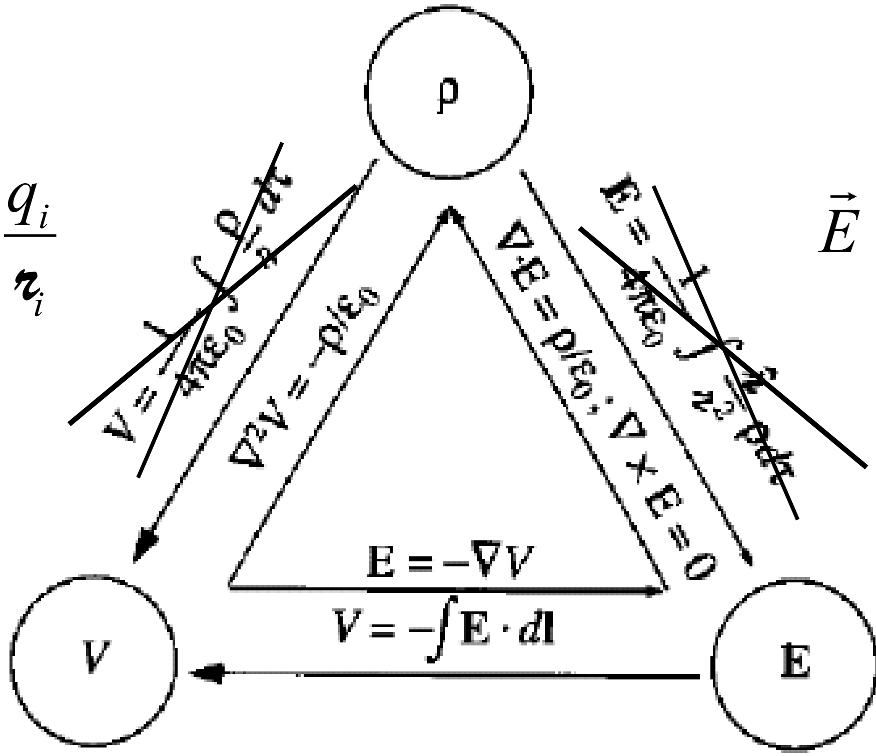
Wed. Thurs Fri.	2.4.1-.4.2 Work & Energy in Electrostatics T3 Contour Plots 2.4.3-4.4 Work & Energy in Electrostatics	 HW2
Mon. Wed. 	2.5 Conductors Summer Science Research Poster Session: Hedco7pm~9pm 	 HW3

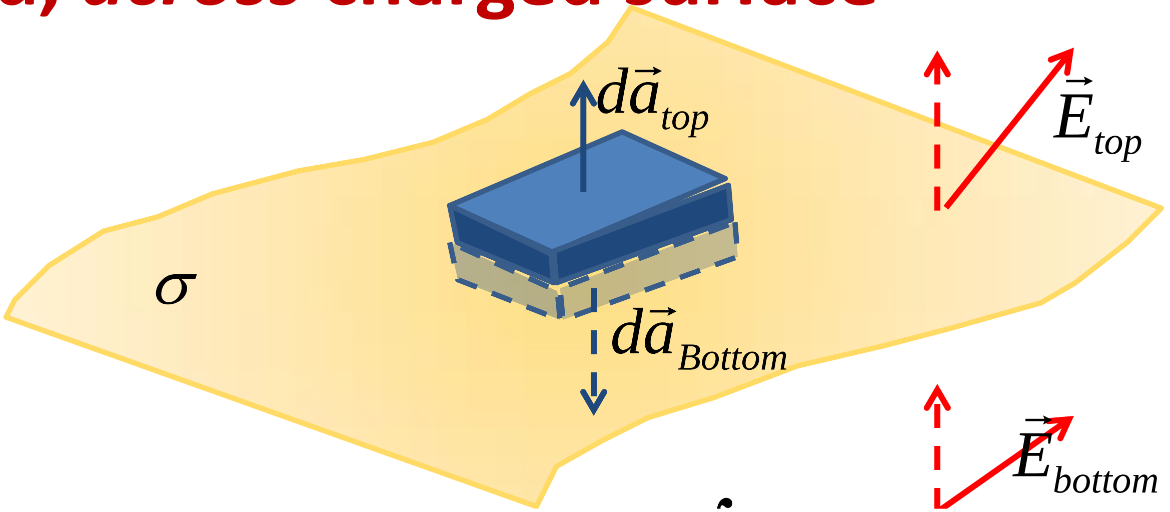
Electro-static Relations

$$V = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{\mathscr{r}_i}$$



$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{\mathscr{r}_i^{\,2}}\,\hat{\mathscr{r}}_i$$

Boundary Conditions
Electric field, *across* charged surface



$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{encl}}{\varepsilon_o}$$

$$\int \vec{E}_{top} \cdot d\vec{a}_{top} + \int \vec{E}_{bottom} \cdot d\vec{a}_{bottom} + \int \vec{E}_{sides} \cdot d\vec{a}_{sides}$$

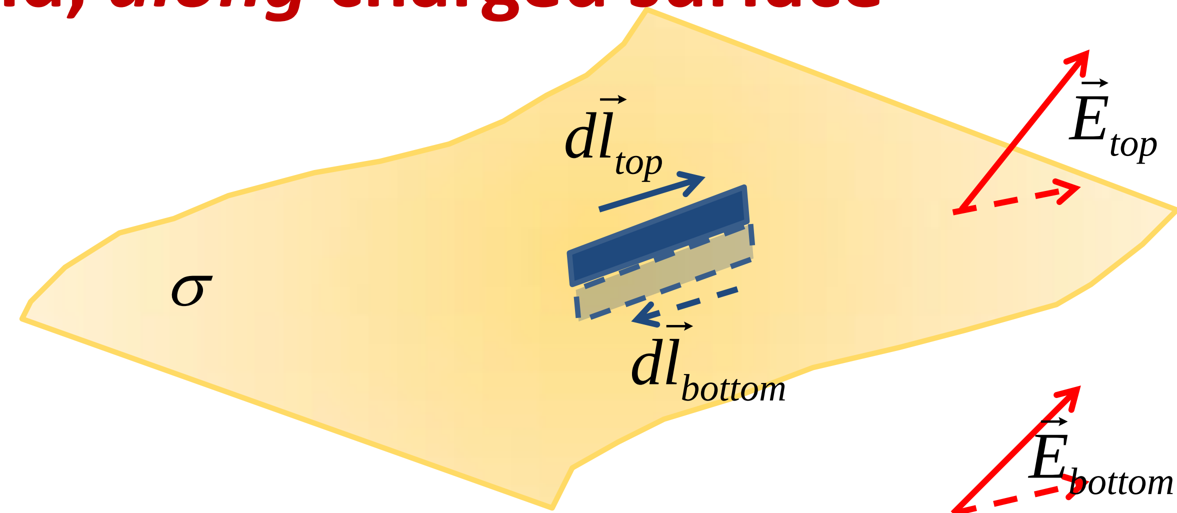
Send side height / area to 0

$$\int \vec{E}_{top} \cdot d\vec{a}_{top} + \int \vec{E}_{bottom} \cdot d\vec{a}_{bottom} = \frac{\int \sigma da_{surface}}{\varepsilon_o}$$

$$E_{\perp top} A + E_{\perp bottom} A(-1) = \frac{\sigma A}{\varepsilon_o}$$

$$E_{\perp top} - E_{\perp bottom} = \frac{\sigma}{\varepsilon_o}$$

Boundary Conditions
Electric field, *along* charged surface



$$\oint \vec{E} \cdot d\vec{l} = 0$$

$$\int \vec{E}_{top} \cdot d\vec{l}_{top} + \int \vec{E}_{bottom} \cdot d\vec{l}_{bottom} + \int \vec{E}_{sides} \cdot d\vec{l}_{sides} = 0$$

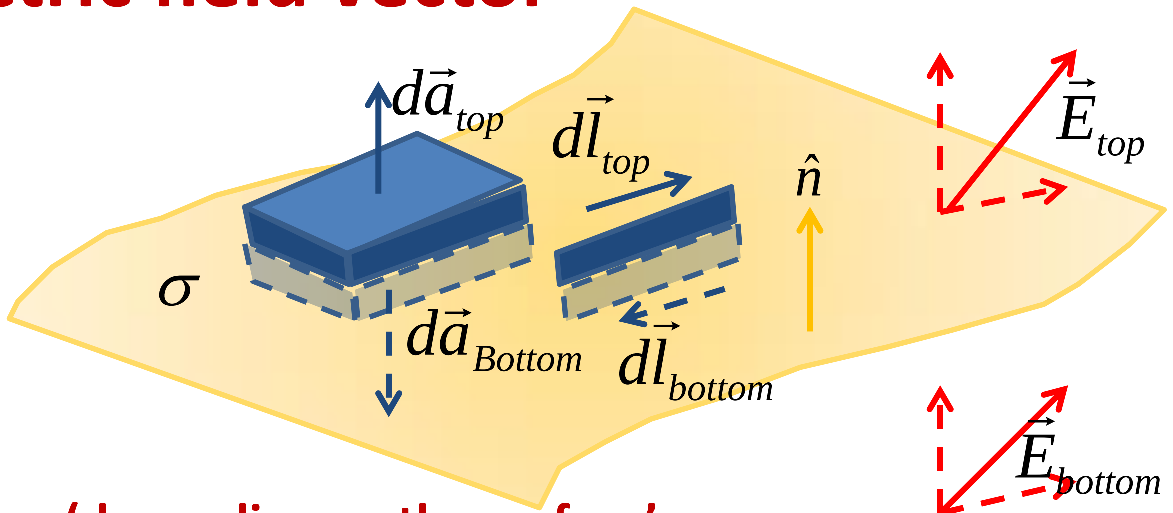
Send side height to 0

$$\int \vec{E}_{top} \cdot d\vec{l}_{top} + \int \vec{E}_{bottom} \cdot d\vec{l}_{bottom} = 0$$

$$E_{\|top} L + E_{\|bottom} L(-1) = 0$$

$$E_{\|top} - E_{\|bottom} = 0$$

Boundary Conditions
Electric field vector



Along

$$E_{\|top} - E_{\|bottom} = 0$$

Across, generically call $\hat{n}$ direction (depending on the surface's orientation, it could be x, y, z, some random angle between them,…

$$E_{\perp top} - E_{\perp bottom} = \frac{\sigma}{\varepsilon_o}$$

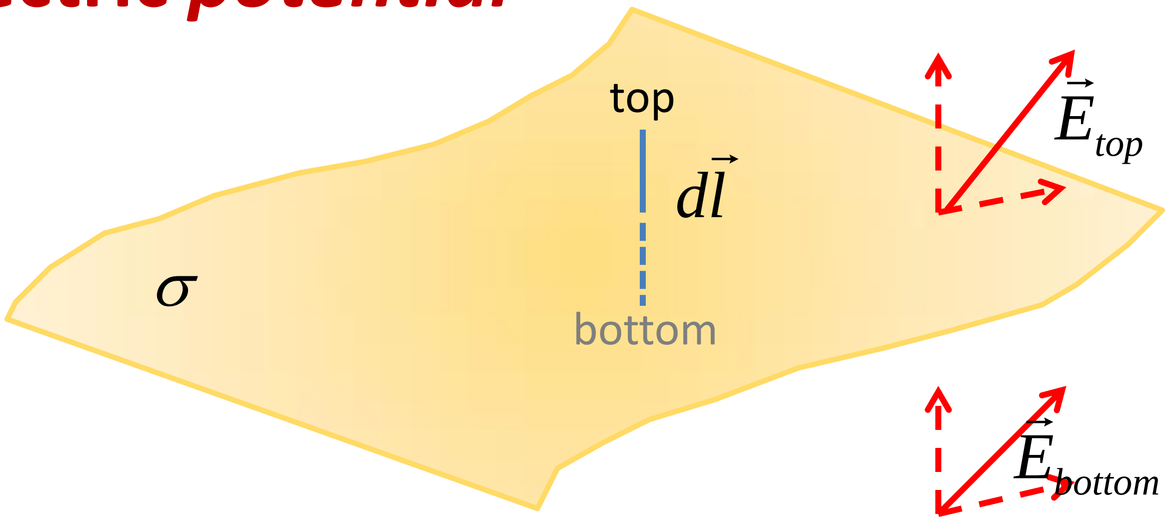
To be concrete: if surface vector points in z direction,

$$\vec{E}_{top} - \vec{E}_{bottom} = \frac{\sigma}{\varepsilon_o}\hat{z}$$

Combined

$$\vec{E}_{top} - \vec{E}_{bottom} = \frac{\sigma}{\varepsilon_o}\hat{n}$$

Boundary Conditions
Electric *potential*



$$V_{top} - V_{bottom} = \int_{bottom}^{top} \vec{E} \cdot d\vec{l}$$

Imagine choosing a shorter and shorter dl until it vanishes

$$V_{top} - V_{bottom} = \int_{bottom}^{top} \vec{E} \cdot d\vec{l} \rightarrow 0$$

$$V_{top} - V_{bottom} = 0$$

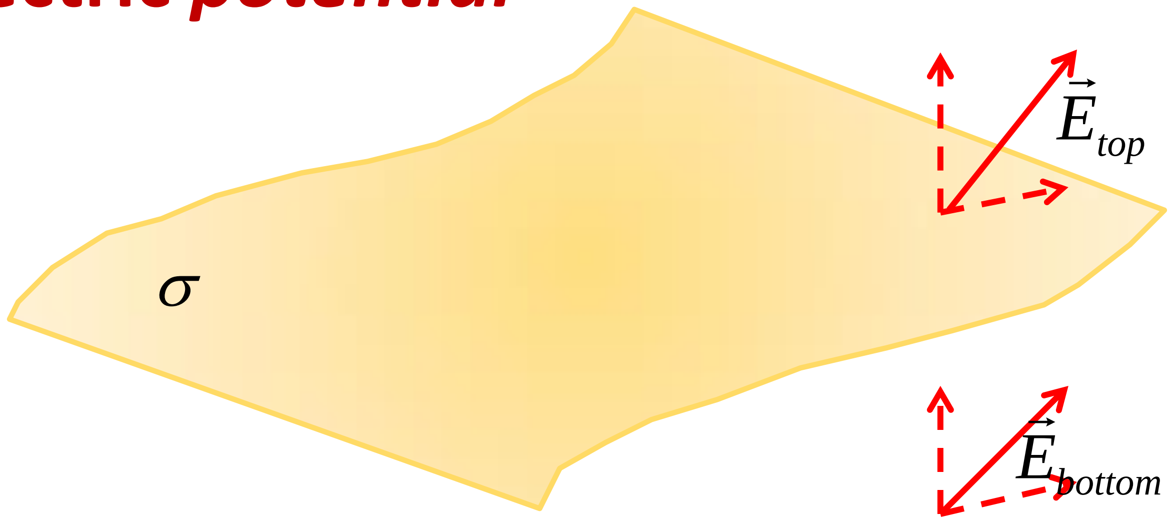
Boundary Conditions
Electric *potential*

$$\vec{E}_{top} - \vec{E}_{bottom} = \frac{\sigma}{\varepsilon_o}\hat{n}$$

$$\vec{\nabla}V_{top} - \vec{\nabla}V_{bottom} = \frac{\sigma}{\varepsilon_o}\hat{n}$$

$$\left(\frac{\partial}{\partial"n"}V_{top} - \frac{\partial}{\partial"n"}V_{bottom}\right)\hat{n} = \frac{\sigma}{\varepsilon_o}\hat{n}$$

$$\left(\frac{\partial}{\partial"n"}V_{top} - \frac{\partial}{\partial"n"}V_{bottom}\right) = \frac{\sigma}{\varepsilon_o}$$

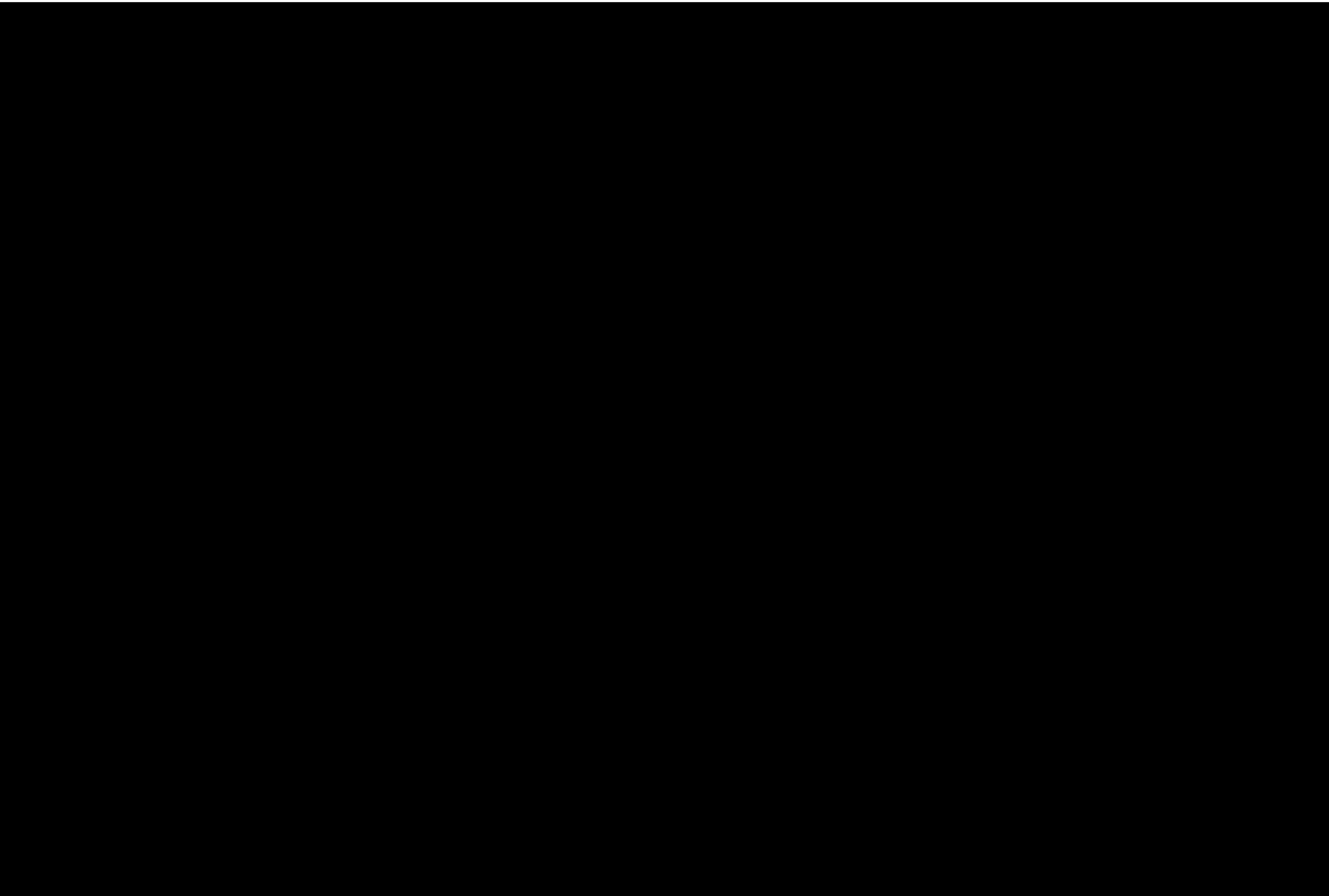


To be concrete: if surface vector points in z direction

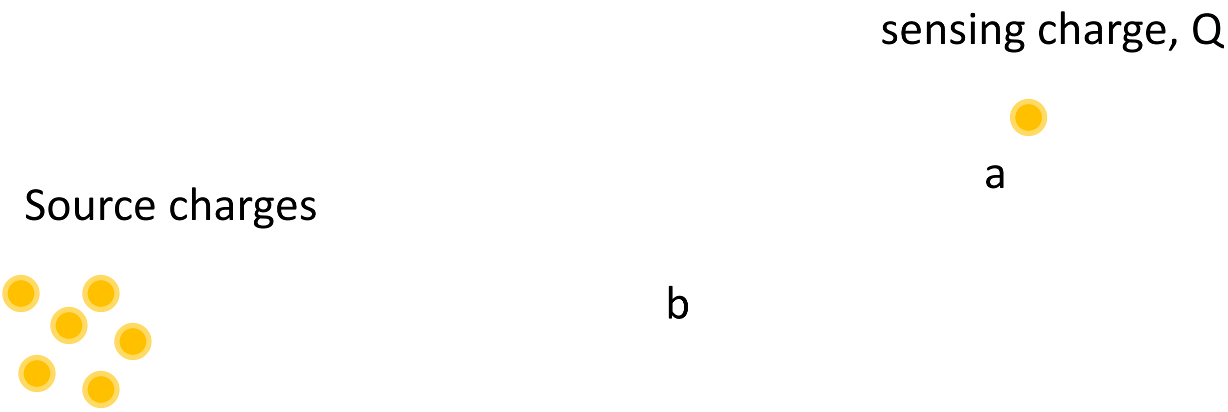
$$\left(\frac{\partial}{\partial z}V_{top} - \frac{\partial}{\partial z}V_{bottom}\right)\hat{z} = \frac{\sigma}{\varepsilon_o}\hat{z}$$

$$\left(\frac{\partial}{\partial z}V_{top} - \frac{\partial}{\partial z}V_{bottom}\right) = \frac{\sigma}{\varepsilon_o}$$

Exercise: Boundary of charged sphere

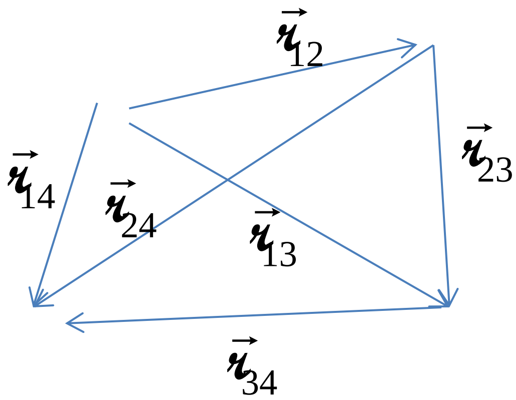


Work to construct charge distribution



$$W(a \rightarrow b) = \int_a^b \vec{F}_{you} \cdot d\vec{l} = -\int_a^b \vec{F}_E \cdot d\vec{l} = -\int_a^b Q\vec{E} \cdot d\vec{l} = Q\left(-\int_a^b \vec{E} \cdot d\vec{l}\right) = Q\Delta V$$

Work to construct charge distribution



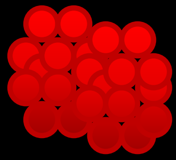
$$W = W_1 + W_2 + W_3 + W_4$$

$$W = 0 + \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{|\mathscr{r}_{12}|} + \left(\frac{1}{4\pi\varepsilon_o}\frac{q_1 q_3}{|\mathscr{r}_{13}|} + \frac{1}{4\pi\varepsilon_o}\frac{q_2 q_3}{|\mathscr{r}_{23}|}\right) + \left(\frac{1}{4\pi\varepsilon_o}\frac{q_1 q_4}{|\mathscr{r}_{14}|} + \frac{1}{4\pi\varepsilon_o}\frac{q_2 q_4}{|\mathscr{r}_{24}|} + \frac{1}{4\pi\varepsilon_o}\frac{q_3 q_4}{|\mathscr{r}_{34}|}\right)$$

$$W = \sum_{i=2}^{n}\sum_{j=1}^{i-1}\frac{1}{4\pi\varepsilon_o}\frac{q_i q_j}{|\mathscr{r}_{ij}|} = \sum_{i}\sum_{j<i}\frac{1}{4\pi\varepsilon_o}\frac{q_i q_j}{|\mathscr{r}_{ij}|}$$

Example: Work/Energy released fissioning U-238

Let's estimate how much energy is released when a ${}^{238}_{92}\mathrm{U}$ nucleus (92 protons and 238 total nucleons) fissions. Uranium can break into a variety of products, but we'll assume that it goes into two identical nuclei with 46 protons and 119 total nucleons each (${}^{119}_{46}\mathrm{Pd}$, a Palladium isotope). The radius for a uranium nucleus is about $10\ \mathrm{f}m = 10\times10^{-15}\mathrm{m} = 10^{-14}\mathrm{m}$, so let's assume that the two "daughter" nuclei start a distance $d = 2\times10^{-14}\mathrm{m}$ apart. For simplicity, we'll treat the nuclei as point charges.



Exercise: Work to assemble charge triangle