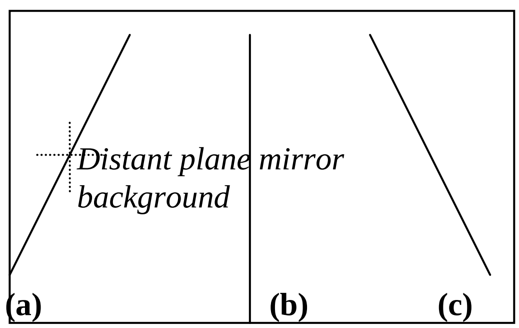
Example of an *early* draft. To see how things improve through revision, compare with *published* draft.

Optics in the Garden: the Helical Mirror

Over the last few years, the spinner has taken its place along side the gazing ball, sun dial, and blown glass humming bird feeder. This reflective, tapering, (often left-handed) helical strip of plastic decorates many an eave and garden bow. As it revolves in the breeze, a distant observer is struck by its surprising fluidity; but drawn in for a close look, one discovers intriguing optical properties: each segment of the spinner reflects roughly the same scene; objects are reflected sideways; bi-optic viewing is disconcertingly dizzying; and while reflections have exaggerated depth, they are otherwise relatively clear and distortion free. With their curb appeal and catalog of interesting properties, spinners' present a teaching opportunity, whether at a garden party or in the classroom.

By extending the two dimensional reasoning of introductory optics to three dimensions, we briefly discuss some of the spinner's more interesting optical properties. These are presented in terms of observations and questions that students can explore. In many regards, the spinner is similar to the cylindrical mirror, which we have previously treated [ref 1 & 2]; therefore, our present discussion of those points is rather brief, and we direct the interested reader to those previous treatments.



(replace with photo)

Figure 1. Spinners in three orientations. (a) Diagonal, with convex (concave) axis horizontal (vertical) **(b)** Vertical, as is normally displayed **(c)** Diagonal, with concave (convex) axis horizontal (vertical).

As is often the case, the subject is best understood form an alternative perspective. While a spinner is typically hung vertically (Fig. 1 b), we begin by considering it hung diagonally (Fig. 1a). Focusing on just one segment of the spinner, the natural axes emerge. A horizontal line traced across the surface is concave while a vertical one is convex; their intersection forms a saddle point. The asymmetry of these axes is responsible for many of the spinner's optical properties.

How does an image's orientation compare with that of the object? At first glance, a vertically hung spinner appears to reflect a 90° rotated image; however, comparing Fig. 1(b) with the background, planer reflection shows that the image is actually flipped along the diagonal. Any mirror reflects images flipped along the axis perpendicular to its surface, for example, while you face into a mirror, your image faces out of it. The concave mirror (for an object outside its focal length) also flips the image across the surface; flat and convex mirrors do not. A segment of a spinner is a hybrid concave-convex mirror. Thus the image is flipped parallel to the concave axis and is unchanged parallel to the convex axis. Different orientations of the mirror's axes relative to the object will therefore flip the image left to right (Fig 1a), top to bottom (Fig. 1c), or diagonally (Fig. 1b). (ref. Paper 1).

Investigate an image's clarity. The image is fairly clear, but not perfectly so. An image is unclear when the reflected rays from individual object points fail to converge to individual image points. For the spinner, one object point reflects off the concave axis to an image point in front of the surface but simultaneously reflects off the convex axis to an image point behind the surface.[1] This can easily be seen by tracking an object's reflection while first moving your head parallel to one and then the other axis. Focusing to one or the other extreme would result in an image well resolved along one axis but extremely blurred along the other. For typical objects, an individual eye will reconcile this extremely astigmatic situation by focusing somewhere in between. This is dubbed the point of least confusion, where horizontal and vertical features are roughly equally, but only mildly, blurred. How blurry depends on how much of a compromise is required. Considering the limiting case of a distant object, the two extreme image points become the concave and convex cross sections' focal points. For a typical spinner, these are only 2 to 4 cm apart. In contrast, a distant object reflected in a concave cylindrical mirror requires compromising between image points tending toward the concave cross-section's focal point and the full distance to the object, resulting in a far blurrier reflection.

Why is it worse with your eyes along the concave axis than along the convex axis?? Do triangulation and focus agree better

[1] Intermediary cross sections fill in between these two extremes.

Investigate an image's depth into/out of the surface. The depth is dizzyingly ambiguous. This is because the two cues we use to judge distance disagree. These are the triangulation of our two eyes' views and the tension in the ancillary muscles that focus the retina to reduce blurriness. When directly viewing a real object, the cues are redundant and necessarily agree. Reflection from an asymmetric surface, such as a concave-convex mirror, gives rise to disagreement. As noted in the previous section, an individual eye will place the image at the point of least confusion. However, each eye sees the object reflected from slightly different segments of the mirror and places this point at a different location; biopic triangulation therefore places the image at yet a third point. Depending on the head's alignment relative to the spinner's axes, this point may be near the image point of one or the other axis, somewhere in between, or nowhere (the two eye's views needn't converge at all).

Why do images in the spinner have exaggerated depth but good clarity and little horizontal-vertical distortion? The lack of significant distortion, i.e., different magnification in different directions, follows from the typical spinner's concave and convex axes having nearly equal and opposite concavities. Their focal points are nearly equidistant before and behind the surface. Equations 1 and 2 show that, while near objects will be asymmetrically magnified, those well outside the focal length ($d_{obj} >> f$) will suffer little distortion.

$$M_{convex} = \frac{f}{d_{obj} + f} \approx \frac{f}{d_{obj}} \qquad (1)$$

$$M_{concave} = \frac{-f}{d_{obj} - f} \approx -\frac{f}{d_{obj}} \qquad (2)$$

The sign flip indicates that one axis flips the image while the other does not; but the equal magnitudes indicate that the image is undistorted. The exaggerated depth is also apparent in these relationships. The presence of d_{obj} in the denominator means that more distant objects will be less magnified, thus amplifying their apparent depth.

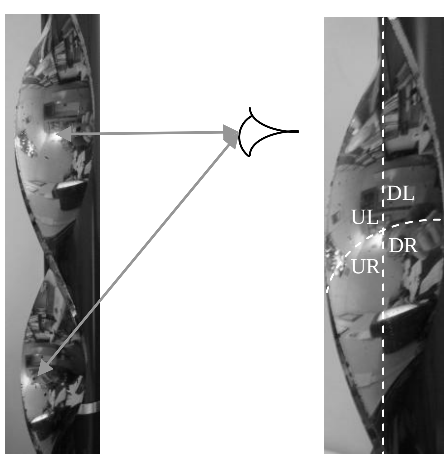


Figure 2. Reflections in two segments of a spinner. (a) Two consecutive segments reflect *nearly* the same images. **(b)** The four quadrants of a

spinner’s surface angle four different directions: Up Left, Down Left, Up Right, and Down Right.

Observe the similarities and differences between the reflected scenes from consecutive segments along the length of the spinner. Each segment reflects nearly, but not exactly, the same scene. Taking the observer’s (camera’s) eye as an example, it appears further down and left in each subsequent segment down the length of the (left-handed) spinner in Fig. 2 (a). This can be understood by reconciling the direction of the observer’s gaze with the changing orientation of the spinner’s surface. If the observer’s horizontal gaze reflects perpendicularly back from the center of first segment, so that the eye is imaged there, then the gaze is necessarily angled downward when looking to the next lower segment. For this lowered gaze to be reflected back up to the eye, it to must again hit the surface perpendicularly, i.e., it must hit a point properly angled upward. As is illustrated in Fig. 2 (b), a left-handed spinner twists so that the surface angles more upward (downward) the further left (right) of a segment’s horizontal center. Meanwhile, it angles more leftward (rightward) the further above (below) the segment’s vertical center. So the gaze must be directed left (striking an upward angled surface to compensate for looking down) and down (striking a right-ward angled surface to compensate for looking left). Thus the eye, and by extension, the whole scene is further down left in each lower segment of the spinner.

- Summary