Fri.	6.1 Magnetization	
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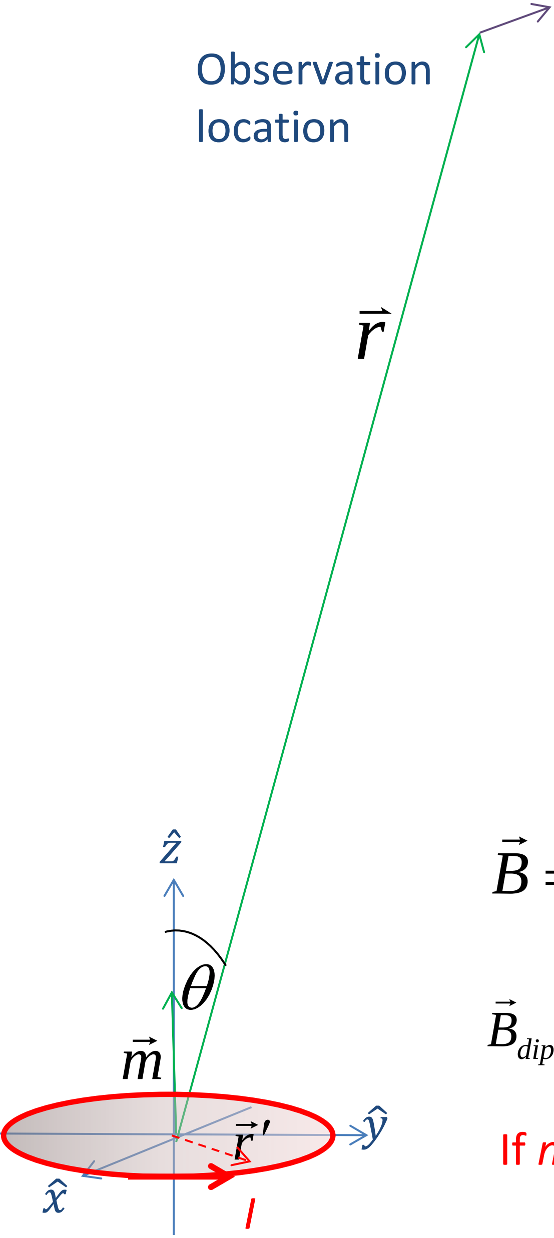
Dipole term for a loop



$$\vec{A}(r) = \frac{\mu_o}{4\pi}\left(\frac{\vec{m}\times\hat{r}}{r^2} + \ldots\right)$$

Magnetic Dipole Moment

$$\vec{m} \equiv I\vec{a}'$$

$$\vec{m} = I\left(\pi R^2 \hat{z}\right)$$

$$\vec{A}(r) = \frac{\mu_o}{4\pi}\left(\frac{I\pi R^2 \hat{z}\times\hat{r}}{r^2} + \ldots\right) = \frac{\mu_o}{4\pi}\left(\frac{I\pi R^2 \sin\theta}{r^2}\hat{\phi} + \ldots\right)$$

Same direction as current

$$\vec{B} = \vec{\nabla}\times\vec{A}$$

$$\vec{B}_{dip}(\vec{r}) = \frac{\mu_o}{4\pi}\frac{3(\vec{m}\cdot\hat{\mathscr{r}})\hat{\mathscr{r}} - \vec{m}}{\mathscr{r}^3}$$

If *m* at origin and pointing up

$$\vec{B}_{dip}(\vec{r}) = \frac{\mu_o}{4\pi}\frac{m}{r^3}\left(2\cos\theta\hat{r} + \sin\theta\hat{\theta}\right)$$

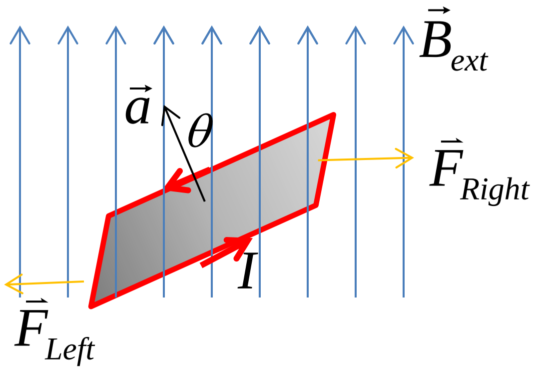
(yes, same form as *E* for *p*)

Magnetic Dipoles

$$\vec{m} \equiv I\vec{a}' \qquad \vec{A}(r) = \frac{\mu_o}{4\pi}\left(\frac{\vec{m}\times\hat{r}}{r^2} + \ldots\right) \qquad \vec{B}_{dip}(\vec{r}) = \frac{\mu_o}{4\pi}\frac{m}{r^3}\left(2\cos\theta\hat{r} + \sin\theta\hat{\theta}\right)$$

From a distance much greater than the current distribution's size, the dipole term dominates

Torque for real current loop



$$N = ILB_{ext}w\sin\theta$$

$$\vec{N} = I\vec{a}\times\vec{B}_{ext}$$

In terms of dipole moment

$$\vec{N} = \vec{m}\times\vec{B}_{ext}$$

Like for *electric* dipoles

$$\vec{N} = \vec{p}\times\vec{E}_{ext}$$

Change in **energy**

$$\Delta U = -\int \vec{F}_R\cdot d\vec{l}_R - \int \vec{F}_L\cdot d\vec{l}_L$$

$$= -2\int \vec{F}_{right}\cdot d\vec{l}_{right}$$

$$= -2\int (IL)\vec{B}\cdot \tfrac{w}{2}\, d\vec{\theta}$$

$$= -2\int (IL)B\sin\theta\, \tfrac{w}{2}\, d\theta$$

$$= -(ILw)B\cos\theta\Big|_{\theta_i}^{\theta_f}$$

$$\Delta U = -mB\cos\theta\Big|_{\theta_i}^{\theta_f}$$

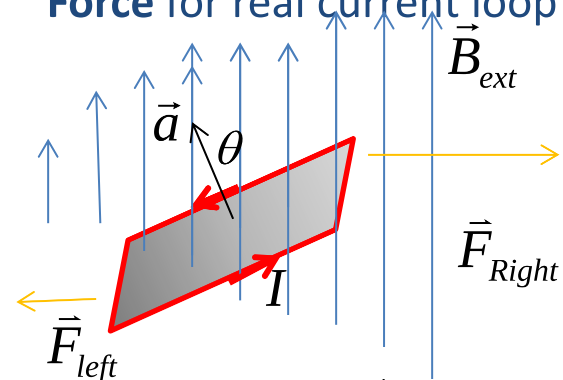
In terms of dipole moment

$$\Delta U = -\Delta\left(\vec{m}\cdot\vec{B}\right)$$

Like for *electric* dipoles

$$\Delta U = -\Delta\left(\vec{p}\cdot\vec{E}\right)$$

Force for real current loop



$$\vec{F}_{net} = \vec{F}_{Right} + \vec{F}_{left} = \left(ILB_{Right} - ILB_{Left}\right)\hat{x}$$

$$\vec{F}_{net} = IL\Delta x\frac{B(x+\Delta x) - B(x)}{\Delta x}\hat{x}$$

$$\vec{F}_{net} = IL\left(w\cos\theta\right)\frac{dB}{dx}\hat{x} = \frac{d\left(\vec{m}\cdot\vec{B}\right)}{dx}\hat{x}$$

In terms of dipole moment

$$\vec{F} = \vec{\nabla}\left(\vec{m}\cdot\vec{B}_{ext}\right) = m\times\left(\vec{\nabla}\times\vec{B}_{ext}\right) + \left(\vec{m}\cdot\vec{\nabla}\right)\vec{B}_{e}$$

Like for *electric* dipoles

$$\vec{F} = \left(\vec{p}\cdot\vec{\nabla}\right)\vec{E}_{ext}$$

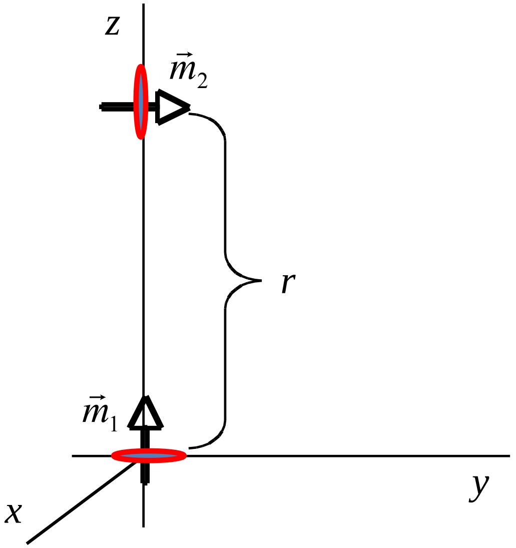
Magnetic Dipoles

$$\vec{m} \equiv I\vec{a}' \qquad \vec{A}(r) = \frac{\mu_o}{4\pi}\left(\frac{\vec{m}\times\hat{r}}{r^2} + \dots\right) \qquad \vec{B}_{dip}(\vec{r}) = \frac{\mu_o}{4\pi}\frac{m}{r^3}\left(2\cos\theta\hat{r} + \sin\theta\hat{\theta}\right)$$

From a distance much greater than the current distribution's size, the dipole term dominates

$$\vec{N} = \vec{m}\times\vec{B}_{ext} \qquad \Delta U = -\Delta\left(\vec{m}\cdot\vec{B}\right) \qquad \vec{F} = \vec{\nabla}\left(\vec{m}\cdot\vec{B}_{ext}\right)$$

Exercise: you have two *magnetic* dipoles; find the torque m_1 applies on m_2.



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Magnetic Dipoles

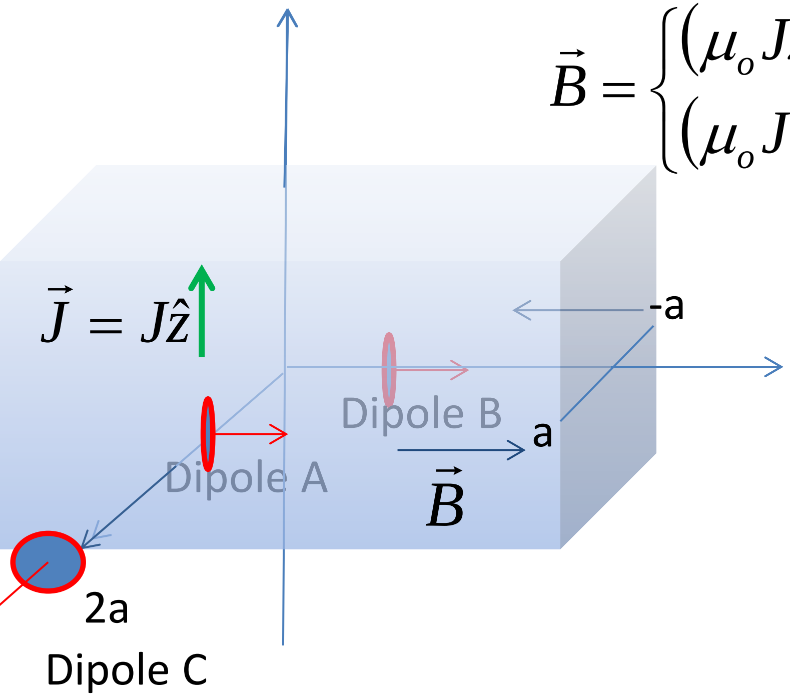
$$\vec{m} \equiv I\vec{a}' \qquad \vec{A}(r) = \frac{\mu_o}{4\pi}\left(\frac{\vec{m}\times\hat{r}}{r^2} + \dots\right) \qquad \vec{B}_{dip}(\vec{r}) = \frac{\mu_o}{4\pi}\frac{m}{r^3}\left(2\cos\theta\hat{r} + \sin\theta\hat{\theta}\right)$$

From a distance much greater than the current distribution's size, the dipole term dominates

$$\vec{N} = \vec{m}\times\vec{B}_{ext} \qquad \Delta U = -\Delta\left(\vec{m}\cdot\vec{B}\right) \qquad \vec{F} = \vec{\nabla}\left(\vec{m}\cdot\vec{B}_{ext}\right)$$

Exercise: find the force on dipoles A, B, C in and near a slab of uniform current

$$\vec{B} = \begin{cases} (\mu_o Jx)\hat{y} & \text{inside} \\ (\mu_o Ja)\hat{y} & \text{infront} \end{cases}$$



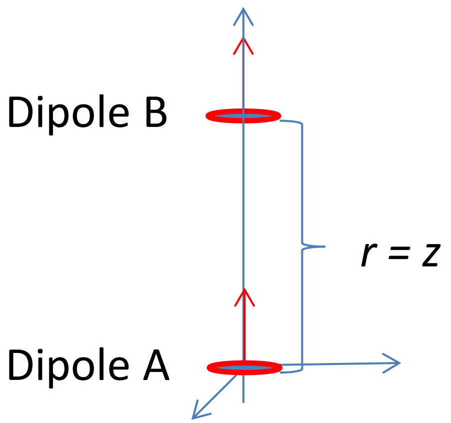
Magnetic Dipoles

$$\vec{m} \equiv I\vec{a}' \qquad \vec{A}(r) = \frac{\mu_o}{4\pi}\left(\frac{\vec{m}\times\hat{r}}{r^2} + \ldots\right) \qquad \vec{B}_{dip}(\vec{r}) = \frac{\mu_o}{4\pi}\frac{m}{r^3}\left(2\cos\theta\hat{r} + \sin\theta\hat{\theta}\right)$$

From a distance much greater than the current distribution's size, the dipole term dominates

$$\vec{N} = \vec{m}\times\vec{B}_{ext} \qquad \Delta U = -\Delta\left(\vec{m}\cdot\vec{B}\right) \qquad \vec{F} = \vec{\nabla}\left(\vec{m}\cdot\vec{B}_{ext}\right)$$

Exercise: Force between two dipoles (read, "bar magnets"). What's the force A exerts on B?



Considering 'real' dipoles (with real radii), roughly sketch the field and resulting forces on the current loops

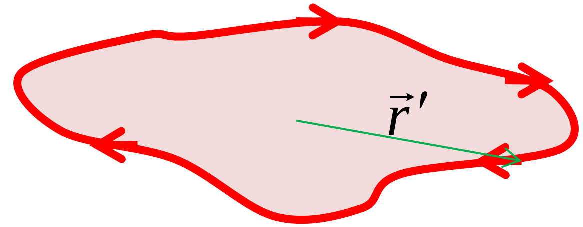
Atomic Dipoles

Angular momentum – dipole moment relation

Where there's a circulation and charge, there's current loop and magnetic moment

Consider a loop of electrons circulating

$$\vec{L}_{loop} = \sum \vec{r}\,' \times m_e \vec{v} = \sum \vec{r}\,' \times (\lambda_m dl') \vec{v} \;\; = \oint \vec{r}\,' \times \vec{v} \lambda_m dl'$$



Assuming a constant mass-to-charge ratio

$$\vec{L}_{loop} = \oint \vec{r}\,' \times \vec{v} \tfrac{m_e}{e} \lambda_q dl' \;\; = \tfrac{m_e}{e} \oint \vec{r}\,' \times \vec{v} \lambda_q dl'$$

As the velocity is confined to point *along* the loop, we can rewrite as

$$\vec{L}_{loop} = \tfrac{m_e}{e} \oint (v\lambda_q) \vec{r}\,' \times d\vec{l}\,' = \tfrac{m_e}{e} \oint I \vec{r}\,' \times d\vec{l}\,' = \tfrac{m_e}{e} I \oint \vec{r}\,' \times d\vec{l}\,' = \tfrac{m_e}{e} I 2\vec{a} \;\; = 2 \tfrac{m_e}{e} \vec{m}$$

Assuming constant current   May recognize the integral

or

$$\vec{m} = \tfrac{e}{2m_e} \vec{L}_{loop}$$

Where there's charge and angular momentum, there's a magnetic dipole

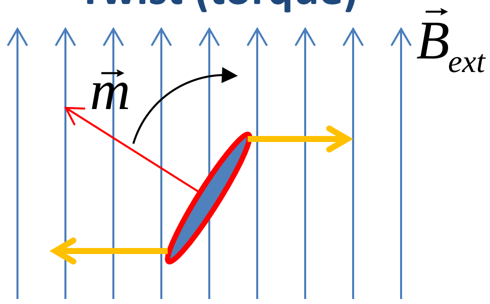
Electron Spin and Orbit

$$L_{loop} \approx \hbar \quad \text{so} \quad m \approx \tfrac{e}{2m_e} \hbar \;\; \approx 9.3 \times 10^{-24} \, \mathrm{A\,m^2}$$

Magnetic Field Effects on Atomic Dipoles

Disclaimer: we get only qualitative insight from considering current loops; spinning and orbiting electrons aren't *really* current loops

Twist (torque)



$$\vec{N} = \vec{m} \times \vec{B}_{ext}$$

$$\Delta U = -\Delta\left(\vec{m} \cdot \vec{B}_{ext}\right)$$

Paramagnetism

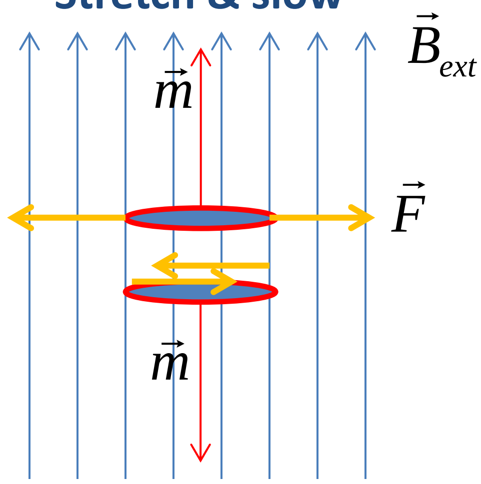
Current loop would be torqued toward alignment, thus decreasing energy.

Zeeman Effect: in a pair of opposite spin electrons, one will be aligned with the field – lowered energy, and one anti-aligned – raised energy

Magnetic Field Effects on Atomic Dipoles

Disclaimer: we get only qualitative insight from considering current loops; spinning and orbiting electrons aren't *really* current loops

Stretch & slow



$$m = \tfrac{e}{2m_e} L$$

$$\Delta m = \tfrac{e}{2m_e} \Delta L$$

$$\Delta L = \int N dt$$

$$N = (eE)r$$

From __future__ import time-varying magnetic

$$\vec{\nabla} \times \vec{E} = -\frac{d}{dt}\vec{B}$$

Diamagnetism

Introducing a B means curling an E which slows/speeds the current

So, by Stoke's

$$E 2\pi r = -\frac{\Delta B}{\Delta t}\pi r^2$$

$$E = -\frac{dB}{dt}\tfrac{1}{2} r$$

$$\Delta m = -\tfrac{e}{2m_e}\int\left(e\frac{dB}{dt}\tfrac{1}{2}r\right) r dt \approx -\tfrac{e^2}{4m_e} r^2 \int\left(\frac{dB}{dt}\right) dt$$

Approximating r as *fairly* constant,

$$\Delta m \approx -\tfrac{e^2}{4m_e} r^2 \Delta B$$

Magnetic Field Effects on Atomic Dipoles

Better Derivation

Consider a charged particle moving in the presence of a magnetic field.
The ‘momentum’ in the particle + field system: $\vec{p}_{system} = \vec{p}_{kin} + \vec{p}_{field} = m\vec{v}_1 + q\vec{A}$

$$\vec{p}_{kin} = \vec{p} - q\vec{A}$$

For an electron, $M = m_e$, $q = -e$

$$H_{Hamiltonian} = \frac{p_{kin}^2}{2M} = \frac{1}{2M}\left(\vec{p} - q\vec{A}\right)^2 = \frac{1}{2m_e}\left(\vec{p} + e\vec{A}\right)^2 = \frac{p^2}{2m_e} + \frac{e}{2m_e}\left(2\vec{A}\cdot\vec{p}\right) + \frac{e^2}{2m_e}A^2$$

Magnetic field uniform and in the z direction gives $\vec{A} = \frac{B_z s}{2}\hat{\phi} = \frac{\vec{B}\times\vec{r}}{2}$

$$H = \frac{p^2}{2m_e} + \frac{e}{2m_e}\left(\vec{B}\times\vec{r}\right)\cdot\vec{p} + \frac{e^2}{8m_e}\left(\vec{B}\times\vec{r}\right)^2$$

Product Rule 2

$$\vec{B}\cdot\left(\vec{r}\times\vec{p}\right)$$
$$\vec{B}\cdot\vec{L}$$

Product Rules 1 & 2 and define B in z direction, s in x-y plane, reduces to

$$\vec{B}\cdot\left(\vec{B}s^2\right) = \left(Bs\right)^2$$

$$H = \frac{p^2}{2m_e} + \left(\frac{e}{2m_e}\vec{L}\cdot\vec{B} + \frac{e^2}{8m_e}\left(Bs\right)^2\right)$$

So if $m = \frac{dH}{dB} = \frac{e}{2m_e}L_z + \frac{e^2}{4m_e}Bs^2$

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