Mon.	(C 21.1-.5,.8) 1.2, 2.2.1-.2.2 Gauss & Div, T2 Numerical Quadrature	
Wed.	(C 21.1-.5,.8) 2.2.3 Using Gauss	
Thurs		HW1
Fri.	(C21.1-.5,.8) 2.2.3-.2.4 Using Gauss	

Gauss’s Law

Last Time

$$\vec{E}_{net} = \sum_{i=1} \frac{1}{4\pi\varepsilon_o} \frac{q_i}{\mathscr{r}_i^{\,2}} \hat{\mathscr{r}}_i \xrightarrow[\lim q \to dq]{} \int_{charge} \frac{1}{4\pi\varepsilon_o} \frac{\hat{\mathscr{r}}}{\mathscr{r}_i^{\,2}} \, dq$$

Line charge

Linear charge density

$$\lambda(r') \equiv \frac{dq}{dl'} \Rightarrow dq = \lambda(r') dl'$$

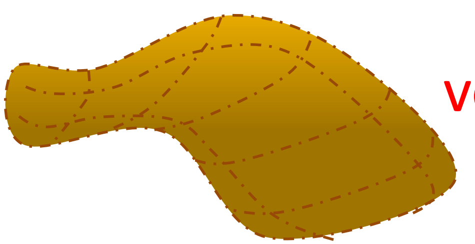
May vary with position

Surface charge

Surface charge density

$$\sigma(r') \equiv \frac{dq}{da'} \Rightarrow dq = \sigma(r') da'$$

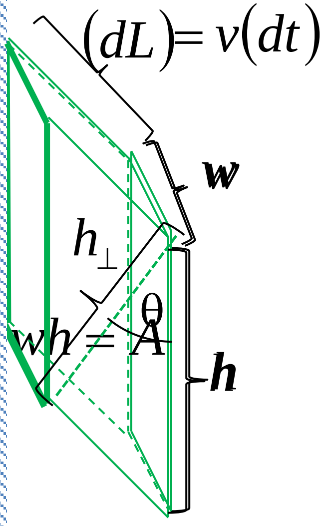
Volume charge



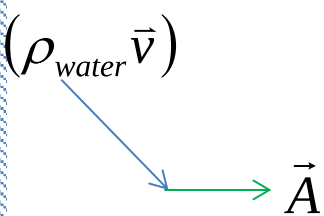
volume charge density

$$\rho(r') \equiv \frac{dq}{d\tau'} \Rightarrow dq = \rho(r') d\tau'$$

Tangible Flux Example: rain through window



$$dVol = (dL)wh_\perp$$
$$\Downarrow$$
$$(h\cos\theta)$$



$$\Phi_{water\to window} = \frac{dm_{water}}{dt}$$

$$\Phi_{water\to window} = \left(\frac{dm_{water}}{dVol}\right)\left(\frac{dVol}{dt}\right)$$

$$\Phi_{water\to window} = \rho_{water}\left(\frac{(dL)w(h\cos\theta)}{dt}\right)$$

$$\Phi_{water\to window} = \rho_{water}(vw(h\cos\theta))$$

$$\Phi_{water\to window} = \rho_{water}vA(\cos\theta)$$

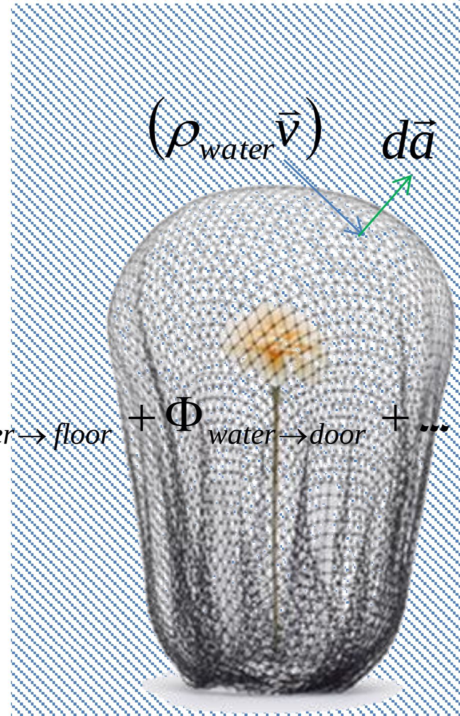
$$\Phi_{water\to window} = (\rho_{water}\vec{v})\cdot\vec{A}$$

If multiple entry areas

$$\Phi_{water\to room} = \Phi_{water\to window1} + \Phi_{water\to window2} + \Phi_{water\to floor} + \Phi_{water\to door} + \dots$$

$$\Phi_{water\to room} = \sum(\rho_i\vec{v}_i)\cdot\vec{A}_i$$

$$\Phi_{water.into.room} = \oint(\rho\vec{v})\cdot d\vec{a}$$

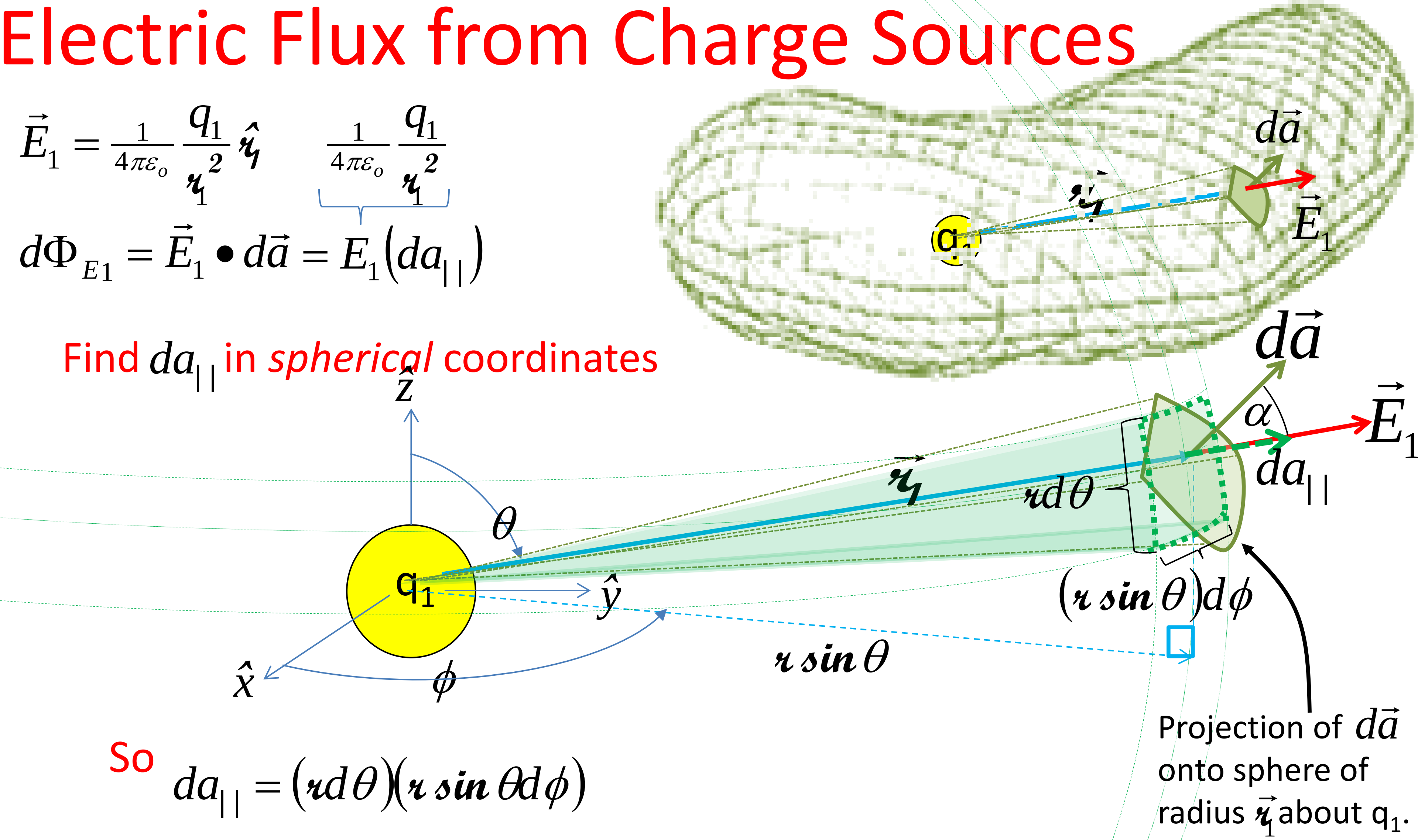


Electric Flux from Charge Sources

$$\vec{E}_1 = \frac{1}{4\pi\varepsilon_o}\frac{q_1}{\mathscr{r}_1^{\,2}}\hat{\mathscr{r}}_1 \qquad \underbrace{\frac{1}{4\pi\varepsilon_o}\frac{q_1}{\mathscr{r}_1^{\,2}}}$$

$$d\Phi_{E1} = \vec{E}_1 \bullet d\vec{a} = E_1\left(da_{||}\right)$$

Find $da_{||}$ in *spherical* coordinates



So

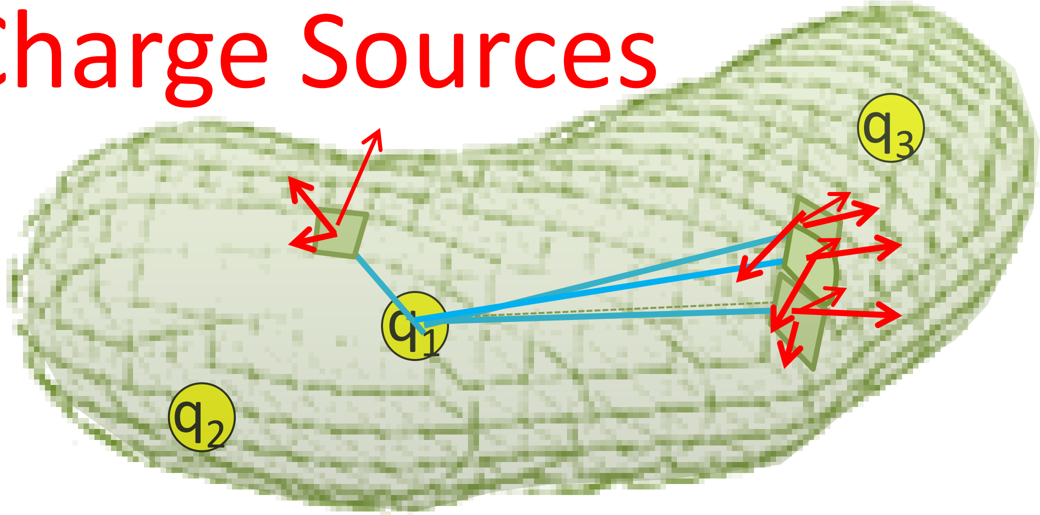
$$da_{||} = (\mathscr{r}d\theta)(\mathscr{r}\,sin\,\theta d\phi)$$

Putting together:

$$d\Phi_{E1} = \vec{E}_1 \bullet d\vec{a} = E_1\left(da_{||}\right) = \left(\frac{1}{4\pi\varepsilon_o}\frac{q_1}{\mathscr{r}_1^{\,2}}\right)(\mathscr{r}d\theta)(\mathscr{r}\,sin\,\theta d\phi) = \left(\frac{1}{4\pi\varepsilon_o}q_1\right)(d\theta)(sin\,\theta d\phi)$$

No distance dependence!

Electric Flux from Charge Sources



out whole, closed surface

$$\oint \vec{E}_1 \cdot d\vec{a} = \Phi_{E1}$$

$$\Phi_{E1} = \oint E_1 da_{||}$$

$$\Phi_{E1} = \frac{q_1}{4\pi\varepsilon_o} \int_{\phi=-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{\theta=0}^{2\pi} d\phi \sin\theta d\theta$$

$$\Phi_{E1} = \frac{q_1}{4\pi\varepsilon_o} 4\pi$$

$$\Phi_{E1} = \frac{q_1}{\varepsilon_o}$$

$$\oint \vec{E}_1 \cdot d\vec{a} = \Phi_{E1} = \frac{q_1}{\varepsilon_o}$$

Ditto for q_2, q_3, …

$$\oint \vec{E}_1 \cdot d\vec{a} = \Phi_{E1} = \frac{q_1}{\varepsilon_o}$$

$$\oint \vec{E}_2 \cdot d\vec{a} = \Phi_{E2} = \frac{q_2}{\varepsilon_o}$$

$$+ \quad \oint \vec{E}_3 \cdot d\vec{a} = \Phi_{E3} = \frac{q_3}{\varepsilon_o}$$

$$\oint \vec{E}_{net} \cdot d\vec{a} = \Phi_{E\,net} = \frac{Q_{net.enclosed}}{\varepsilon_o}$$

Gauss's Law (integral form)

Gauss goes Differential

$$\Phi_E = \oint \vec{E} \cdot \hat{n}\, dA = \frac{Q_{enclosed}}{\varepsilon_0}$$

Divergance

$$\frac{\Phi_\mathrm{E}}{\mathrm{Vol}} = \frac{\oint \vec{E} \cdot \hat{n}\, dA}{Vol} = \frac{1}{\varepsilon_o} \frac{Q_{enclosed}}{Vol} = \frac{1}{\varepsilon_o} \rho_{ave}$$

$$\mathrm{div}\left(\vec{E}\right) \equiv \lim_{Vol \to 0} \frac{\Phi_\mathrm{E}}{\mathrm{Vol}} = \lim_{Vol \to 0} \frac{1}{\varepsilon_o} \rho_{ave} = \frac{1}{\varepsilon_o} \rho_{local}$$

$$\mathrm{div}\left(\vec{E}\right) = \lim_{Vol \to 0} \frac{\oint \vec{E} \cdot \hat{n}\, dA}{Vol}$$

Gauss goes Differential (and relativistic)

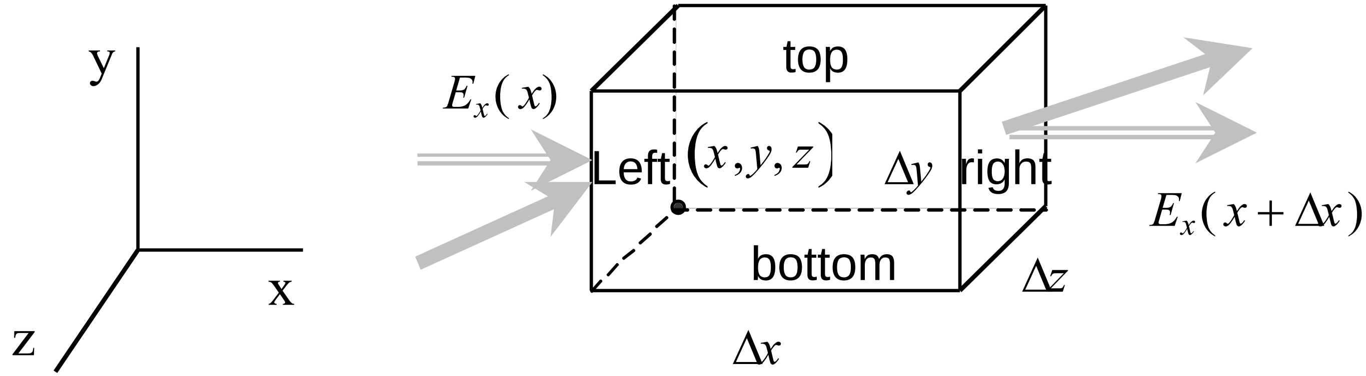
$$\Phi_E = \oint \vec{E} \cdot \hat{n}\, dA = \frac{Q_{enclosed}}{\varepsilon_0}$$

Divergance

$$\frac{\Phi_{\mathrm{E}}}{\mathrm{Vol}} = \frac{\oint \vec{E} \cdot \hat{n}\, dA}{Vol} = \frac{1}{\varepsilon_o} \frac{Q_{enclosed}}{Vol} = \frac{1}{\varepsilon_o} \rho_{ave}$$

$$\mathrm{div}\left(\vec{E}\right) \equiv \lim_{Vol \to 0} \frac{\Phi_{\mathrm{E}}}{\mathrm{Vol}} = \lim_{Vol \to 0} \frac{1}{\varepsilon_o} \rho_{ave} = \frac{1}{\varepsilon_o} \rho_{local}$$

$$\mathrm{div}\left(\vec{E}\right) = \lim_{Vol \to 0} \frac{\oint \vec{E} \cdot \hat{n}\, dA}{Vol}$$



$$\operatorname{div}\left(\vec{E}\right) = \lim_{Vol \to 0} \frac{\oint \vec{E} \cdot \hat{n}\, dA}{Vol} = \frac{1}{\varepsilon_o} \rho_{ave}$$

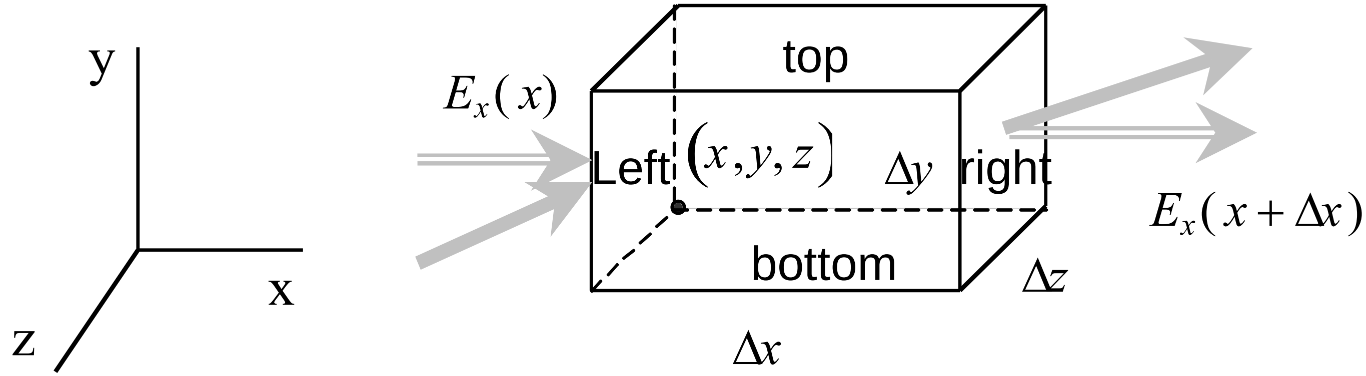
Where as $\text{Vol} \Rightarrow \Delta x \Delta y \Delta z \to 0$

$$\oint \vec{E} \cdot \hat{n}\, dA = \int_{Right} \vec{E} \cdot \hat{n}\, dA + \int_{Left} \vec{E} \cdot \hat{n}\, dA \Rightarrow E_x(x+\Delta x)\Delta y \Delta z - E_x(x)\Delta y \Delta z$$

$$\left[E_x(x+\Delta x) - E_x(x)\right]\Delta y \Delta z$$

$$+ \int_{Front} \vec{E} \cdot \hat{n}\, dA + \int_{Back} \vec{E} \cdot \hat{n}\, dA$$

$$+ \int_{Top} \vec{E} \cdot \hat{n}\, dA + \int_{Bottom} \vec{E} \cdot \hat{n}\, dA$$



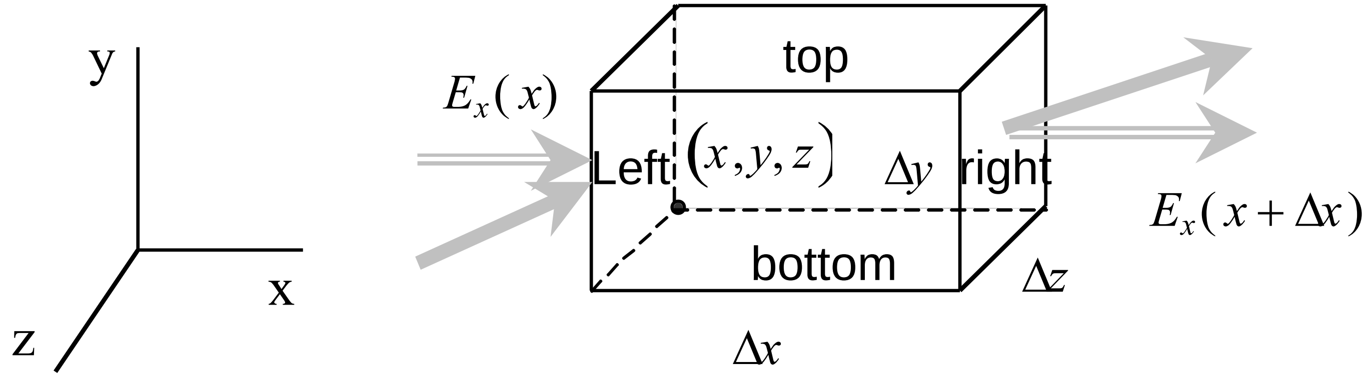
$$\operatorname{div}\left(\vec{E}\right) = \lim_{Vol \to 0} \frac{\oint \vec{E}\cdot\hat{n}\,dA}{Vol} = \frac{1}{\varepsilon_o}\rho_{ave}$$

Where as $\mathrm{Vol} \Rightarrow \Delta x \Delta y \Delta z \to 0$

$$\oint \vec{E}\cdot\hat{n}\,dA = \int_{Right} \vec{E}\cdot\hat{n}\,dA + \int_{Left} \vec{E}\cdot\hat{n}\,dA \Rightarrow \left[E_x(x+\Delta x) - E_x(x)\right]\Delta y \Delta z$$

$$+ \int_{Front} \vec{E}\cdot\hat{n}\,dA + \int_{Back} \vec{E}\cdot\hat{n}\,dA \Rightarrow \left[E_z(z+\Delta z) - E_z(z)\right]\Delta y \Delta x$$

$$+ \int_{Top} \vec{E}\cdot\hat{n}\,dA + \int_{Bottom} \vec{E}\cdot\hat{n}\,dA \Rightarrow \left[E_y(y+\Delta y) - E_y(y)\right]\Delta z \Delta x$$

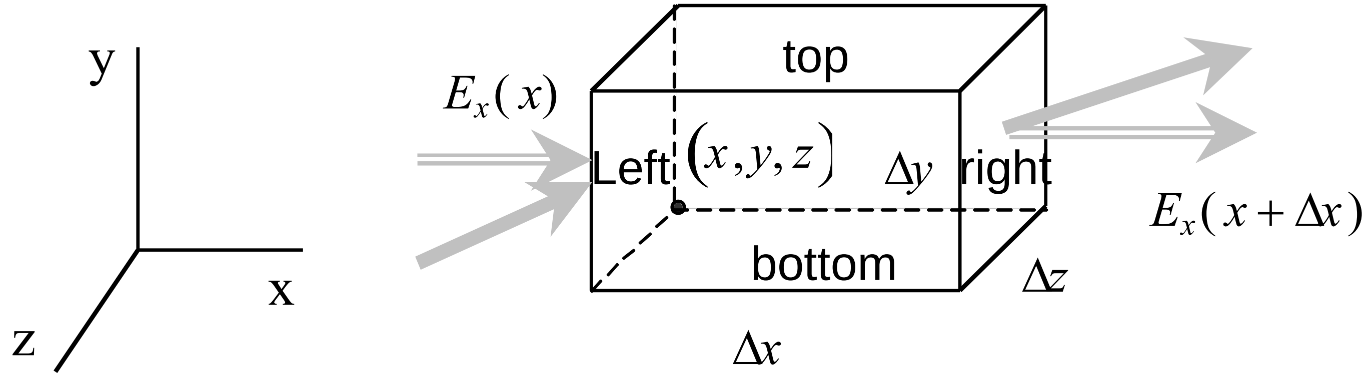


$$\operatorname{div}\left(\vec{E}\right) = \lim_{Vol \to 0} \frac{\oint \vec{E} \cdot \hat{n}\, dA}{Vol} = \tfrac{1}{\varepsilon_o}\, \rho_{ave}$$

$$\frac{\oint \vec{E} \cdot \hat{n}\, dA}{Vol} = \frac{\left[E_x(x+\Delta x) - E_x(x)\right]\cancel{\Delta y}\cancel{\Delta z}}{\Delta x \cancel{\Delta y}\cancel{\Delta z}}$$

$$+ \frac{\left[E_z(z+\Delta z) - E_z(z)\right]\Delta y \Delta x}{\Delta x \Delta y \Delta z}$$

$$+ \frac{\left[E_y(y+\Delta y) - E_y(y)\right]\Delta z \Delta x}{\Delta x \Delta y \Delta z}$$

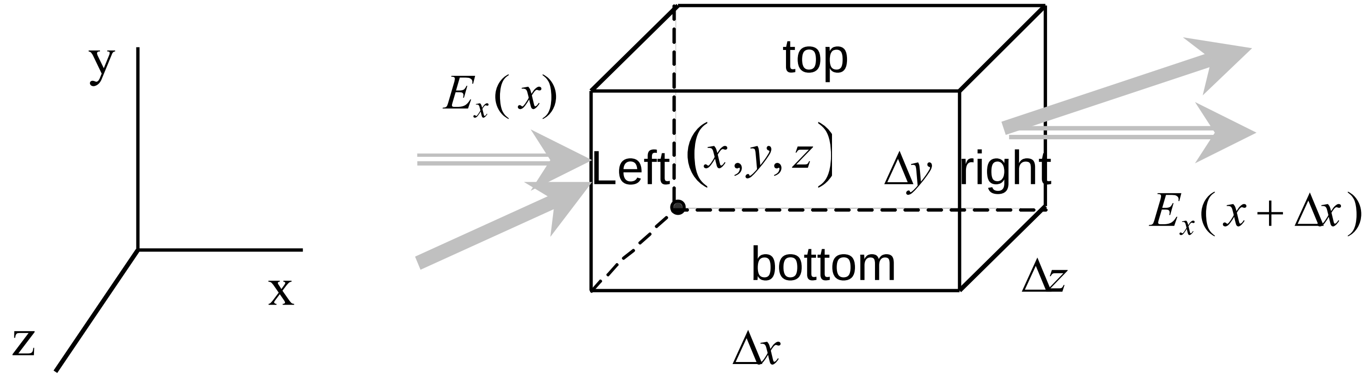


$$\operatorname{div}\left(\vec{E}\right) = \lim_{Vol \to 0} \frac{\oint \vec{E} \cdot \hat{n}\, dA}{Vol} = \frac{1}{\varepsilon_o} \rho_{ave}$$

$$\frac{\oint \vec{E} \cdot \hat{n}\, dA}{Vol} = \frac{\left[E_x(x+\Delta x) - E_x(x)\right]}{\Delta x}$$

$$+ \frac{\left[E_z(z+\Delta z) - E_z(z)\right]}{\Delta z}$$

$$+ \frac{\left[E_y(y+\Delta y) - E_y(y)\right]}{\Delta y}$$

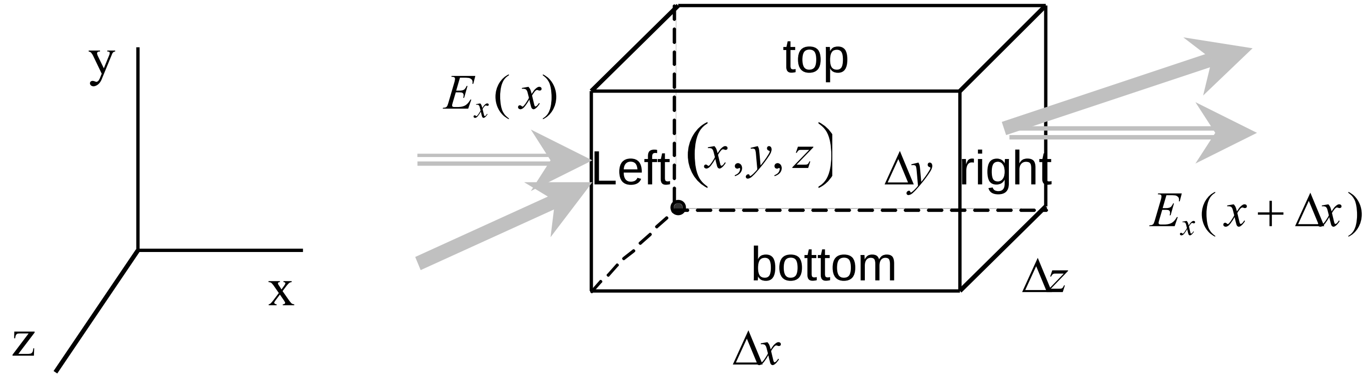


$$\operatorname{div}\left(\vec{E}\right) = \lim_{Vol\to 0} \frac{\oint \vec{E}\cdot\hat{n}\, dA}{Vol} = \tfrac{1}{\varepsilon_o}\,\rho_{ave}$$

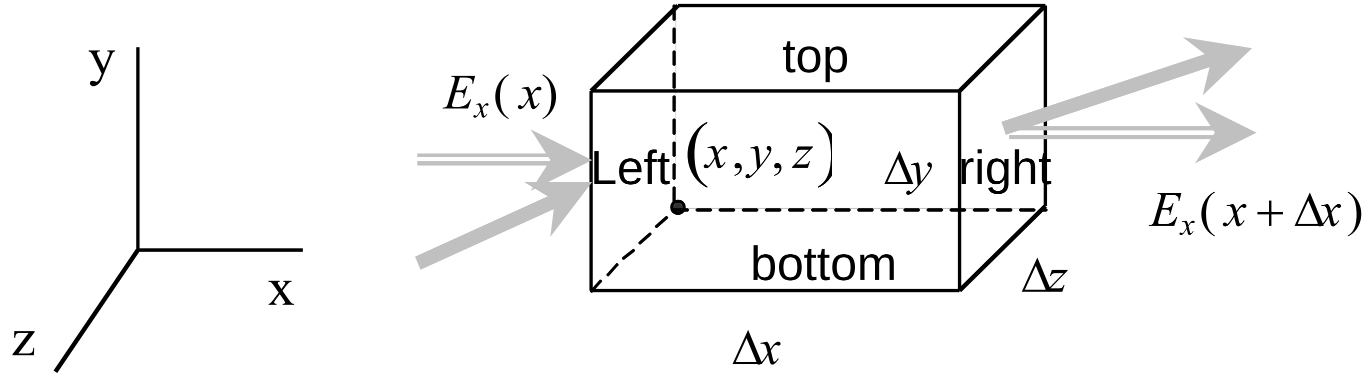
$$\frac{\oint \vec{E}\cdot\hat{n}\, dA}{Vol} = \frac{\left[E_x(x+\Delta x) - E_x(x)\right]}{\Delta x} + \frac{\left[E_z(z+\Delta z) - E_z(z)\right]}{\Delta z} + \frac{\left[E_y(y+\Delta y) - E_y(y)\right]}{\Delta y}$$

$$\lim_{Vol\to 0} \frac{\oint \vec{E}\cdot\hat{n}\, dA}{Vol} = \frac{\partial E_x}{\partial x} + \frac{\partial E_z}{\partial z} + \frac{\partial E_y}{\partial y}$$

$$\operatorname{div}\left(\vec{E}\right) = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = \tfrac{1}{\varepsilon_o}\,\rho_{ave}$$



$$\mathrm{div}\left(\vec{E}\right) = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = \frac{1}{\varepsilon_o}\rho_{ave}$$



$$\mathrm{div}\left(\vec{E}\right) = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = \frac{1}{\varepsilon_o}\rho_{ave}$$

if

$$\vec{\nabla} \equiv \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle$$

then

$$\mathrm{div}\left(\vec{E}\right) = \vec{\nabla} \cdot \vec{E} = \frac{1}{\varepsilon_o}\rho_{ave}$$

Differential form of Gauss's

Exercise

Suppose the electric field (in cylindrical coordinates; using *s* instead of ρ for obvious reasons) is

$$\vec{E} = \begin{cases} Cs\hat{s} & s < a \\ \left(Ca^2/s\right)\hat{s} & s > a \end{cases}$$

What is the charge density in each region?

[redacted]