~} 0.001$ '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
(Dispersion parameter for binomial family taken to be 1)
Null deviance: 2769.5 on 2200 degrees of freedom
Residual deviance: 2312.8 on 2197 degrees of freedom AIC: 2320.8

Number of Fisher Scoring iterations: 4
This model may also be expressed as
> titanic.logistic.agesex2=glm(SURVIVED~AGE+SEX+AGE:SEX,

+ family=binomial(logit),data=titanic)
> summary(titanic.logistic.agesex2)
Call:
glm(formula = SURVIVED ~ AGE + SEX + AGE:SEX, family = binomial(logit), data $=$ titanic)

Deviance Residuals:

| Min | $1 Q$ | Median | 3Q | Max |
| ---: | ---: | ---: | ---: | ---: |
| -1.6497 | -0.6732 | -0.6732 | 0.7699 | 1.7865 |

Coefficients:

|  | Estimate | Std. Error | $z$ | value |
| :--- | ---: | ---: | ---: | ---: |
|  | $\operatorname{Pr}(>\|z\|)$ |  |  |  |
| (Intercept) | 0.4990 | 0.3075 | 1.623 | 0.1046 |
| AGEAdult | 0.5654 | 0.3269 | 1.729 | 0.0837. |
| SEXMale | -0.6870 | 0.3970 | -1.731 | 0.0835 | .

---
Signif. codes: $0{ }^{\prime * * * '} 0.001{ }^{\prime * * '} 0.01 '^{\prime *} 0.05 '^{\prime} 0.1^{\prime} 1$
(Dispersion parameter for binomial family taken to be 1)

Null deviance: 2769.5 on 2200 degrees of freedom
Residual deviance: 2312.8 on 2197 degrees of freedom AIC: 2320.8

Number of Fisher Scoring iterations: 4
The odds associated with the model are

```
> coefs=summary(titanic.logistic.agesex2)$coef
> est=exp(coefs[,1])
> upper.ci=exp(coefs[,1]+1.96*coefs[,2])
> lower.ci<-exp(coefs[,1]-1.96*coefs[,2])
> cbind(est,lower.ci,upper.ci)
```

|  | est | lower.ci | upper.ci |
| :--- | ---: | ---: | ---: |
| (Intercept) | 1.6470588 | 0.90154072 | 3.0090740 |
| AGEAdult | 1.7601573 | 0.92740960 | 3.3406529 |
| SEXMale | 0.5030612 | 0.23104993 | 1.0953069 |
| AGEAdult:SEXMale | 0.1743855 | 0.07705575 | 0.3946531 |
|  |  |  |  |
| > rm(coefs) |  |  |  |

### 7.3 CLASS, AGE and SEX

A model to test for the difference in odds of survival as determined by class, age and sex may be fitted using the glm function with binomial error and logit link.

```
> titanic.logistic.classagesex=glm(SURVIVED~AGE*SEX+CLASS*SEX+CLASS:AGE,
+ family=binomial(logit),data=titanic)
> summary(titanic.logistic.classagesex)
Call:
glm(formula = SURVIVED ~ AGE * SEX + CLASS * SEX + CLASS:AGE,
    family = binomial(logit), data = titanic)
```

| Deviance | Residuals: |  |  |  |
| ---: | ---: | ---: | ---: | ---: |
| Min | $1 Q$ | Median | $3 Q$ | Max |
| -2.6771 | -0.7099 | -0.5952 | 0.2374 | 2.2293 |

Coefficients: (1 not defined because of singularities)

| (Intercept) | 1.86087 | 0.73347 | 2.537 | $0.01118 *$ |
| :--- | ---: | ---: | ---: | :--- |
| AGEAdult | 0.03625 | 0.39325 | 0.092 | 0.92655 |
| SEXMale | -2.46011 | 0.81614 | -3.014 | $0.00258 * *$ |
| CLASS1 | 17.99982 | 920.38680 | 0.020 | 0.98440 |
| CLASS2 | 17.11036 | 405.66287 | 0.042 | 0.96636 |
| CLASS3 | -2.05502 | 0.63854 | -3.218 | $0.00129 * *$ |
| AGEAdult:SEXMale | -0.68679 | 0.52541 | -1.307 | 0.19116 |
| SEXMale:CLASS1 | -1.13608 | 0.82048 | -1.385 | 0.16616 |
| SEXMale:CLASS2 | -1.06807 | 0.74658 | -1.431 | 0.15254 |
| SEXMale:CLASS3 | 1.66387 | 0.65601 | 2.536 | $0.01120 *$ |
| AGEAdult:CLASS1 | -16.34159 | 920.38645 | -0.018 | 0.98583 |
| AGEAdult:CLASS2 | -17.19040 | 405.66230 | -0.042 | 0.96620 |
| AGEAdult:CLASS3 | NA | NA | NA | NA |

Signif. codes: $0{ }^{\prime * * * '} 0.001{ }^{\prime * * '} 0.01 '^{\prime \prime} 0.05$ '.' $0.1^{\prime} 1$
(Dispersion parameter for binomial family taken to be 1)
Null deviance: 2769.5 on 2200 degrees of freedom
Residual deviance: 2097.5 on 2189 degrees of freedom AIC: 2121.5

Number of Fisher Scoring iterations: 15
The odds associated with the model are

```
> coefs=summary(titanic.logistic.classagesex)$coef
> est=exp(coefs[,1])
> upper.ci=exp(coefs[,1]+1.96*coefs[,2])
> lower.ci<-exp(coefs[,1]-1.96*coefs[,2])
> cbind(est,lower.ci,upper.ci)
```

|  | est | lower.ci | upper.ci |
| :--- | ---: | ---: | ---: |
| (Intercept) | $6.429309 \mathrm{e}+00$ | 1.52693798 | 27.0711771 |
| AGEAdult | $1.036918 \mathrm{e}+00$ | 0.47973665 | 2.2412280 |
| SEXMale | $8.542543 \mathrm{e}-02$ | 0.01725325 | 0.4229640 |
| CLASS1 | $6.564808 \mathrm{e}+07$ | 0.00000000 | Inf |
| CLASS2 | $2.697330 \mathrm{e}+07$ | 0.00000000 | Inf |
| CLASS3 | $1.280899 \mathrm{e}-01$ | 0.03664230 | 0.4477617 |

```
AGEAdult:SEXMale 5.031883e-01 0.17967631 1.4091922
SEXMale:CLASS1 3.210755e-01 0.06429782 1.6033118
SEXMale:CLASS2 3.436711e-01 0.07955044 1.4847160
SEXMale:CLASS3 5.279697e+00 1.45950127 19.0991297
AGEAdult:CLASS1 7.997187e-08 0.00000000 Inf
AGEAdult:CLASS2 3.422187e-08 0.00000000 Inf
> rm(coefs)
```


## 8 Fitting Logistic Models Using LRM

The models fitted here are the equivalent of those fitted above using GLM. Maybe the greatest advantage of the use of LRM is the ability to generate nomograms - if you like them that is.

### 8.1 CLASS

A model to test for the difference in odds of survival as determined by class may be fitted using the lrm function.

```
> library(rms)
> dd = datadist(titanic)
> options(datadist="dd")
> attach(titanic)
> titanic.lrm.class=lrm(SURVIVED~CLASS, x=TRUE, y=TRUE)
> titanic.lrm.class
```

Logistic Regression Model
$\operatorname{lrm}($ formula $=$ SURVIVED $\sim$ CLASS, $\mathrm{x}=$ TRUE, $\mathrm{y}=$ TRUE)

|  |  | Model Likelihood |  | Discrimination |  | Rank Discrim. |  |
| :---: | ---: | :--- | ---: | :--- | ---: | :--- | ---: |
|  |  | Ratio Test |  | Indexes |  | Indexes |  |


|  | Coef | S.E. | Wald Z | $\operatorname{Pr}(>\mid$ Z\|) |
| :--- | ---: | :--- | ---: | :--- |
| Intercept | -1.1552 | 0.0788 | -14.67 | $<0.0001$ |
| CLASS $=1$ | 1.6643 | 0.1390 | 11.97 | $<0.0001$ |
| CLASS=2 | 0.8078 | 0.1438 | 5.62 | $<0.0001$ |
| CLASS=3 | 0.0678 | 0.1171 | 0.58 | 0.5624 |
|  |  |  |  |  |
| > anova(titanic.lrm.class) |  |  |  |  |


| Factor | Chi-Square | d.f. | P |
| :--- | :--- | :--- | :--- |
| CLASS | 173.23 | 3 | $<.0001$ |
| TOTAL | 173.23 | 3 | $<.0001$ |

Note that the (log) odds of survival do not differ for classes 0 (viewed as baseline) and 3 . However, classes 1 and 2 differ from 0 (and thus 3 ) as well as from eachother. This can most easily be seen using the odds ratios.

```
> summary(titanic.lrm.class,CLASS='0')
    Effects Response : SURVIVED
    Factor Low High Diff. Effect S.E. Lower 0.95 Upper 0.95
        CLASS - 1:0 1 2 NA 1.66 0.14 1.39 1.94
        Odds Ratio 1 2 NA 5.28 NA 4.02 6.94
        CLASS - 2:0 1 3 NA 0.81 0.14 0.53 1.09
        Odds Ratio 1 3 NA 2.24 NA 1.69 2.97
CLASS - 3:0 1 4 NA 0.07 0.12 -0.16 0.30
    Odds Ratio 1 4 NA 1.07 NA 0.85 1.35
> summary(titanic.lrm.class,CLASS='3')
    Effects Response : SURVIVED
Factor Low High Diff. Effect S.E. Lower 0.95 Upper 0.95
CLASS - 0.3 4 1 NA -0.07 0.12 -0.30 0.16
\begin{tabular}{rrlrrrr} 
Odds Ratio 4 & 1 & NA & 0.93 & NA & 0.74 & 1.18 \\
CLASS - 1.3 4 & 2 & NA & 1.60 & 0.14 & 1.31 & 1.88 \\
Odds Ratio 4 & 2 & NA & 4.94 & NA & 3.72 & 6.54 \\
CLASS - 2.3 4 & 3 & NA & 0.74 & 0.15 & 0.45 & 1.03 \\
Odds Ratio 4 & 3 & NA & 2.10 & NA & 1.57 & 2.80
\end{tabular}
> summary(titanic.lrm.class,CLASS='1')
    Effects Response : SURVIVED
\begin{tabular}{llllllrrr} 
Factor & Low & High & Diff. & Effect & S.E. Lower & 0.95 & Upper & 0.95 \\
CLASS - \(0: 1\) & 2 & 1 & NA & -1.66 & 0.14 & -1.94 & -1.39 \\
Odds Ratio & 2 & 1 & NA & 0.19 & NA & 0.14 & 0.25 \\
CLASS - \(2: 1\) & 2 & 3 & NA & -0.86 & 0.17 & -1.18 & -0.53 \\
Odds Ratio & 2 & 3 & NA & 0.42 & NA & 0.31 & 0.59 \\
CLASS - 3:1 & 2 & 4 & NA & -1.60 & 0.14 & -1.88 & -1.31 \\
Odds Ratio 2 & 4 & NA & 0.20 & NA & 0.15 & 0.27
\end{tabular}
> summary(titanic.lrm.class,CLASS='2')
```

|  | Effects |  |  | Response : SURVIVED |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Factor | Low | High | Diff. | Effect | S.E. | Lower | 0.95 | Upper | 0.95 |
| CLASS - 0:2 | 3 | 1 | NA | -0.81 | 0.14 | -1.09 |  | -0.53 |  |
| Odds Ratio | 3 | 1 | NA | 0.45 | NA | 0.34 |  | 0.59 |  |
| CLASS - 1:2 | 3 | 2 | NA | 0.86 | 0.17 | 0.53 |  | 1.18 |  |
| Odds Ratio | 3 | 2 | NA | 2.35 | NA | 1.70 |  | 3.26 |  |
| CLASS - 3:2 | 3 | 4 | NA | -0.74 | 0.15 | -1.03 |  | -0.45 |  |
| Odds Ratio | 3 | 4 | NA | 0.48 | NA | 0.36 |  | 0.64 |  |

While the odds for class 3 relative to class 0 are essentially $1: 1$, class 1 has a 5.28:1 odds of survival and class 2 has a 2.24:1 odds of survival relative to class 0 .

The probability of survival for the different classes may be plotted (Figure 20).

```
> print(plot(Predict(titanic.lrm.class, fun=plogis),
+ ylab='Probability of Survival'))
```

And we should validate the model.
> validate(titanic.lrm.class, $B=80$ )

|  | index.orig | training | test | optimism | index.corrected |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Dxy | $n$ |  |  |  |  |
| R2 | 0.2833 | 0.2833 | 0.2798 | 0.0034 | 0.279980 |
| Intercept | 0.1102 | 0.1089 | 0.1090 | -0.0001 | 0.1103 |
| 80 |  |  |  |  |  |
| Slope | 0.0000 | 0.0000 | 0.0134 | -0.0134 | 0.013480 |
| Emax | 1.0000 | 1.0000 | 1.0088 | -0.0088 | 1.008880 |
| D | 0.0000 | 0.0000 | 0.0044 | 0.0044 | 0.004480 |
| U | 0.0817 | 0.0808 | 0.0808 | 0.0000 | 0.0818 |
| 80 |  |  |  |  |  |
| Q | -0.0009 | -0.0009 | 0.0002 | -0.0011 | 0.000280 |
| B | 0.0826 | 0.0817 | 0.0806 | 0.0011 | 0.081680 |
| g | 0.1998 | 0.1995 | 0.2002 | -0.0007 | 0.200480 |
| gp | 0.5450 | 0.5463 | 0.5469 | -0.0006 | 0.545580 |
| D | 0.1240 | 0.1236 | 0.1241 | -0.0004 | 0.124480 |

A nomogram may be helpful at this point (Figure 21.

```
> nom = nomogram(titanic.lrm.class, fun=plogis)
> print(plot(nom))
```

NULL

### 8.2 AGE and SEX

A model to test for the difference in odds of survival as determined by age and sex may be fitted using the lmr function.

```
> dd = datadist(titanic)
> options(datadist="dd")
> attach(titanic)
```

The following object(s) are masked from 'titanic (position 4)':
AGE, CLASS, SEX, SURVIVED
> titanic.lrm.agesex=lrm(SURVIVED~AGE*SEX, x=TRUE, $y=T R U E)$
> titanic.lrm.agesex
Logistic Regression Model
$\operatorname{lrm}($ formula $=$ SURVIVED $\sim$ AGE $*$ SEX, $x=$ TRUE, $y=$ TRUE)

|  |  | Model Likelihood Ratio Test |  | Discrimination Indexes |  | Rank Discrim. Indexes |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Obs | 2201 | LR chi2 | 456.68 | R2 | 0.262 | C | 0.713 |
| No | 1490 | d.f. | 3 | g | 0.841 | Dxy | 0.427 |
| Yes | 711 | Pr(> chi2) | <0.0001 | gr | 2.320 | gamma | 0.787 |
| max \|deriv| | $1 \mathrm{e}-10$ |  |  | gp | 0.187 | tau-a | 0.187 |
|  |  |  |  | Brier | 0.171 |  |  |

    Coef S.E. Wald \(Z \operatorname{Pr}(>|Z|)\)
    $\begin{array}{lllll}\text { Intercept } & 0.4990 & 0.3075 & 1.62 & 0.1046\end{array}$
$\begin{array}{lllll}\text { AGE=Adult } & 0.5654 & 0.3269 & 1.73 & 0.0837\end{array}$
$\begin{array}{llllll}\text { SEX=Male } & -0.6870 & 0.3970 & -1.73 & 0.0835\end{array}$
AGE=Adult * SEX=Male -1.7465 0.4167 -4.19 <0.0001
> anova(titanic.lrm.agesex)

Wald Statistics Response: SURVIVED

| Factor | Chi-Square d.f. P |  |  |
| :--- | ---: | :--- | :--- |
| AGE (Factor+Higher Order Factors) | 23.88 | 2 | $<.0001$ |
| All Interactions | 17.57 | 1 | $<.0001$ |
| SEX (Factor+Higher Order Factors) | 371.97 | 2 | $<.0001$ |
| All Interactions | 17.57 | 1 | $<.0001$ |
| AGE * SEX (Factor+Higher Order Factors) | 17.57 | 1 | $<.0001$ |
| TOTAL | 391.59 | 3 | $<.0001$ |

The odds associated with the model are

```
> summary(titanic.lrm.agesex, AGE='Adult', SEX='Male')
```



The probability of survival for the different combinations of sex and age group may be plotted (Figure 22).

```
> Predict(titanic.lrm.agesex, fun=plogis,
+ AGE=c('Child','Adult'), SEX=c('Female','Male'))
    AGE SEX yhat lower upper
1 Child Female 0.6222222 0.4741134 0.7505638
2 Adult Female 0.7435294 0.6998704 0.7828084
3 Child Male 0.4531250 0.3362143 0.5754460
4 Adult Male 0.2027594 0.1841419 0.2227454
Response variable (y):
Limits are 0.95 confidence limits
> print(plot(Predict(titanic.lrm.agesex, fun=plogis,
+ AGE=c('Child','Adult'), SEX=c('Female','Male'))))
```

And we should validate the model.

```
> validate(titanic.lrm.agesex, B=80)
```

|  | index.orig | training | test |  | optimism |
| :--- | ---: | ---: | ---: | ---: | ---: |
| index. corrected | $n$ |  |  |  |  |
| Dxy | 0.4267 | 0.4277 | 0.4264 | 0.0013 | 0.4254 |
| 80 |  |  |  |  |  |
| R2 | 0.2618 | 0.2636 | 0.2608 | 0.0029 | 0.2589 |
| 80 |  |  |  |  |  |
| Intercept | 0.0000 | 0.0000 | -0.0012 | 0.0012 | -0.001280 |
| Slope | 1.0000 | 1.0000 | 0.9951 | 0.0049 | 0.995180 |
| Emax | 0.0000 | 0.0000 | 0.0013 | 0.0013 | 0.0013 |
| 80 |  |  |  |  |  |
| D | 0.2070 | 0.2088 | 0.2062 | 0.0026 | 0.2044 |
| 80 |  |  |  |  |  |
| U | -0.0009 | -0.0009 | -0.0001 | -0.0008 | -0.000180 |
| Q | 0.2079 | 0.2097 | 0.2062 | 0.0035 | 0.2045 |
| 80 |  |  |  |  |  |
| B | 0.1713 | 0.1708 | 0.1716 | -0.0008 | 0.1720 |
| 80 |  |  |  |  |  |
| g | 0.8414 | 0.8451 | 0.8370 | 0.0080 | 0.833480 |
| Gp | 0.1867 | 0.1871 | 0.1858 | 0.0013 | 0.185480 |

A nomogram may be helpful at this point (Figure 23).
> nom = nomogram(titanic.lrm.agesex, fun=plogis)
> print(plot(nom))
NULL

### 8.3 CLASS, AGE and SEX

A model to test for the difference in odds of survival as determined by class, age and sex may be fitted using the lmr function.

```
> dd = datadist(titanic)
> options(datadist="dd")
> attach(titanic)
```

The following object(s) are masked from 'titanic (position 4)':
AGE, CLASS, SEX, SURVIVED
The following object(s) are masked from 'titanic (position 6)':
AGE, CLASS, SEX, SURVIVED
> titanic.lrm.classagesex=lrm(SURVIVED~CLASS*SEX+AGE*SEX, x=TRUE, y=TRUE)
> titanic.lrm.classagesex
Logistic Regression Model


Obs
2201

| Model Likelihood |  |
| :---: | :---: |
| Ratio Test |  |
| LR chi2 634.70 |  |

Discrimination Rank Discrim.
Indexes Indexes
$\begin{array}{llll}\text { R2 } & 0.350 & C & 0.766\end{array}$

| No | 1490 | d.f. | 9 | g | 1.341 | Dxy | 0.532 |
| :--- | ---: | :--- | ---: | :--- | :--- | :--- | :--- |
| Yes | 711 | $\operatorname{Pr}(>$ chi2 $)<0.0001$ | gr | 3.823 | gamma | 0.638 |  |
| max \|deriv| | $5 \mathrm{e}-10$ |  |  | gp | 0.233 | tau-a | 0.233 |


|  | Coef | S.E. | Wald $\mathrm{Z} \mathrm{Pr}(>\mid \mathrm{Z\mid})$ |  |
| :--- | ---: | :--- | ---: | :--- |
| Intercept | 2.0775 | 0.7171 | 2.90 | 0.0038 |
| CLASS=1 | 1.6642 | 0.8003 | 2.08 | 0.0376 |
| CLASS=2 | 0.0497 | 0.6874 | 0.07 | 0.9424 |
| CLASS=3 | -2.0894 | 0.6381 | -3.27 | 0.0011 |
| SEX=Male | -1.7888 | 0.7728 | -2.31 | 0.0206 |
| AGE=Adult | -0.1803 | 0.3618 | -0.50 | 0.6182 |
| CLASS=1 * SEX=Male | -1.1033 | 0.8199 | -1.35 | 0.1784 |
| CLASS=2 * SEX=Male | -0.7647 | 0.7271 | -1.05 | 0.2929 |
| CLASS=3 * SEX=Male | 1.5623 | 0.6562 | 2.38 | 0.0173 |
| SEX=Male * AGE=Adult | -1.3581 | 0.4551 | -2.98 | 0.0028 |
|  |  |  |  |  |
| > anova(titanic.lrm. classagesex) |  |  |  |  |

Wald Statistics Response: SURVIVED

| Factor | Chi-Square d.f. P |  |  |
| :--- | ---: | :--- | :--- |
| CLASS (Factor+Higher Order Factors) | 124.28 | 6 | $<.0001$ |
| All Interactions | 48.25 | 3 | $<.0001$ |
| SEX (Factor+Higher Order Factors) | 254.38 | 5 | $<.0001$ |
| All Interactions | 63.47 | 4 | $<.0001$ |
| AGE (Factor+Higher Order Factors) | 31.30 | 2 | $<.0001$ |
| All Interactions | 8.91 | 1 | 0.0028 |
| CLASS * SEX (Factor+Higher Order Factors) | 48.25 | 3 | $<.0001$ |
| SEX * AGE (Factor+Higher Order Factors) | 8.91 | 1 | 0.0028 |
| TOTAL INTERACTION | 63.47 | 4 | $<.0001$ |
| TOTAL | 311.38 | 9 | $<.0001$ |

The odds associated with the model are
> summary(titanic.lrm.classagesex, CLASS='0', AGE='Adult', SEX='Male')
Effects Response : SURVIVED

Factor
Low High Diff. Effect S.E. Lower 0.95 Upper 0.95
CLASS - 1:0
Odds Ratio
CLASS - 2:0
Odds Ratio
CLASS - 3:0
Odds Ratio

| 1 | 2 | NA |
| :--- | :--- | :--- |
| 1 | 2 | NA |
| 1 | 3 | NA |
| 1 | 3 | NA |
| 1 | 4 | NA |
| 1 | 4 | NA |

0.56
0.18
0.21
0.91
1.75 NA $1.24 \quad 2.48$
$\begin{array}{llll}-0.72 & 0.24 & -1.18 & -0.25\end{array}$
$\begin{array}{llll}0.49 & \mathrm{NA} & 0.31 & 0.78\end{array}$
$\begin{array}{llll}-0.53 & 0.15 & -0.83 & -0.23\end{array}$
0.59 NA $0.44 \quad 0.80$

| SEX - Female:Male 2 | 1 | NA | 3.15 | 0.62 | 1.92 | 4.37 |  |
| :---: | ---: | :--- | :--- | ---: | ---: | ---: | ---: |
| Odds Ratio | 2 | 1 | NA | 23.26 | NA | 6.84 | 79.12 |
| AGE - Child:Adult | 2 | 1 | NA | 1.54 | 0.28 | 1.00 | 2.08 |
| Odds Ratio | 2 | 1 | NA | 4.66 | NA | 2.71 | 8.00 |

Adjusted to: CLASS=0 SEX=Male AGE=Adult
> summary(titanic.lrm.classagesex, CLASS='3', AGE='Child', SEX='Female') Effects Response : SURVIVED

| Factor | Low | High Diff. | Effect | S.E. | Lower | 0.95 | Upper 0.95 |
| :--- | :--- | :--- | :--- | ---: | ---: | ---: | ---: |
| CLASS - 0:3 | 4 | 1 | NA | 2.09 | 0.64 | 0.84 | 3.34 |
| Odds Ratio | 4 | 1 | NA | 8.08 | NA | 2.31 | 28.22 |
| CLASS - 1:3 | 4 | 2 | NA | 3.75 | 0.53 | 2.72 | 4.79 |
| Odds Ratio | 4 | 2 | NA | 42.67 | NA | 15.11 | 120.56 |
| CLASS - $2: 3$ | 4 | 3 | NA | 2.14 | 0.33 | 1.49 | 2.79 |
| Odds Ratio | 4 | 3 | NA | 8.49 | NA | 4.45 | 16.20 |
| SEX - Male:Female | 1 | 2 | NA | -0.23 | 0.42 | -1.06 | 0.60 |
| Odds Ratio | 1 | 2 | NA | 0.80 | NA | 0.35 | 1.83 |
| AGE - Adult:Child | 1 | 2 | NA | -0.18 | 0.36 | -0.89 | 0.53 |
| Odds Ratio | 1 | 2 | NA | 0.83 | NA | 0.41 | 1.70 |

Adjusted to: CLASS=3 SEX=Female AGE=Child
The probability of survival for the different combinations of sex and age group may be plotted (Figure 24).

```
> Predict(titanic.lrm.classagesex, fun=plogis,
+ CLASS=c('0','1','2','3'), AGE=c('Child','Adult'),
+ SEX=c('Female','Male'))
```

    CLASS AGE SEX yhat lower upper
    $1 \quad 0$ Child Female 0.88869350 .661946380 .9701987
$2 \quad 1$ Child Female 0.97683480 .925753760 .9930367
$3 \quad 2$ Child Female 0.89351600 .780560160 .9519104
$4 \quad 3$ Child Female 0.49701480 .338279640 .6563539
$5 \quad 0$ Adult Female 0.86956520 .664548280 .9573283
$6 \quad 1$ Adult Female 0.97238310 .928742100 .9895961
$7 \quad 2$ Adult Female 0.87509990 .795971430 .9263784
$8 \quad 3$ Adult Female 0.45207600 .378659060 .5276396
$9 \quad 0$ Child Male 0.57167290 .431505000 .7012113
$10 \quad 1$ Child Male 0.70047000 .559278050 .8116614
11 2 Child Male 0.39499690 .256264400 .5529915
123 Child Male 0.44068090 .321905590 .5666583
130 Adult Male 0.22273780 .196198930 .2517422

| 14 | 1 | Adult | Male | 0.3342723 | 0.26910345 | 0.4064468 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 15 | 2 | Adult | Male | 0.1229466 | 0.08312897 | 0.1781310 |
| 16 | 3 Adult | Male | 0.1446912 | 0.11604927 | 0.1789709 |  |

Response variable (y):

Limits are 0.95 confidence limits
> print(plot(Predict(titanic.lrm.classagesex, fun=plogis,

+ CLASS=c('0','1','2','3'),
$+\quad S E X=c(' F e m a l e ', ' M a l e ')$,
$+\quad A G E=c(' C h i l d ', ' A d u l t '))$,
$+\quad \operatorname{pch}=c(2,1), \operatorname{col}=c(1,2)$, layout=c $(1,2)))$
And we should validate the model.
> validate(titanic.lrm.classagesex, $B=80$ )

|  | index.orig | training | test | optimism | index.corrected |
| :--- | ---: | ---: | ---: | ---: | ---: | n

A nomogram may be helpful at this point (Figure 25).

```
> nom = nomogram(titanic.lrm.classagesex, fun=plogis)
> print(plot(nom))
```

NULL

## 9 Fitting CART

The CARTs fitted here are analogous to the logistic models fitted in SAS and R.

### 9.1 CLASS

A classification tree to look at the predictive nature of class when looking at survival may be fitted using the rpart function.

```
> #install.packages(c("rpart", "rpart.plot", "rpartOrdinal"))
> library(rpart)
> library(rpart.plot)
> library(rpartOrdinal)
> titanic.rpart.class=rpart(SURVIVED~CLASS,data=titanic)
> summary(titanic.rpart.class)
Call:
rpart(formula = SURVIVED ~ CLASS, data = titanic)
    n= 2201
```

|  | CP | nsplit rel error | xerror | xstd |
| ---: | ---: | ---: | ---: | ---: |
| 1 | 0.05696203 | 0 | 1.0000000 | 1.0000000 |
| 2 | 0.03085662 |  |  |  |
| 2 | 0.01000000 | 2 | 0.8860759 | 0.8860759 |

Node number 1: 2201 observations, complexity param=0.05696203
predicted class=No expected loss=0.323035
class counts: 1490711
probabilities: 0.6770 .323
left son=2 (1591 obs) right son=3 (610 obs)
Primary splits:
CLASS splits as LRRL, improve=69.6841, (0 missing)
Node number 2: 1591 observations
predicted class=No expected loss=0.2451288
class counts: 1201390
probabilities: 0.7550 .245
Node number 3: 610 observations, complexity param=0.05696203
predicted class=Yes expected loss $=0.4737705$
class counts: 289321
probabilities: 0.4740 .526
left son=6 (285 obs) right son=7 (325 obs)
Primary splits:
CLASS splits as -RL-, improve=13.46678, (0 missing)
Node number 6: 285 observations
predicted class=No expected loss=0.4140351
class counts: 167118
probabilities: 0.5860 .414

```
Node number 7: 325 observations
    predicted class=Yes expected loss=0.3753846
    class counts: 122 203
    probabilities: 0.375 0.625
```

A plot of the tree (Figure 26) may be created using

```
> plot(titanic.rpart.class)
> text(titanic.rpart.class)
```


### 9.2 AGE and SEX

A classification tree to look at the predictive nature of age and sex when looking at survival may be fitted using the rpart function.

```
> titanic.rpart.agesex=rpart(SURVIVED~AGE+SEX,data=titanic)
> summary(titanic.rpart.agesex)
Call:
rpart(formula = SURVIVED ~ AGE + SEX, data = titanic)
    n= 2201
```

|  | CP nsplit rel error | xerror | xstd |  |  |
| ---: | ---: | ---: | ---: | ---: | ---: |
| 1 | 0.3066104 | 0 | 1.0000000 | 1.0000000 | 0.03085662 |
| 2 | 0.0100000 | 1 | 0.6933896 | 0.6933896 | 0.02750982 |

Node number 1: 2201 observations, complexity param=0.3066104
predicted class=No expected loss=0.323035
class counts: 1490711
probabilities: 0.6770 .323
left son=2 (1731 obs) right son=3 (470 obs)
Primary splits:
SEX splits as RL, improve=199.821600, (0 missing)
AGE splits as RL, improve= 9.165241, (0 missing)
Node number 2: 1731 observations
predicted class=No expected loss=0.2120162
class counts: 1364367
probabilities: 0.7880 .212
Node number 3: 470 observations
predicted class=Yes expected loss=0.2680851
class counts: 126344
probabilities: 0.2680 .732

A plot of the tree (Figure 27) may be created using

```
> plot(titanic.rpart.agesex)
> text(titanic.rpart.agesex)
```


### 9.3 CLASS, AGE and SEX

A classification tree to look at the predictive nature of class, age and sex when looking at survival may be fitted using the rpart function.

```
> titanic.rpart.classagesex=rpart(SURVIVED~CLASS+AGE+SEX,data=titanic)
> summary(titanic.rpart.classagesex)
Call:
rpart(formula = SURVIVED ~ CLASS + AGE + SEX, data = titanic)
    n= 2201
```

|  | CP nsplit rel error | xerror | xstd |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: |
| 1 | 0.30661041 | 0 | 1.0000000 | 1.0000000 | 0.03085662 |
| 2 | 0.02250352 | 1 | 0.6933896 | 0.6933896 | 0.02750982 |
| 3 | 0.01125176 | 2 | 0.6708861 | 0.6976090 | 0.02756915 |
| 4 | 0.01000000 | 4 | 0.6483826 | 0.6779184 | 0.02728864 |

Node number 1: 2201 observations, complexity param=0. 3066104 predicted class=No expected loss=0.323035
class counts: 1490711
probabilities: 0.6770 .323
left son=2 (1731 obs) right son=3 (470 obs)
Primary splits:
SEX splits as RL, improve=199.821600, (0 missing)
CLASS splits as LRRL, improve= 69.684100, ( 0 missing)
AGE splits as RL, improve= 9.165241, ( 0 missing)

Node number 2: 1731 observations, complexity param=0.01125176 predicted class=No expected loss=0.2120162
class counts: 1364367
probabilities: 0.7880 .212
left son=4 (1667 obs) right son=5 (64 obs)
Primary splits:
AGE splits as RL, improve=7.726764, ( 0 missing)
CLASS splits as LRLL, improve=7.046106, (0 missing)
Node number 3: 470 observations, complexity param=0. 02250352 predicted class=Yes expected loss=0.2680851
class counts: 126344
probabilities: 0.2680 .732
left son=6 (196 obs) right son=7 (274 obs)

Primary splits:
CLASS splits as RRRL, improve=50.015320, ( 0 missing)
AGE splits as LR, improve= 1.197586, ( 0 missing)
Surrogate splits:
AGE splits as LR, agree=0.619, adj=0.087, (0 split)
Node number 4: 1667 observations
predicted class=No expected loss=0.2027594
class counts: 1329338
probabilities: 0.7970 .203
Node number 5: 64 observations, complexity param=0.01125176 predicted class=No expected loss=0.453125
class counts: 3529
probabilities: 0.5470 .453
left son=10 (48 obs) right son=11 (16 obs)
Primary splits:
CLASS splits as -RRL, improve=12.76042, (0 missing)
Node number 6: 196 observations predicted class=No expected loss=0.4591837
class counts: 10690
probabilities: 0.5410 .459
Node number 7: 274 observations predicted class=Yes expected loss=0.0729927
class counts: 20254
probabilities: 0.0730 .927
Node number 10: 48 observations predicted class=No expected loss=0.2708333
class counts: 3513
probabilities: 0.7290 .271
Node number 11: 16 observations predicted class=Yes expected loss=0
class counts: 016
probabilities: 0.0001 .000
A plot of the tree (Figure 28) may be created using

```
> plot(titanic.rpart.classagesex)
> text(titanic.rpart.classagesex)
```


### 9.4 Additional Functions

The documentation for the function rpart shows how to prune classification trees. There are also a number of sites on the web that show how to interpret rpart output.


Figure 19: Default diagnostic plots for the full model fitted to the htwt data.

CLASS=ad


Figure 20: Estimated probability of survival (and 95\% CIs) based upon class.


Figure 21: Estimated probability of survival based upon class.


Figure 22: Estimated probability of survival based upon sex and age group.

## Points



Figure 23: Nomogram for survival based upon sex and age group.


Figure 24: Estimated probability of survival based upon class, sex and age group.


CLASS
(SEX=Female AGE=Child)


CLASS (SEX=Male AGE=Child)

CLASS (SEX=Female AGE=Adult)


CLASS
(SEX=Male AGE=Adult)


Predicted Value


Figure 25: Nomogram for survival based upon class, sex and age group.


Figure 26: Classification tree for survival based upon class.


Figure 27: Classification tree for survival based upon age and sex.


Figure 28: Classification tree for survival based upon class, age and sex.

