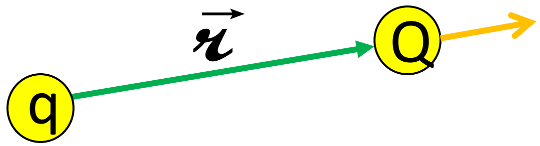
Force between stationary charges

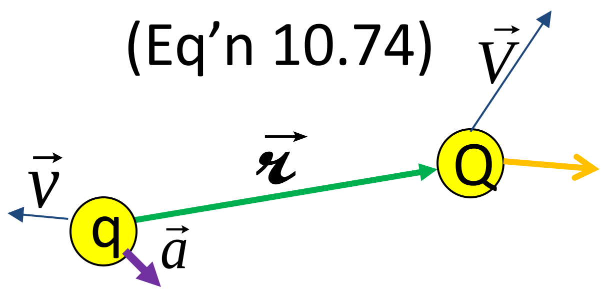
(Coulomb's Law: Eq'n 2.1)



$$\vec{F}_{q\to Q} = \frac{1}{4\pi\varepsilon_o}\frac{qQ}{\mathscr{r}^2_{q\to Q}}\hat{\mathscr{r}} = \frac{qQ}{4\pi\varepsilon_o}\frac{\vec{\mathscr{r}}}{(\mathscr{r})^3}$$

Force between moving charges

(Eq'n 10.74)



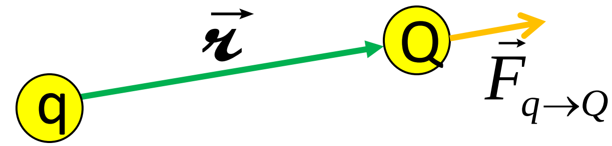
$$\vec{u} \equiv c\hat{\mathscr{r}} - \vec{v}$$

$$\vec{F}_{Q\leftarrow q} = \frac{qQ}{4\pi\varepsilon_o}\frac{\mathscr{r}}{(\vec{\mathscr{r}}\cdot\vec{u})^3}\left\{\left[(c^2-v^2)\vec{u} + \vec{\mathscr{r}}\times(\vec{u}\times\vec{a})\right] + \frac{\vec{V}}{c}\times\left[\hat{\mathscr{r}}\times\left[(c^2-v^2)\vec{u} + \vec{\mathscr{r}}\times(\vec{u}\times\vec{a})\right]\right]\right\}$$

"The entire theory of classical electrodynamics is contained in that equation…but you see why I preferred to start out with Coulomb's law." - Griffiths

Force between stationary charges

(Coulomb's Law: Eq'n 2.1)

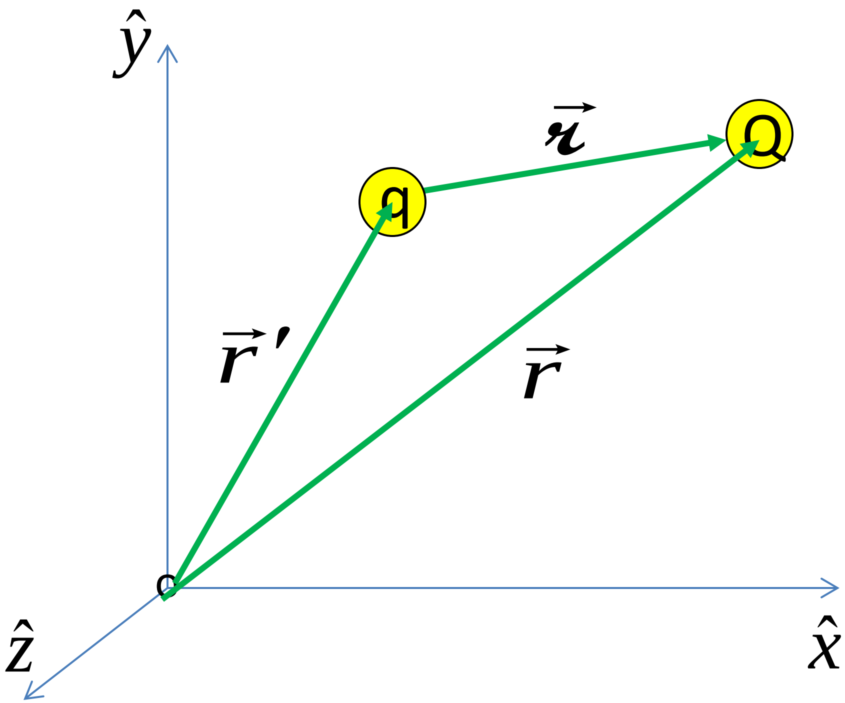


$$\vec{F}_{q\to Q} = \frac{1}{4\pi\varepsilon_o}\frac{qQ}{\mathscr{r}_{q\to Q}^{2}}\hat{\mathscr{r}}$$

$$\varepsilon_o = 8.85\times10^{-12}\,\frac{C^2}{N\cdot m^2}$$

$$\tfrac{1}{4\pi\varepsilon_o} = 8.99\times10^{9}\,\frac{N\cdot m^2}{C^2}$$

Notational Note on Position vectors



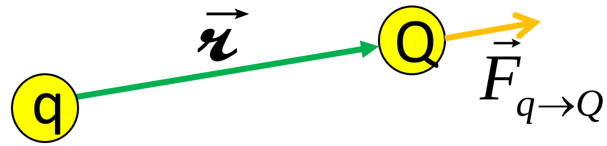
$\vec{r}\,'$ = location of a *source*

$\vec{r}$ = location where we'll evaluate

$\vec{\mathscr{r}} \equiv \vec{r}-\vec{r}\,'$ = where we'll evaluate relative to source

Force between stationary charges

(Coulomb's Law: Eq'n 2.1)



$$\vec{F}_{q\to Q} = \frac{1}{4\pi\varepsilon_o}\frac{qQ}{\mathscr{r}_{q\to Q}^{\,2}}\,\hat{\mathscr{r}}$$

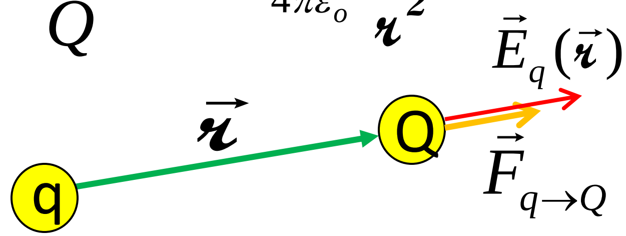
$$\varepsilon_o = 8.85\times10^{-12}\,\frac{C^2}{N\cdot m^2}$$

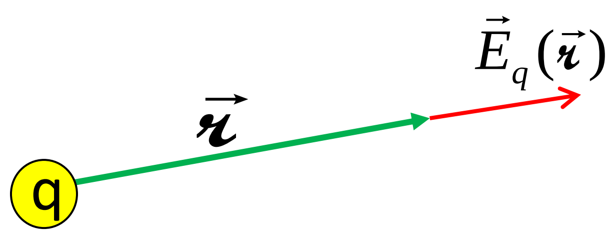
$$\frac{1}{4\pi\varepsilon_o} = 8.99\times10^{9}\,\frac{N\cdot m^2}{C^2}$$

Field of stationary charge

$$\vec{E}_q(\vec{\mathscr{r}}) = \frac{\vec{F}(\vec{\mathscr{r}})_{Q\leftarrow q}}{Q} = \frac{1}{4\pi\varepsilon_o}\frac{q}{\mathscr{r}^2}\,\hat{\mathscr{r}}$$

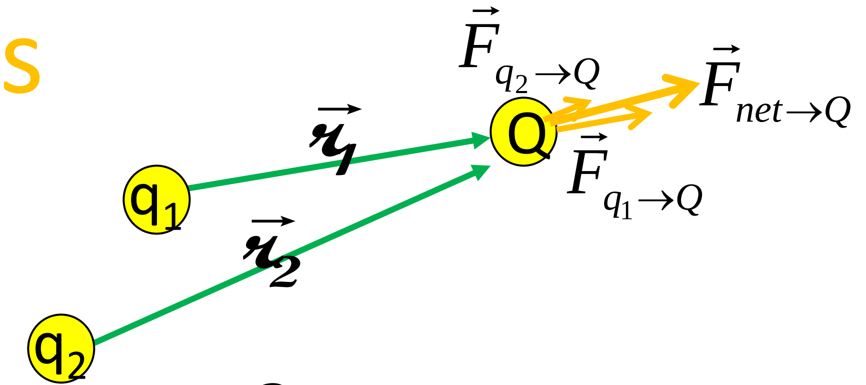
Whether Q is there...





...or not

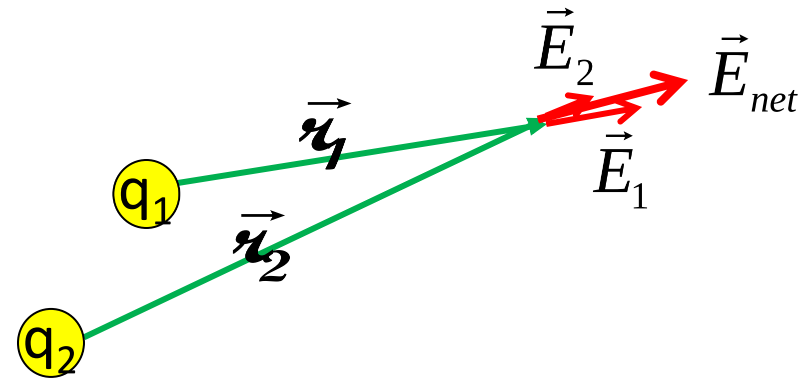
Superposition of Forces



$$\vec{F}_{net \to Q} = \vec{F}_{q_1 \to Q} + \vec{F}_{q_2 \to Q} + \ldots = \sum_{i=1} \vec{F}_{q_i \to Q}$$

$$\vec{F}_{net \to Q} = \frac{1}{4\pi\varepsilon_o} \frac{q_1 Q}{\mathscr{r}_1^{\,2}} \hat{\mathscr{r}}_1 + \frac{1}{4\pi\varepsilon_o} \frac{q_2 Q}{\mathscr{r}_2^{\,2}} \hat{\mathscr{r}}_2 + \ldots = \sum_{i=1} \frac{1}{4\pi\varepsilon_o} \frac{q_i Q}{\mathscr{r}_i^{\,2}} \hat{\mathscr{r}}_i$$

Superposition of Fields



$$\vec{E}_{net} = \vec{E}_1 + \vec{E}_2 + \ldots$$

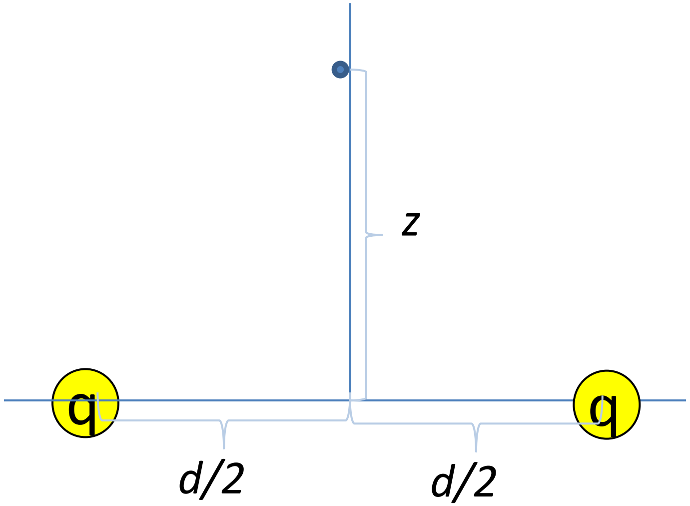
$$\vec{E}_{net} = \frac{1}{4\pi\varepsilon_o} \frac{q_1}{\mathscr{r}_1^{\,2}} \hat{\mathscr{r}}_1 + \frac{1}{4\pi\varepsilon_o} \frac{q_2}{\mathscr{r}_2^{\,2}} \hat{\mathscr{r}}_2 + \ldots$$

Exercises

2.2

(a) Find the electric field (magnitude and direction) a distance *z* above the midpoint between two equal charges, *q,* a distance *d* apart. Check that your result is consistent with what you'd expect when $z \gg d$.

(b) Repeat part (a), only this time make the right-hand charge $-q$ instead of $+q$.



Exercises

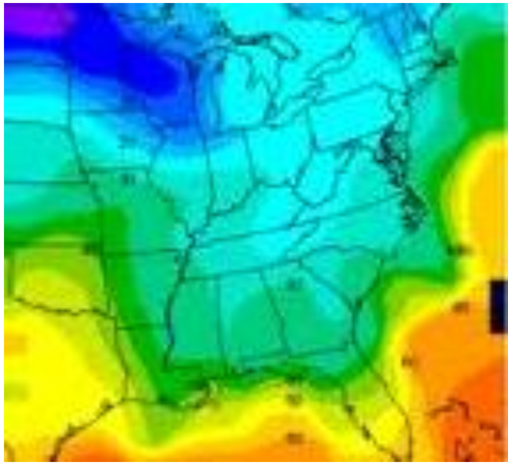
2.1

(a) Twelve equal charges, *q*, are situated at the corners of a regular 12-sided polygon (for instance, one on each numeral of a clock face). What is the net force on a test charge Q at the center?

(b) Suppose *one* of the 12 *q*'s is removed (the one at "6 o'clock"). What is the force on Q? Be prepared to explain your reasoning carefully.

Del Operator

$$\vec{\nabla} \equiv \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$$



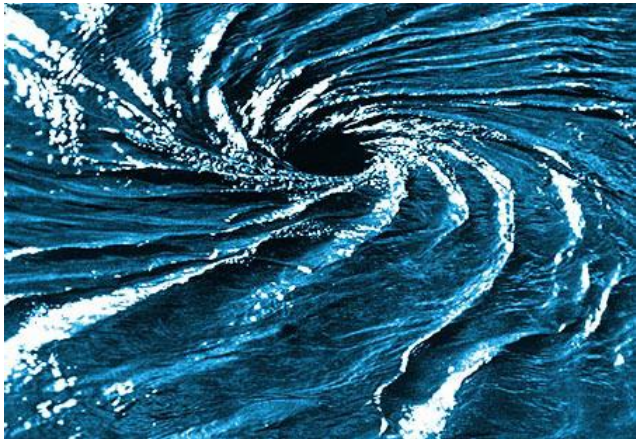
Gradient – vector representing the local slope of a scalar field.

$$\vec{\nabla} T = \frac{\partial T}{\partial x}\hat{x} + \frac{\partial T}{\partial y}\hat{y} + \frac{\partial T}{\partial z}\hat{z}$$

Divergence – scalar representing in/out flow from a point in a vector field.

$$\vec{\nabla} \cdot \vec{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

Curl – vector representing circulation of a vector field.



$$\vec{\nabla} \times \vec{v} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ v_x & v_y & v_z \end{vmatrix} = \hat{x}\left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right) + \ldots$$

Exercises

1.15 ab – find the divergences of the following vector functions

(a) $\vec{v}_a = x^2\hat{x} + 3xz^2\hat{y} - 2xz\,\hat{z}$

(b) $\vec{v}_b = xy\,\hat{x} + 2yz\,\hat{y} + 3zx\,\hat{z}$

1.18 ab – find the curls of the functions above.