Wed. 9/5 Thurs 9/6 Fri. 9/7	**1.1-.3** Intro to Mechanics & to Computation in Mechanics **1.4,.6** Mass, Force, Newton's 1st & 2nd.	**HW0** (Computational Exercise)
Mon. 9/10 Tues 9/11 Wed. 9/12 Thurs. 9/13 Fri. 9/14	**1.5,.7** Newton's 3rd & Polar Coordinates **2.1-.2** Air Resistance - Linear **2.3** Trajectory and Range with Linear Resistance	**HW1** (1.A-.G) **HW2a** (2.A)

Materials:
- White boards
- Group Problem
- Python Code – go over modifications

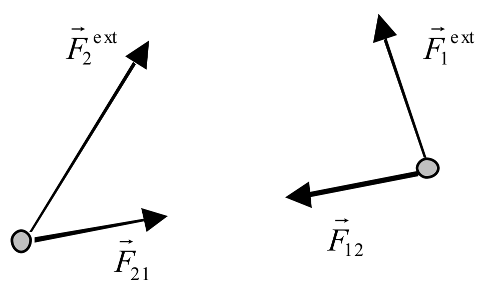
<u>**Newton's Laws**</u>: as they apply to a *point mass* or *particle*

1. **Newton's 1st.** In the absence of forces, a particle moves with constant velocity $\vec{v}$.
    - The velocity is constant in <u>both</u> magnitude and direction.
    - This defines an *inertial frame*, which is a reference frame in which this law holds.
    - A frame in which this does not hold in a *noninertial frame* (e.g. accelerating or rotating relative to an inertial frame – see Ch. 9).
2. **Newton's 2nd.** For any particle of mass *m*, the <u>net</u> force $\vec{F}$ on the particle is always equal to the mass time the particle's acceleration: $\vec{F} = m\vec{a}$.
    - The net force is sometimes written as $\vec{F}_{net}$ or $\Sigma\vec{F}$ to emphasize that it is the sum of all forces
    - This is a <u>vector</u> equation! Don't treat it like a scalar unless the problem is 1-D.
    - It is a second order differential equation.
    - We can introduce *dot notation* for time derivatives:

    $$\vec{a} = \frac{d\vec{v}}{dt} = \dot{\vec{v}} = \frac{d^2\vec{r}}{dt^2} = \ddot{\vec{r}} \qquad \left(\text{and} \quad \vec{v} = \frac{d\vec{r}}{dt} = \dot{\vec{r}}\ \right)$$

    - So, Newton's 2nd can be written as $\vec{F}_{net} = m\ddot{\vec{r}}$ where, of course, the force itself is apt to be a function of time, position, and velocity.
    - If we define the particle's *momentum* as $\vec{p} = m\vec{v}$, then the Second Law can be rewritten as $\vec{F} = \dot{\vec{p}}$, which is sometime convenient.
3. **Newton's 3rd.** If object 1 exerts a force $\vec{F}_{21}$ on object 2, then object 2 always exerts a reaction force $\vec{F}_{12}$ on object 1 given by: $\vec{F}_{12} = -\vec{F}_{21}$.
    - The reaction force is equal in magnitude, but in the opposite direction.
    - A force and the associated reaction force <u>always</u> act on different objects, so they do <u>not</u> cancel.

- I like to think of this as saying *when two objects share and interaction, they have equal and opposite perspectives on it* – 'you're pulling me left', 'no, *you're* pulling *me right.'*
- Completely Tangential to our subject, but of some general interest: Break-down. The book points out that, as intuitive as this may be, when the two parties can't even agree on times and distances, they're not bound to agree about their shared interactions either, that is, when relativistic speeds are involved.
    a. The book's discussion leads to the right moral, but it gives an erroneous impression that the magnetic force is unique in this. The magnetic interaction is the most obvious case, but the same thing happens with the electric force when the two charges are in motion – Griffith's equation 10.67 shows how the force depends on velocity and acceleration, and it isn't symmetric between the two charges that are interacting. The same must be true of gravitation and, I bet it happens with the strong force. The effect is significant only when speeds are near c.
    b. In all cases, the central issue is a relativistic one – different observes going at different speeds will measure the duration of an interaction and the distance of which it occurs differently, so they'll disagree about the forces. So, it's not so much that Newton's 3rd Law breaks down as it is that the agreement about space & time measurements, upon which it's predicated, breaks down.
    c. There are two fixes to the problems this causes for conservation of momentum:
        i. In special relativity we refine our defition of momentum
        ii. With field-mediated interactions we extend our definition to include what I think of as "potential momentum" (in the same spirt as we have potential energy.)
- **<u>Conservation of Momentum</u>**: (see ppt)
- Suppose two particles interact with each other and are each acted on by external forces as shown below.



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- Newton's second law for each particle is:
    - $\dot{\vec{p}}_1 = \vec{F}_1 = \vec{F}_{12} + \vec{F}_1^{\,ext}$
    - $\dot{\vec{p}}_2 = \vec{F}_2 = \vec{F}_{21} + \vec{F}_2^{\,ext}$

- We can define the total momentum on the to particles:
    - $\vec{P} = \vec{p}_1 + \vec{p}_2$.
        - The rate of change of the total momentum is:
    - $\dot{\vec{P}} = \dot{\vec{p}}_1 + \dot{\vec{p}}_2$.
- Using the second law and the third law, $\vec{F}_{12} = -\vec{F}_{21}$, gives:
    - $\dot{\vec{P}} = \left(\vec{F}_{12} + \vec{F}_1^{ext}\right) + \left(\vec{F}_{21} + \vec{F}_2^{ext}\right) = \vec{F}_1^{ext} + \vec{F}_2^{ext} \equiv \vec{F}^{ext}$.
- The change in the total momentum is equal to the total external force on the particles (the vector sum of the external forces on each particle).
- This leads to the Principle of Conservation of Momentum:

• If $\vec{F}^{ext} = 0$, then $\vec{P} = \text{constant}$.

- The same result holds for systems with any number of particles, where the total momentum is:
    - $\vec{P} = \sum_{\alpha} \vec{p}_{\alpha}$,
- and the total external force on the system is:
    - $\vec{F}^{ext} = \sum_{\alpha} \vec{F}_{\alpha}^{ext}$.
- Any force between a pair of particles in the system is an internal force. We will look at conservation of momentum again in Chapter 3.

<u>Differential Equations</u>:

As we said, Newton's 2nd law is a differential equation; depending on the functional dependence of the net force, we'll apply different mathematical techniques to solving Newton's 2nd. Differential equations vary widely in difficulty. We already covered numerical solutions (Euler-Cromer Method) for those that we can't or just don't want to solve analytically. When the dependence on different variables can be brought to separate sides of an equation, the differential equation can be integrated.

<u>Example</u>: Constant Force F_0 in one dimension (call it *x*)

This is the easiest example and the results should be familiar from PHYS 231.

Newton's Second Law can be rewritten as (a scalar equation):

$$\ddot{x} = \frac{F_o}{m}$$
$$\frac{d\dot{x}}{dt} = \frac{F_o}{m}$$
$$d\dot{x} = \left(\frac{F_o}{m}\right)dt$$
$$\int_{\dot{x}_o}^{\dot{x}_f} d\dot{x} = \int_{t_o}^{t_f}\left(\frac{F_o}{m}\right)dt$$
$$\dot{x}_f - \dot{x}_o = \left(\frac{F_o}{m}\right)(t_f - t_o)$$

Rephrasing that, we have

$$\dot{x} - v_o = \left(\frac{F_o}{m}\right)(t - t_o)$$
$$\dot{x} = v_o + \left(\frac{F_o}{m}\right)(t - t_o)$$

similarly:

$$\frac{dx}{dt} = v_o + \left(\frac{F_o}{m}\right)(t - t_o)$$
$$dx = \left(v_o + \left(\frac{F_o}{m}\right)(t - t_o)\right)dt$$
$$\int_{x_o}^{x} dx = \int_{t_o}^{t}\left(v_o + \left(\frac{F_o}{m}\right)(t - t_o)\right)dt = \int_{0}^{t-t_o}\left(v_o + \left(\frac{F_o}{m}\right)t'\right)dt'$$
$$x - x_o = \left[v_o t' + \tfrac{1}{2}\left(\frac{F_o}{m}\right)t'^2\right]_0^{t-t_o}$$
$$\boxed{x = x_o + v_o(t - t_o) + \tfrac{1}{2}\left(\frac{F_o}{m}\right)(t - t_o)^2}$$

Position & velocity dependent forces. In general, a force on a particle can depend on the time, its position, and its velocity. You will do the case of force in one dimension that depends on the position (2.12) for HW#1. Velocity dependent forces are the topic of Chapter 2.

Newton's Second Law in Cartesian Coordinates

In Cartesian coordinates, the Second Law, $\vec{F} = m\ddot{\vec{r}}$, is equivalent to three equations (one for each component):

$$\vec{F} = m\ddot{\vec{r}}$$

$$\begin{pmatrix} F_x \\ F_y \\ F_z \end{pmatrix} = \begin{pmatrix} m\ddot{x} \\ m\ddot{y} \\ m\ddot{z} \end{pmatrix}$$

Much of General Physics I (PHYS 231) is spent on problems with constant forces, especially gravity. You will do a few problems (1.36, 1.38, 1.40) to refresh your memories. Comments:

- In the textbooks notation, $g = +9.8\ \mathrm{m/s^2}$ and $\vec{g}$ is a vector that points downward (what "downward" is in terms of your coordinate system depends on your choice of coordinates).
- It is often helpful to choose the coordinates so that the acceleration in one or two component directions is zero.

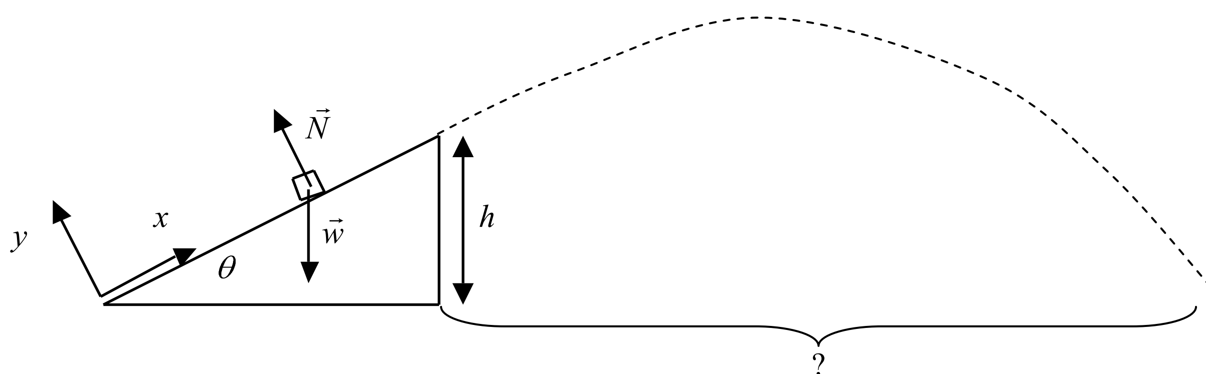
Example: An ice cube is kicked with an initial speed $v_0 = 5$ m/s straight up a ramp with slope of $\theta = 30^\circ$ and height $h = 20\,\mathrm{cm}$. If friction and air resistance can be ignored, how far from the end of the ramp does the ice cube land?

Draw picture, and then Ask them for some ideas on strategy, tools, etc.

Just using force (not energy too – like fighting with one hand behind our backs)

Solution: This problem is essentially two dimensional, since there is no force to the side. It must be divided into two parts: (1) find the speed of the ice cube at the top of the ramp (the direction of the velocity is still along the ramp) and (2) find how far it travels in the air.

(1) For the motion on the ramp, it is convenient to choose coordinates with x upward along the ramp and y perpendicular to the ramp with the origin at the initial position of the ice cube. With this choice, $\ddot{y} = 0$ because the ice slides along the ramp.



The two forces on the ice cube, the weight and the normal force, are drawn in the diagram above. The weight of the ice is $\vec{w} = m\vec{g}$, where in the chosen coordinates $\vec{g} = (-g\cos\theta, -g\sin\theta)$. Newton's Second Law for the two components is:

$$m\ddot{x} = -mg\sin\theta \qquad \text{and} \qquad m\ddot{y} = 0 = -mg\cos\theta + N$$

Integrating the first equation once gives:

$$\int_{v_0}^{\dot{x}(t)} d\dot{x}' = \dot{x}(t) - v_0 = -g\sin\theta \int_0^t dt' = -gt\sin\theta$$

$$\dot{x}(t) = v_0 - gt\sin\theta.$$

Integrate again to get:

$$\int_0^{x(t)} dx' = x(t) = \int_0^t (v_0 - gt'\sin\theta)\, dt' = v_0 t - \tfrac{1}{2} g t^2 \sin\theta$$

Define the time the cube reaches the top of the ramp as t_1 and the speed there as v_1. The *x* coordinate of that point is $x_1 = h/\sin\theta$, so

$$h/\sin\theta = v_0 t_1 - \tfrac{1}{2} g t_1^2 \sin\theta$$

$$\left(\tfrac{1}{2} g\sin\theta\right) t_1^2 + (-v_0) t_1 + h/\sin\theta = 0$$

The solution to this quadratic equation is:

$$t_1 = \frac{v_0 \pm \sqrt{v_0^2 - 4\left(\tfrac{1}{2} g\sin\theta\right)(h/\sin\theta)}}{2\left(\tfrac{1}{2} g\sin\theta\right)}$$

$$t_1 = \frac{v_0 \pm \sqrt{v_0^2 - 2gh}}{g\sin\theta} = \frac{5\text{ m/s} \pm \sqrt{(5\text{ m/s})^2 - 2(9.8\text{ m/s}^2)(0.2\text{ m})}}{(9.8\text{ m/s}^2)\sin 30^\circ}$$

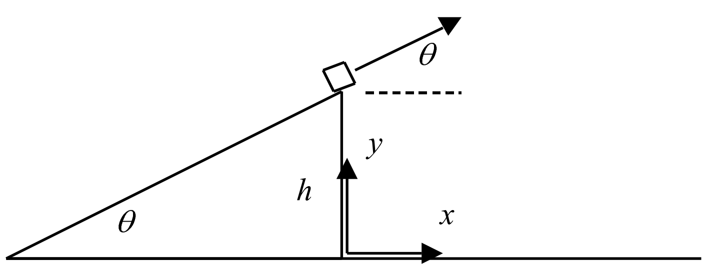
$$t_1 = 0.0834\text{ s}, 1.957\text{ s}.$$

The smaller solution corresponds is the desired solution. The other corresponds to the time it would take the cube to slide back down to that point if the ramp was long. At the top of the ramp, the speed is:

Plug in symbolically rather than numerically, that gives $v_1 = \pm\sqrt{v_o^2 - 2gh}$ *which is exactly what we'd get (more easily) using an energy approach.*

$$v_1 = v_0 - g t_1 \sin\theta = 5\text{ m/s} - (9.8\text{ m/s}^2)(0.0834\text{ s})\sin 30^\circ = 4.59\text{ m/s}$$

(2) For the second part of this problem it is more natural to use coordinates with *x* horizontal and *y* vertical. Choose the origin at ground level below the edge of the ramp.



With the new coordinates, $\vec{g} = (0, -g)$. Since the only force in the air is the weight, Newton's Second Law for the two components is:

$$m\ddot{x} = 0 \qquad \text{and} \qquad m\ddot{y} = -mg\ .$$

The initial conditions for the ice are $\vec{r}_o = (0, h)$ and $\vec{v}_o = (v_1\cos\theta, v_1\sin\theta)$. Integrating the equations above twice gives:

$$\dot{x}(t) = v_1\cos\theta \qquad \text{and} \qquad \dot{y}(t) = v_1\sin\theta - gt$$

$$x(t) = v_1 t\cos\theta \qquad \text{and} \qquad y(t) = h + v_1 t\sin\theta - \tfrac{1}{2}gt^2$$

Define the time t_2 when the ice hits the ground at $y = 0$, which is given:

$$0 = h + v_1 t_2\sin\theta - \tfrac{1}{2}gt_2^2$$

The solution to this quadratic equation is:

$$t_2 = \frac{-v_1\sin\theta \pm \sqrt{(v_1\sin\theta)^2 - 4\left(-\tfrac{1}{2}g\right)h}}{2\left(-\tfrac{1}{2}g\right)} = \frac{-v_1\sin\theta \pm \sqrt{(v_1\sin\theta)^2 + 2gh}}{-g}$$

$$t_2 = \frac{-(4.59\ \text{m/s})\sin 30^\circ \pm \sqrt{\left[(4.59\ \text{m/s})\sin 30^\circ\right]^2 + 2(9.8\ \text{m/s}^2)(0.2\ \text{m})}}{-9.8\ \text{m/s}^2}$$

$$t_2 = 0.687\ \text{s},\ -0.219\ \text{s}$$

The positive solution is the physical one for this problem, so the distance from the ramp when the ice lands is:

$$x(t_2) = v_1 t_2\cos\theta = (4.59\ \text{m/s})(0.687\ \text{s})\cos 30^\circ$$

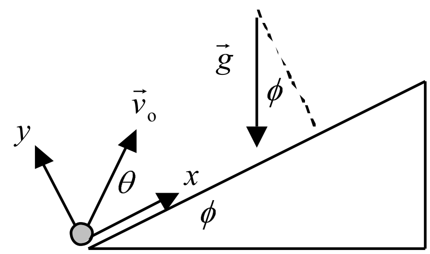
$$\boxed{x(t_2) = 2.73\ \text{m}}.$$

<u>Group Problem</u>**: is this a problem I assigned (by another name?)**(1.39) A golf ball is hit with initial speed v_o up an inclined plane. The plane is inclined at an angle ϕ above the horizontal, and the ball's initial velocity is at an angle θ above the plane. Choose axes with *x* measured up the slope, *y* normal to the slope and *z* across it. (a) Draw a diagram of the situation including coordinate axes and angles. (b) Ignore air resistance. Write down Newton's second law using these axes and find the ball's position as a function of time. (c) Show that the ball lands a distance $R = 2v_o^2\sin\theta\cos(\theta + \phi)/(g\cos^2\phi)$ along the slope. (d) Show that for given v_o and ϕ, the maximum possible range up the inclined plane is $R_{max} = v_o^2/[g(1 + \sin\phi)]$.

<u>Hint</u>: Parts (c) and (d) will require some trigonometric identities. See the front cover of the book.

<u>Solution</u>:

(a) This is essentially a 2-D problem.



(b) The weight of the ball is $\vec{w} = m\vec{g}$, where in the chosen coordinates $\vec{g} = (-g\sin\phi, -g\cos\phi)$. Newton's Second Law for the two components is:

$$m\ddot{x} = -mg\sin\phi \qquad \text{and} \qquad m\ddot{y} = -mg\cos\phi$$

Integrating these equations twice with the initial conditions are $\vec{r}_o = (0,0)$ and $\vec{v}_o = (v_o\cos\theta, v_o\sin\theta)$ gives:

$$\dot{x}(t) = v_o\cos\theta - gt\sin\phi \qquad \text{and} \qquad \dot{y}(t) = v_o\sin\theta - gt\cos\phi$$

$$\boxed{x(t) = v_o t\cos\theta - \tfrac{1}{2}gt^2\sin\phi} \qquad \text{and} \qquad \boxed{y(t) = v_o t\sin\theta - \tfrac{1}{2}gt^2\cos\phi}$$

(c) The ball lands when $y = 0$, so

$$0 = v_o t\sin\theta - \tfrac{1}{2}gt^2\cos\phi = t\left(v_o\sin\theta - \tfrac{1}{2}gt\cos\phi\right),$$

which has solutions $t = 0$ and the desired solution:

$$t = \frac{2v_o\sin\theta}{g\cos\phi}.$$

Plug that time into $x(t)$ to find the distance along the slope to where the ball lands is:

$$R = v_o\left(\frac{2v_o\sin\theta}{g\cos\phi}\right)\cos\theta - \tfrac{1}{2}g\left(\frac{2v_o\sin\theta}{g\cos\phi}\right)^2\sin\phi$$

$$R = \frac{2v_o^2\sin\theta}{g\cos^2\phi}\left(\cos\theta\cos\phi - \sin\theta\sin\phi\right)$$

Use the trig identity $\cos(\theta+\phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$ to get the desired result:

$$R = \frac{2v_o^2\sin\theta\cos(\theta+\phi)}{g\cos^2\phi}.$$

(d) Maximize the range with respect to θ:

$$\frac{dR}{d\theta} = 0 = \left(\frac{2v_o^2}{g\cos^2\phi}\right)\frac{d}{d\theta}\left[\sin\theta\cos(\theta+\phi)\right],$$

so:

$$0 = \frac{d}{d\theta}\left[\sin\theta\cos(\theta+\phi)\right] = \cos\theta\cos(\theta+\phi) - \sin\theta\sin(\theta+\phi),$$

Using the same identity as in part (c) gives:

$$\cos(2\theta + \phi) = 0.$$

The angle of the slope must satisfy $\phi < \pi/2$ and the other angles must satisfy $\theta < \pi/2$ or the ball is not going up the slope, so the appropriate solution is:

$$2\theta + \phi = \pi/2$$

$$\theta = \pi/4 - \phi/2.$$

Substitute this into the result from part (c):

$$R_{\max} = \frac{2v_o^2 \sin(\pi/4 - \phi/2)\cos(\pi/4 - \phi/2 + \phi)}{g\cos^2\phi} = \frac{2v_o^2 \sin(\pi/4 - \phi/2)\cos(\pi/4 + \phi/2)}{g\cos^2\phi}.$$

Use the trig identity $\sin A \cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$ to get:

$$R_{\max} = \frac{2v_o^2}{g\cos^2\phi} \cdot \frac{1}{2}\left[\sin(\pi/2) + \sin(-\phi)\right] = \frac{v_o^2}{g\cos^2\phi}(1 - \sin\phi).$$

Multiply and divide by $(1 + \sin\phi)$ and simplify to get the desired result:

$$R_{\max} = \frac{v_o^2}{g\cos^2\phi}(1 - \sin\phi)\frac{(1 + \sin\phi)}{(1 + \sin\phi)} = \frac{v_o^2}{g\cos^2\phi}\frac{(1 - \sin^2\phi)}{(1 + \sin\phi)} = \frac{v_o^2}{g(1 + \sin\phi)}.$$

Next two classes:

- Monday – Polar coordinates & momentum conservation
- Wednesday – Start Ch. 2 (air resistance)