Physics 391/491: Junior/Senior Seminar
Equations

In Physics, mathematics is used as a language to concisely and precisely express ideas. Below are some examples of mathematics properly integrated into prose; they're followed by specific rules, most of which follow from this simple idea: *you should include mathematical symbols and expressions just as you do other words and phrases, albeit foreign ones.*

After all the rules are some exercises Dr. DeWeerd collected to give you practice employing the rules.

> Since the aluminum slug does not undergo phase changes and its temperature change, ΔT_{sl}, is relatively small, we can approximate that
>
> $$\Delta U_{sl} = m_{sl} c_{Al} \Delta T_{sl}\,, \qquad (1)$$
>
> where ΔU_{sl} is the change in the slug's internal energy, m_{sl} is it's mass, and c_{Al} is the specific heat for aluminum near room temperature.

Rule 1: Integrate into your prose.

As a general rule, you want your writing to flow smoothly and coherently. That holds just as well for sections that include mathematics as those that do not. For example, you'd never write

> My main point is expressed in the next sentence. Garfunkel's contribution to the partnership is often underestimated.

That's just plain clunky. Instead you'd write

> My main point is that Garfunkel's contribution to the partnership is often underestimated.

Similarly, you shouldn't write something like

> My central result is expressed in equation 10.
>
> $$c_{Al} = \frac{\Delta U_{sl}}{m_{sl} \Delta T_{sl}} \qquad (10)$$

Instead, you should write something like

> My central result is that
>
> $$c_{Al} = \frac{\Delta U_{sl}}{m_{sl} \Delta T_{sl}}\,. \qquad (10)$$

Now, there are occasions when you *would* write something like "My point is simply this: You're underestimating Garfunkel's contribution." or even "My point is simple. You're underestimating Garfunkel's contribution." Similarly, there are occasions when an equation *would* use a colon to introduce an equation, or when an equation would even be a sentence all by itself, but they're rare.

Rule 2: Punctuate and capitalize properly.

Of course, if your equation is integrated into a sentence, then the proper punctuation should surround it. In case you're not certain about the punctuation required, try reading the sentence as if the equation were in English. In the example above, that would go like "My central result is that the slug's change in internal energy is equal to the product of its mass, specific heat, and change in temperature." In this all-English version, it should be clear that you don't need any punctuation before "the slug's change in internal energy" but you do need a period after "temperature." Ditto for the mixed English and mathematics version. Regarding capitalization, notice that the "where" just after Eq. 1 is *not* capitalized, that's because it's a part of the same sentence (note: text editor autocorrects often mis-correct this.)

Rule 3: Italicize.

In general, whenever you switch from writing in English to writing in another language, you're supposed to italicize the second language to cue the reader that you're stepping out of English. For example, you might write "The attempted *coup d'état* was unsuccessful." Since *coup d'état* isn't actually English (it's French), you italicize it. Look at the example involving equation 1; the same rule holds for the language of mathematics – whenever you use a symbol, whether it's alone or in a whole string of them (i.e., an equation), you should italicize.

Rule 4: Define.

Unless it's universally understood by English speakers (as is *coup d'état*), the first time you introduce a foreign word, you need to also provide your readers with a translation of it to English. Again, that rule holds for mathematics. That's why ΔU_{sl}, m_{sl}, c_{Al}, and ΔT_{sl} are all translated into English in the example around Eq. 1. Of course, all those symbols are reused in equation 10, so there was no need for me to define them a second time (though, if equations 1 and 10 were separated by a number of pages, it might be considerate of me to do so.)

These above rules all follow from the fact that you should be integrating English and mathematics as seamlessly as you would English and any other language. The next couple of rules follow from the special role of mathematics in your paper – you and others will want to be able to refer to an individual equation concisely, find it quickly, and read it smoothly.

Rule 5: Separate (and center the equation).

While an equation should logically read as part of a sentence, that doesn't mean that it should be physically on the same line as the English. It takes extra focus to read an equation, so give an equation some uncluttered personal space – that means centering the equation on the page and giving a blank line above and below it.

Rule 6: Number (and right justify the number).

There's a good chance that somewhere in your prose after you've introduced an equation you'll want to refer to it again; even if that isn't the case, your reader may wish to do so when communicating with someone else about your work. So give each of your equations a number (not just the equations that you anticipate latter referencing, but *each* of them.) That way you can say something like "observe that Eq. 10 is simply Eq. 1 solved for the slug's specific heat." To make it particularly easy for your reader to then go back and hunt up Eq's 1 and 10, you should place the number far right on the page – that way, (s)he can flip through your voluminous work just scanning the right edge of the page to locate the two equations.

Rule 7: Reference it properly.

As already mentioned, you're going to reference some of your equations. You could refer to them as Equation 1 and Equation 10, but most journals prefer the abbreviation "Eq." as in Eq. 1 and Eq. 10. That's not just because they're stingy of using unnecessary characters; it's because of the special way people read equations. It's common for a reader to skim through a paper looking for references to a particular equation; an abbreviation like "Eq. 1" stands out more than "Equation 1" does, so it's easier to find when skimming text (for the same reason "Fig. 3" is often preferable to "Figure 3.")

Rule 8: *Really* format your equations.

Let's face it - math is harder to read than English is, so make it a *little* easier on your reader by making the equations look nice. That means using an equation editor. In MS Word, there are two ways to insert an equation. Both approaches make equally pretty equations, but only one approach codes them in a way that allows you to later copy and paste your equations into PowerPoint (which is convenient if you'll later be giving a presentation related to the paper.)

- **Mechanism 1 (which does play well with PowerPoint).** Hit the "Insert" tab. In the "Text" section (second from the right) is a button for inserting an "Object"; click that. In the Dialog box that comes up, select "Microsoft Equation." Sometimes this brings up a new blank page in which you can create the equation, sometimes it just creates a box within your current document. If it does the former, the non-intuitive part is that, once you've written your equation, close that new window, and the equation will be sitting in your original document.

- **Mechanism 2 (which doesn't play well with PowerPoint).** Hit the "Insert" tab. In the "Symbols" section (far right) click the giant π "Equation" button.

There are two more rules, completely unrelated to formatting, to consider when including math.

Rule 9: *Don't* show every step.

In your homework, we want to see every logical step you take, that's because the homework is supposed to communicate the *activity* of your solving a problem. In a paper, don't wear yourself out (or bore your reader) by including equations for even intermediate and trivial steps. You can use English to simply allude do the process, for example, "solving Eq. 1 for ΔT, substituting it into Eq. 3, and integrating the resulting relation from the initial temperature to the final temperature yields…"

Rule 10: Cite your sources.

For the same reasons as with English, you should cite your sources for mathematics – give credit (or blame) where it is due, and let your reader know where to look for more information.

Exercises

In the exercise below, identify and correct the problems with the how the equations are incorporated into the text. Note which of the 10 rules had been violated.

1) Solving for F_T in Eqs. (3) and (4) and combining them,

$$F_T = F_B - m_{gas}g - m_{balloon}g = F_N - m_{\substack{gram \\ mass}}g. \qquad (5)$$

2) For a mixture of two gases, say, He and air (where air is treated as a "pure" gas), the effective molar mass is given by:

$$\mu_{gas} = \frac{m_{gas}}{m_{gas}}. \qquad (9)$$

3) After the release, Observer 1 sees *M* follow a parabolic trajectory characterized by the equations

$$x = 0.7t \qquad\qquad y = 0 - (1/2)9.8t^2.$$

4) Solving Eq. (1) for $\cos\phi$ and substituting into Eq. (2):

$$h^2 = \frac{d^2}{2} - \frac{L^2}{4} + \frac{(d-\Delta)^2}{2},$$

we obtain, after some rearranging,

$$sin\theta = \frac{d^2-(d-\Delta)^2}{L\cdot\sqrt{2d^2-L^2+2(d-\Delta)^2}}. \qquad (5)$$

5) In the cup, the force arising due to radial pressure difference provides the centripetal force required to keep a fluid parcel moving in a circle, i.e.,

$$\frac{1}{\rho}\frac{\partial p}{\partial r} = \frac{V^2}{r}, \qquad (1)$$

where ρ is the density of the fluid, p is the pressure, V is the fluid velocity, and r is the radial distance from the center.

6) Substituting Eq. (2) in the expression for net inward acceleration,

$$\frac{V^2-v^2}{r},$$

we can rewrite it in terms of v_t and Ω as follows:

$$\frac{V^2-v^2}{r} = -\frac{2Vv_t}{r} - \frac{v_t^2}{r} = -2\Omega v_t - \frac{v_t^2}{r}, \qquad (3)$$

where it has been assumed that $V = \Omega r$ outside the boundary layer.

7) The amount of negative charge will be the rate I coulombs per second that is being sucked out, times the time delay ($T_2 - T_1$) before the current is replaced by the source.

$$Q = I(T_2 - T_1). \qquad (2)$$

8) To calculate the time delay, we note that the trigger signal and the current sink are approaching each other at a relative speed ($c + v$), and cover the contracted distance $L' = L\sqrt{1 - v^2/c^2}$ in a time

$$T_1 = \frac{L'}{c+v}. \qquad (3)$$

9) So, for any total reflection to occur, it is necessary that α be greater than the critical angle i_c. That is,

$$sin\alpha \geq \frac{1}{n_{water}}.$$

10) Eliminating the time t, one can determine the initial speed

$$V_{0x} = \frac{x}{t} = x\sqrt{\frac{g}{2h}}. \qquad (2)$$

Sources:

1) Anthony C. Zable, "Experiments with Helium-Filled Balloons," *Phys. Teach*. **48** (9), 582–586 (2010).
2) Same as above.
3) Albert A. Bartlett, "'Apparent Weight': A Concept that Is Confusing and Unnecessary," *Phys. Teach*. **48** (8), 522–524 (2010).
4) Mark I. Liff, "Another Demo of the Unusual Thermal Properties of Rubber," *Phys. Teach*. **48** (7), 444–446 (2010).
5) Amit Tandon and John Marshall, "Einstein's Tea Leaves and Pressure Systems in the Atmosphere," *Phys. Teach*. **48** (5), 292–295 (2010).
6) Same as above.
7) Elisha Huggins, "Note on Magnetism and Simultaneity," *Phys. Teach*. **47** (9), 587–589 (2009).
8) Same as above.
9) Zhu Yuhua and Shi Fengliang, "Why Does the Goldfish Disappear in the Fishbowl?" *Phys. Teach*. **47** (8), 424–426 (2009).
10) Mikhail M. Agrest, "Physics Labs with Flavor," *Phys. Teach*. **47** (5), 297–301 (2009).