Physics 310
Lecture 6b – Op Amps

Fri. 2/19	**Ch 9.1-.5, .9, App B-3:** Operational Amplifiers	**HW5**: * ; **Lab 5 Notebook**
Mon. 2/22 Wed. 2/24 Thurs. 2/25 Fri. 2/26	**Ch 9,** the rest **Quiz** Ch 9, **Lab 6:** Operational Amplifiers More of the same **Midterm Review** (through Transistors)	 **HW 6:** Ch9 Pr 1,3*, 4*, 5, 21 **Lab 6 Notebook**

Announcements

- Midterm just after break.

Handout (in lecture):

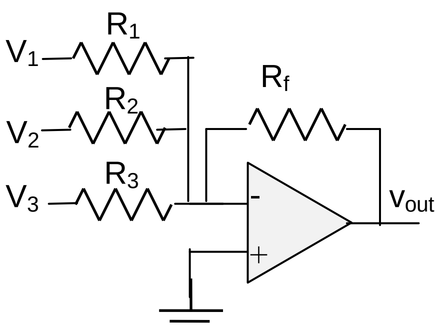
- Lab #6

9-6 Mathematical Functions: Addition, Differentiation, Integration

One of the swell things about op-amps is that you can build circuits with them that perform mathematical operations. On the one hand, that's a cute novelty item – a circuit that will add, integrate, or differentiate for you. No matter how *non-analytical* the input function may be, the circuit happily integrates or differentiates it for you! On the other hand, these are very common signal-processing operations; within the context of a large circuit, you'll often find little 'adder', 'integrator', and 'differentiator' blocks.

9-6.1 Addition

We'll start with the simplest circuit, the adder.



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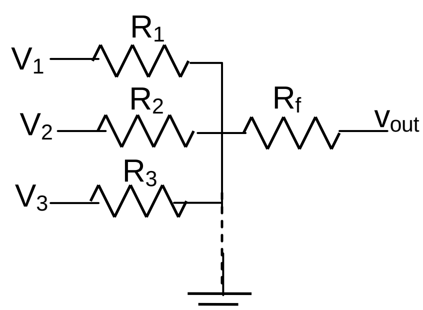


Since the Op-Amp itself draws no current, but forces its negative input to ground, we know that the current flowing through the gain resistor is

$$i_f = i_1 + i_2 + i_3$$

$$\frac{V_{out} - 0}{R_F} = \frac{0 - V_1}{R_1} + \frac{0 - V_2}{R_2} + \frac{0 - V_3}{R_3}$$

Or

$$V_{out} = -R_F\left(\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}\right)$$

Now, in the mathematically simplest case, say you choose your four resistors so they all have the same numeric value: $R_F = R_1 = R_2 = R_3$. Then this expression reduces to

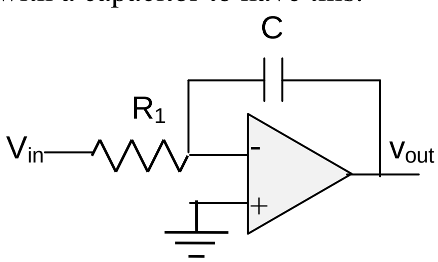
$$V_{out} = -(V_1 + V_2 + V_3)$$

So (aside from inverting, with the negative sign) you've just added the values on each of the three input lines to give the output value.

Of course, you could also weight each input signal differently. For example, say you wanted $V_{out} = -(1*V_1 + 10*V_2 + 100*V_3)$ that's easily achieved by choosing the right resistors.

9-6.2 **Integration** (I do)

We go back to the simple inverting-amplifier configuration and replace the gain resistor with a capacitor to have this.



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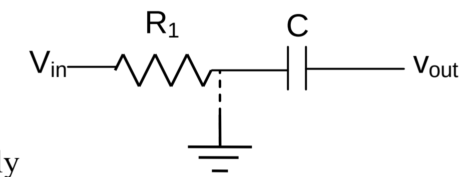


Looking at the flow of charge,

$$\frac{\Delta v_R}{R} = -i$$

$$\frac{0 - v_{in}}{R} = -\frac{dq_c}{dt}$$

(I'm using lower case since we're likely considering time varying voltages, currents, and charges.)

Where what I mean by q_c is the charge on the left side of the capacitor.

Now, as for the capacitor,

$$q_c = -C\Delta v_c$$

$$q_c = -C(v_{out} - 0)$$

(as with the sign in Ohm's law, books usually ignore the sign here, but it means that if you've got a positive charge on the left, then you've got a voltage *drop* from left to right.)

So, substituting this in for the charge in the derivative,

$$\frac{0 - v_{in}}{R_1} = -i = -\frac{dq_c}{dt} = -\frac{d}{dt}\left(C(v_{out} - 0)\right) = C\frac{dv_{out}}{dt}$$

Flipping around to solve for V_{out} gives

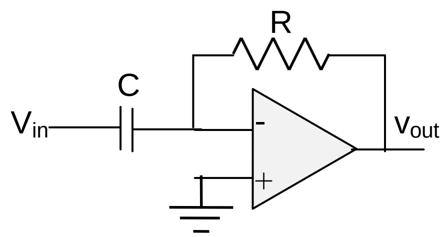
$$v_{out}(t) = -\tfrac{1}{R_1 C}\int_0^t v_{in}(t')dt'$$

The output is the (scaled and inverted) time-integral of the input!

Remember the simple RC circuit which only *approximated* an integrator for the right range of frequencies. Now, this circuit *is* an integrator (to as many sig figs as 1/A = 0 and as long as the frequency isn't insanely high).

9-6.3 Differentiation (They do)

If you flip the R and the C, you flip the job this performs, form integrating to differentiating.



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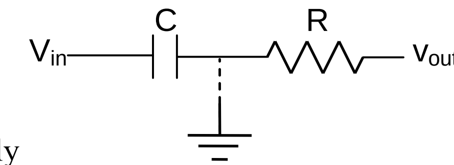


Again, being specific about signs, the relationship between a capacitor's charge and voltage drop is

$$q_c = -C\Delta v_c$$

In this case,

$$\Delta v_c = 0 - v_{in}$$

so,

$$Cv_{in} = q_c$$

While the relationship between a resistor's current and voltage drop is

$$i = -\frac{\Delta v_R}{R}$$

Where the current flowing through the resistor (implicitly to the right) equals the rate of charge flowing onto the capacitor (from the left):

$$i = \frac{dq_c}{dt}$$

$$\frac{v_{out}}{R} = -i = -\frac{dq_{Left}}{dt} = -\frac{dCv_{in}}{dt}$$

$$v_{out} = -RC\frac{dv_{in}}{dt}$$

Tada! The output voltage is equal to the (negative) derivative of the input voltage, times RC.

9-8 Filters

We've seen how an Op-Amp can be used as an Integrator and how one can be used as a Differentiator. Now, you've previously seen similar combinations or R's and C's (without the Op-Amps) in "RC" filters. You can see that the Integrator and Differentiator can perform a similar function – Preferentially 'passing' either high or low frequency signals.

Differentiator – High-Pass Filter / Amplifier

The differentiator's output and input are related by

$$v_{out} = -RC\frac{dv_{in}}{dt}$$

So, if the input voltage varies sinusoidally, $v_{in}(t) = |V_{in}|\sin(\omega t)$, then the output would be

$$v_{out}(t) = -RC\frac{d}{dt}v_{in}(t) = -RC\omega|V_{in}|\cos(\omega t)$$

Notice that the *bigger* ω is, i.e., the *higher* the frequency is, the bigger the output signal is. More specifically,

$$\omega < \tfrac{1}{RC} \Rightarrow |V_{out}| < |V_{in}|$$
$$\omega > \tfrac{1}{RC} \Rightarrow |V_{out}| > |V_{in}|$$

This is essentially a High Pass filter and an amplifier combined.

Integrator – Low-Pass Filter / Amplifier

On the other hand, the Integrator's output and input are related by

$$v_{out}(t) = -\tfrac{1}{R_1 C}\int_0^t v_{in}(t')dt'$$

Again, if the input voltage varies sinusoidally, $v_{in}(t) = |V_{in}|\sin(\omega t)$, then the output would be

$$v_{out}(t) = -\tfrac{1}{R_1 C}\int_0^t v_{in}(t')dt' = \tfrac{1}{R_1 C}\frac{1}{\omega}|V_{in}|\cos(\omega t)$$

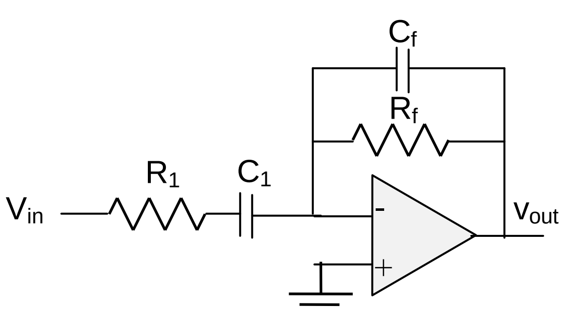
In this case, the *smaller* ω, i.e., the *lower* the frequency is, the bigger the output signal is. More specifically,

$$\omega < \tfrac{1}{RC} \Rightarrow |V_{out}| > |V_{in}|$$
$$\omega > \tfrac{1}{RC} \Rightarrow |V_{out}| < |V_{in}|$$

This is essentially a *Low* Pass filter and amplifier.

9-8.1 Integrator-Differentiator

Now, let's say we put the two together! A Low-Pass Filter Integrator *and* a High-Pass Filter Differentiator: we get a *Band – Pass* Integrator-Differentiator. We might qualitatively guess that most frequencies come through weakly, but right around the sweet spot of $\omega = \dfrac{1}{RC}$ the signal comes through loud and clear. Let's see how this plays out.



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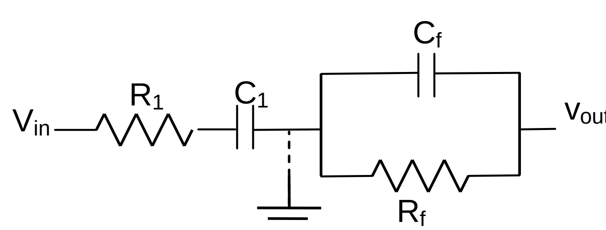


This circuit has a complex enough mixture of impeding elements, resistors and capacitors, that we're going to want to use Phasors to analyze it.

Following the current flow across the circuit,

$$\vec{I}_1 = \vec{I}_f$$

$$-\frac{0-\vec{V}_{in}}{\vec{Z}_1} = -\frac{\vec{V}_{out}-0}{\vec{Z}_f}$$

So,

$$\vec{V}_{out} = -\vec{V}_{in}\frac{\vec{Z}_f}{\vec{Z}_1}$$

Where, the two elements in series add up to $\vec{Z}_1 = R_1 + \vec{Z}_{C1} = R_1 - j\frac{1}{C_1\omega}$

While the two elements in parallel add up to $\vec{Z}_f = \frac{1}{\frac{1}{R_f}+\frac{1}{\vec{Z}_{Cf}}} = \frac{1}{\frac{1}{R_f}+jC_f\omega}$

So,

$$\vec{V}_{out} = -\vec{V}_{in}\frac{1}{R_1 - j\frac{1}{C_1\omega}}\frac{1}{\frac{1}{R_f}+jC_f\omega} = -\vec{V}_{in}\frac{1}{\frac{R_1}{R_f}+\frac{C_f}{C_1}+j\left(R_1C_f\omega - \frac{1}{R_fC_1\omega}\right)}$$

In Amplitude-Phase notation, that's

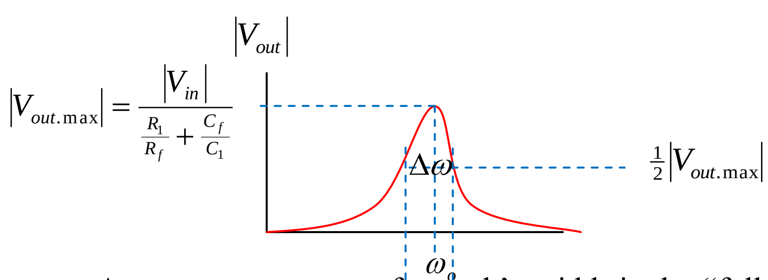
$$\vec{V}_{out} = -\vec{V}_{in}\frac{1}{\sqrt{\left(\frac{R_1}{R_f}+\frac{C_f}{C_1}\right)^2+\left(R_1C_f\omega - \frac{1}{R_fC_1\omega}\right)^2}}e^{-j\tan^{-1}\left(\frac{R_1C_f\omega-\frac{1}{R_fC_1\omega}}{\frac{R_1}{R_f}+\frac{C_f}{C_1}}\right)}$$

Obviously, the denominator minimizes (the output voltage maximizes) *and* the phase shift is just the -1 that's out front, at the frequency ω_o for which

$$\left(R_1C_f\omega_o - \frac{1}{R_fC_1\omega_o}\right) = 0 \text{ or } \omega_o = \sqrt{\frac{1}{R_1C_1R_fC_f}}$$

There we have the predicted sweet-spot frequency.

A plot of V_{out} vs. frequency would look something like



A common measure of a peak's width is the "full-width at half-max." As the name suggests, it's how far apart (in frequency, in this case) are the two points at which the curve drops to half its maximal value. So, it can be found by returning to the expression for V_{out} and setting its amplitude equal to half the peak, then solving for the frequencies that satisfy that condition – the difference between these two frequencies is the width of the peak. Skipping all that work, this peak's width is

$$\Delta\omega = \sqrt{\omega_f^2+\omega_1^2+6\omega_o^2} \qquad \text{where } \omega_f \equiv \frac{1}{R_fC_f} \text{ and } \omega_1 \equiv \frac{1}{R_1C_1};$$

in those terms, $\omega_o^2 = \omega_f \omega_1$

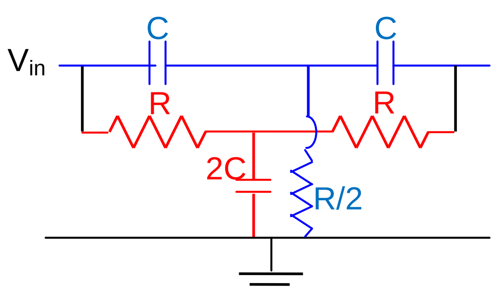
One thing we can read from this is that, in absolute terms, the higher the circuit's designed pass-frequency, ω_o, the wider the peak, but in relative terms (relative to the pass-frequency), the peak isn't exactly dependent on the frequency, rather it depends on how well balanced ω_f and ω_1 are, with a minimum if $\omega_f = \omega_1$:

$$\frac{\Delta\omega}{\omega_o} = \sqrt{\tfrac{\omega_f}{\omega_1} + \tfrac{\omega_1}{\omega_f} + 6} = \sqrt{8}$$

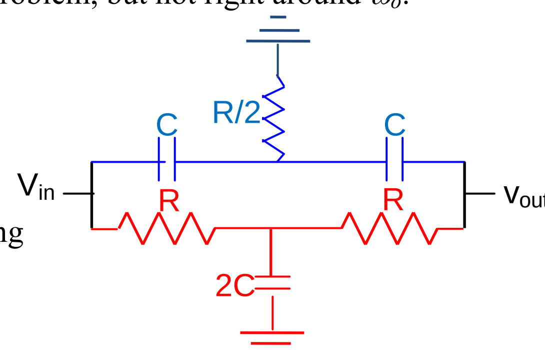
9-8.2 Twin-T Filter

Strictly speaking, this would have fit better in Ch. 2; however, it's good review (test coming up) and we'll use it with an op-amp in a moment. First we'll consider a Twin-T filter all by itself (that we could have done back in chapter 2) and then we'll see how it can be incorporated in an Op-Amp circuit.

All by itself, the Twin-T filter has an impedance that peaks at a specific frequency – that means it's a ***notch-pass*** filter. That is, the output spectrum looks like a notch was cut out of it – most frequency signals pass without a problem, but not right around ω_o.

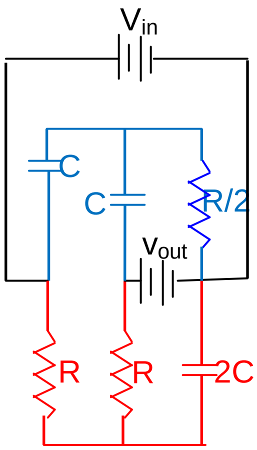


Or, unwrapping it a bit

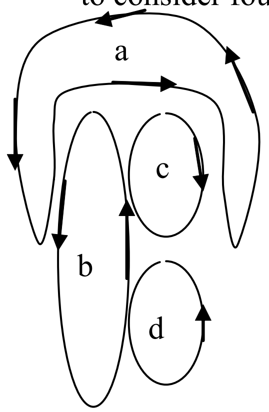


Qualitatively, there are a *high* pass filter (blue) and a *low* pass filter (red) in parallel. So a signal with a high frequency is blocked on the red branch, but, no matter, it passes along the blue branch. Similarly, a low-frequency signal is blocked on the blue branch but passes along the red branch. In contrast, a signal with frequency around $\omega_o = \dfrac{1}{RC}$ has trouble down *either* branch.

Quantitatively, it's easier to analyze the circuit if we look at it like this (all the same connections, just twisted around a bit.)



One way to tackle this is to consider four loops:



Applying the Loop Rule to each gives

a) $v_{in} = (i_a - i_b)Z_C + (i_a + i_c)R/2$ where $Z_C = -j\dfrac{1}{C\omega}$

b) $0 = i_b R + (i_b - i_d)R + (i_b + i_c)Z_C + (i_b - i_a)Z_C$

c) $v_{out} = (i_c + i_b)Z_C + (i_c + i_a)R/2$

d) $v_{out} = (i_d - i_b)R + i_d z_{2C}$

Not counting v_{in} and v_{out}, we have 4 equations and 4 unknowns (the currents), so a relationship can be constructed for v_{out} in terms of v_{in}, R, and Z_C. Without going through all the motions, that relationship is

$$v_{out} = \frac{v_{in}}{\left(1 + j\dfrac{4}{\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}}\right)}$$

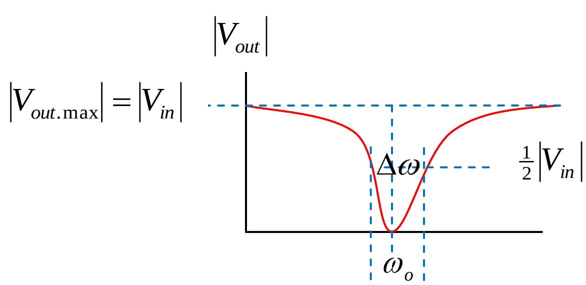
(based on eq'n 3-82 and Fig. 3-17 of J.J. Brophy's Basic Electronics for Scientists, where the C's and R's assume the values to match Fig 9.17 of Diefenderfer's book.)

or, in amplitude – phase notation,

$$v_{out} = \frac{v_{in}}{\sqrt{1 + \left(\dfrac{4}{\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}}\right)^2}}\, e^{-j\tan^{-1}\left(\frac{4}{\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}}\right)}$$

where $\omega_o = \dfrac{1}{RC}$.

Strikingly, v_{out} doesn't just get *small* at ω_o; it *vanishes!* For that matter, when we're far from ω_o, v_{out} approaches v_{in}.



A similar full-width at half-max analysis to that used for the Integrator-Differentiator Filter reveals a width of $\Delta\omega = \frac{4}{\sqrt{3}}\omega_o$, or a relative width of $\frac{\Delta\omega}{\omega_o} = \frac{4}{\sqrt{3}}$ which is slightly narrower than the best that the Integrator-Differentiator could do (when $\omega_1 = \omega_f$), $\sqrt{8} = \frac{4}{\sqrt{2}}$.

Impedance. In terms of impedance, the impedance of Twin-T filter vanishes far from ω_o and explodes at ω_o

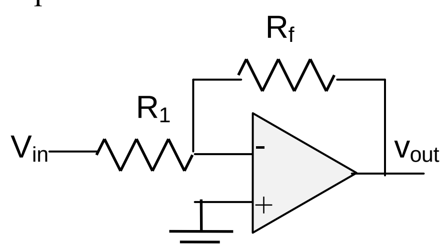
$Z_T \to 0$ for $\omega >> \omega_o$ or $\omega << \omega_o$

$Z_T \to \infty$ as $\omega \to \omega_o$

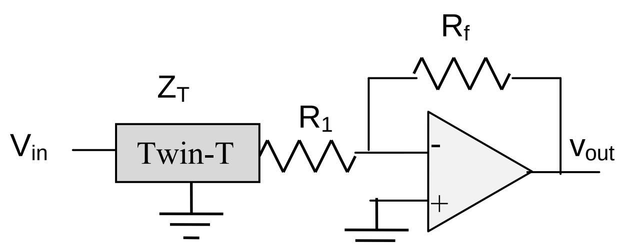
Frequency Switch. This is kind of like a switch: for most frequency signals, the switch is closed – allowing the signal to pas; for signals with frequency ω_o, the switch is flung open – blocking the signal.

With an Op-Amp. This Twin-T filter could be used on its own, or it could be used in conjunction with an Op-Amp (that's the chapter we're in, after all). Again, the reason for going this rout rather than using a simple Integrator-Differentiator configuration is that the Twin-T is more selective (narrower band).

For thinking how it might work with an op-amp, it's handy to think in terms of the impedance. Let's look back at the simple inverting amplifier configuration.



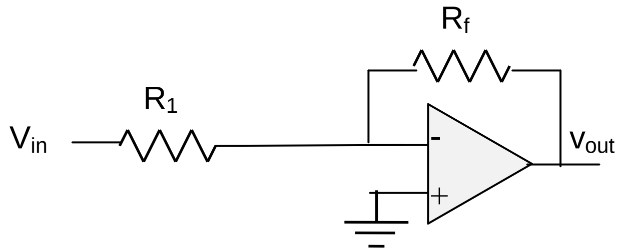
Now, say we augment R1 with a Twin-T,



So, the output voltage should be

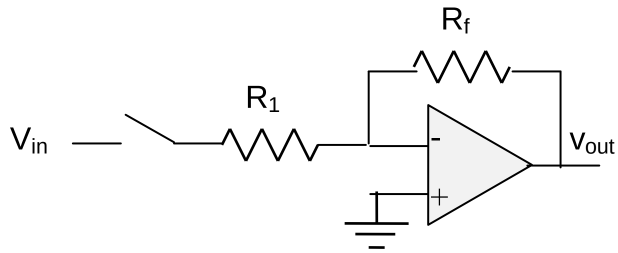
$$V_{out} = -V_{in}\frac{R_f}{(R_1 + Z_T)}$$

Now, for frequencies far from ω_o, the Twin-T is as good as not-there (no impedance), and, just as usual,



$$V_{out} \approx -V_{in}\frac{R_f}{(R_1)} \text{ when } \omega >> \omega_o \text{ or } \omega << \omega_o$$

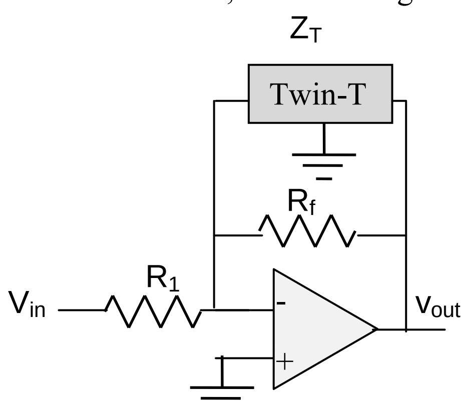
But when in the vicinity of $\omega = \omega_o$ the impedance blows up, killing the output



$$V_{out} \approx -V_{in}\frac{R_f}{\infty} = 0$$

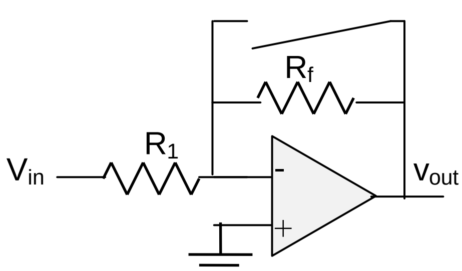
This isn't too surprising, we essentially have a Twin-T in series with an Inverting Amplifier, so of courses the amplifier's signal cuts out when the Twin-T kills its input signal.

On the other hand, we could augment the gain resistor, R_f, with the Twin-T.



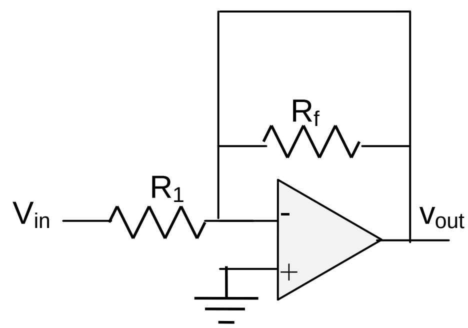
Now, $$V_{out} = -V_{in}\frac{R_f Z_T/(Z_T + R_f)}{R_1}$$

Right around $\omega = \omega_o$, Z_T goes infinite, so it's like having an open switch above the gain resistor. In that case, the we simply have



$$V_{out} = -V_{in}\frac{R_f}{R_1}$$ As if the Twin-T weren't there.

Far from this frequency, the Twin-T has no resistance, so we've essentially got



$V_{out} = 0$

So the gain resistor is shorted out and we've got a follower with an input of ground. Sure, there's a V_{in} on the other side of R_1, but if the Op-Amp's doing it's thing, it's forcing $V_-=V_+$ and $V_+ = 0$.

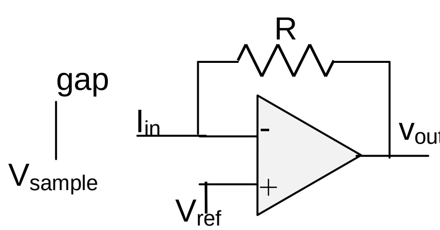
The advantage of using a complicated Twin-T filter rather than a simple Integrator-Differentiator one is that, as already noted, the Twin-T filter is more selective:

9-7 Current Amplifiers

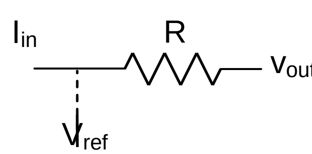
The point of many circuits is manipulating a "signal" that is represented by a voltage – multiply the signal, integrate the signal, add the signal to another one… Sometimes though, the salient property is a current, not a voltage. For example, your measure a current, and you want to translate that "signal" into a voltage (for adding, integrating,…), or perhaps your building a power supply so you want to be able to boost up a current. So, here are a couple of op-amp circuits that focus in on the *current* rather than the *voltage.*

9-7.1 Current to Voltage Conversion

Some transducers output a current (ex. near and dear to my heart, the STM senses a tiny current between a sample and a sharp, metal tip.) Let's look at how the *current* input and *voltage* output are related.



which is equivalent to

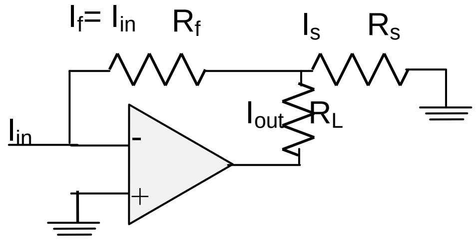


Here, for the sake of generality, I've set the + terminal at "V_{ref}", that 'reference' voltage could be ground (as in the previous examples), but it needn't be. I should also point out that there is effectively a great big (mega-ohm) resistor between the sample and the tip/input – if there weren't we'd have an unpleasant tug of war between V_{sample} and V_{ref} – the sample and the op-amp would be trying to make the – terminal have two different voltages.

$$V_{out} - V_{ref} = -I_{in} R \Rightarrow V_{out} = V_{ref} - I_{in} R$$

9-7.2 Current-to-Current Amplifier

Say you want to amplify a current.



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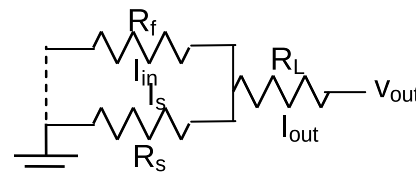


While the actual circuit may not be crystal clear, the equivalent one is. Looking at that, from the Node Rule:

$$I_{out} = I_{in} + I_s ,$$

From the Loop Rule:

$$\Delta V_f = \Delta V_s$$

$$I_{in} R_f = I_s R_s \Rightarrow I_s = I_{in} \frac{R_f}{R_s}$$

Putting this together with the I_{out} expression gives

$$I_{out} = I_{in} + I_{in} \frac{R_f}{R_s}$$

$$I_{out} = I_{in} \left(1 + \frac{R_f}{R_s} \right)$$

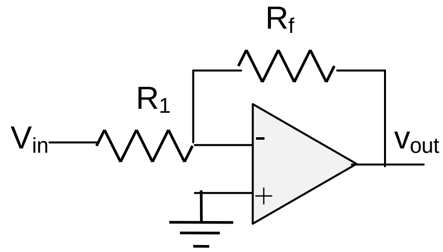
So, how much bigger the output current is than the input current depends on these two resistances. Rather remarkably, the current through the load resistor, I_{out}, is independent of the load resistance (for an ideal op-amp anyway), since the op-amp will adjust its Vout value to be whatever it needs to be in order to maintain equal voltage it its two input terminals, $V_+ = V_-$.

9-10 Real Op-Amps

For a while now, we've been idealizing the Op-Amp: it draws no current($Z_{in} = \infty$), it can source as much current as you want ($Z_{out} = 0$), it has infinite gain, ($A = \infty$), and there's no limit to how big V_{out} can be. In most applications, they're as good as true, but none are absolutely true. Actually, when we talked about the Comparator, we admitted and made use of the fact that V_{out} was bounded by the + and – power lines. Now we'll amend the other idealizations and consider their impact on circuit design. For a given make & model of Op-Amp, the manufacturer produces documentation that, among other things, characterizes these imperfections – to help the user select the appropriate Op-Amp for his/her application.

9-10.1 Gain

Let's return to the simple Inverting Amplifier, and analyze it a little more rigorously to see just how the output depends on the gain value, A – something that's often on the order 10^4, but we usually like to idealize as infinite. How much does the difference between infinity and 10^4 usually matter?



The basic relationship for an Op-Amp is $V_{out} = A(V_+ - V_-)$ where $V_+ = 0$ in this circuit

Recall that, when we *had* assume A was infinite, we'd ended up with $V_{out} \approx -V_{in}\frac{R_f}{R_1}$.

Now we'll see how good an approximation that is.

If we still allow that the op-amp draws negligible current (we'll address that later), then

$$\frac{V_- - V_{in}}{R_1} = \frac{V_{out} - V_-}{R_f}$$

But $V_{out} = A(0 - V_-)$

So

$$\frac{-V_{out}/A - V_{in}}{R_1} = \frac{V_{out} + V_{out}/A}{R_f} \Rightarrow V_{out} = -V_{in}\frac{R_f}{R_1}\left(\frac{1}{1 + \left(1 + \frac{R_f}{R_1}\right)\frac{1}{A}}\right) \approx -V_{in}\frac{R_f}{R_1}\left(1 - \left(1 + \frac{R_f}{R_1}\right)\frac{1}{A}\right)$$

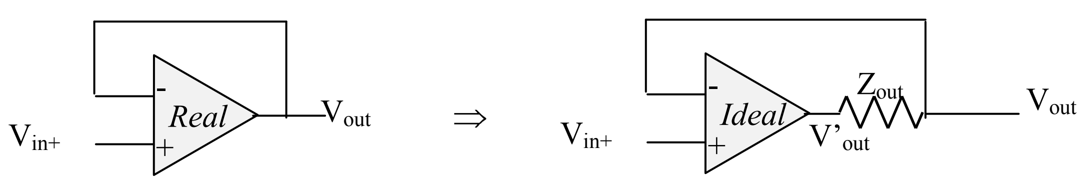
Where the last step came from a binomial expansion.

Since most Op-Amps have an A of about 10^4, as long as $\frac{R_f}{R_1} \le 10$, assuming A is infinite overestimates V_{out} by only about 0.1%.

9-10.2 Output Impedance

We generally assume that our op-amp has no output impedance, thus, it can source as much current as is required in order to maintain the prescribed output voltage, regardless of what's down-line from it. In reality, Op-Amps often have a few kΩ of output impedance. While this *number* sounds pretty non-negligible, we'll see that, in the end, the effect in a typical Op-Amp circuit is pretty insignificant. The simplest way to model an Op-Amp that has output impedance is as an *ideal* Op-Amp with a resistor connected right at its output (this is the same way we model a "real" battery – an ideal one in series with a resistor).

Let's see the effect this resistor has on a simple Op-Amp circuit, say, a follower.



Ideal Follower

Now, a follower with the ideal *no* output impedance, would have an output of $V_{out} = A(V_+ - V_-) = A(V_{in} - V_{out}) \Rightarrow V_{out} = \frac{V_{in}}{\left(1 + \frac{1}{A}\right)}$ and under the further idealization that A is infinite $V_{out} = V_{in}$.

Real Follower

How do things look for the *real* follower?

$$V'_{out} = A(V_+ - V_-) = A(V_{in} - V_{out})$$

but $V_{out} - V'_{out} = I_{out} Z_{out}$

So

$$V_{out} = V'_{out} + I_{out} Z_{out} = A(V_{in} - V_{out}) + I_{out} Z_{out} \Rightarrow V_{out}(1 + A) = AV_{in} + I_{out} Z_{out}$$

$$V_{out} = \frac{V_{in}}{1 + \frac{1}{A}} + I_{out}\left(\frac{Z_{out}}{1 + A}\right)$$

Again, in the limit that *A* is infinite, the output resistor has no effect on the circuit's operation – the voltage it outputs is the same, V_{in}. Backing off from that limit, apparently it's not Z_{out} that matters, it's $Z_{c.o.} \equiv \left(\frac{Z_{out}}{1 + A}\right)$ which defines the output resistance of the full, closed Op-Amp circuit.

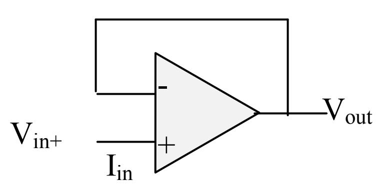
While Z_{out} may well be 5 kΩ, that divided by an *A* of 10^4 spells a resistance around only 0.2Ω!

9-10.3 Input Impedance

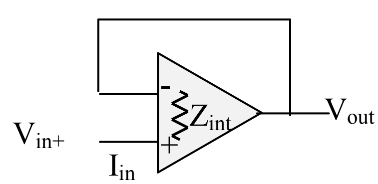
Normally we approximate the Op-Amp's input impedance as infinite – *no* current gets drawn into the device. Of course that's not completely true. For example, in reality, there's an internal impedance between the two inputs of about 50 MΩ for a bi-polar-transistor based Op-Amp, or a whopping $10^{12}\Omega$ for a J-FET based one! But just as with the Output Impedance, the specific impedance hardwired in the Op-Amp is only part of the story. What's most relevant is the Op-Amp *circuit's* input impedance. For example, if you're using the Op-Amp + some resistors as an Inverting Amplifier, then what you really want to know is the whole Inverting Amplifier's input impedance, not just that of the Op-Amp all by itself. Similarly, and a little more simply, if you're using the Op-Amp plus a wire as a Follower, the Follower's Input impedance is what you want to know.

Follower Input Impedance. How does the input impedance of a Follower circuit using a *real* Op-Amp differ from that using an *ideal* Op-Amp?

Ideal



Real



Ideally, the Op-Amp draws no current, regardless of Vin's value, so the input impedance for the circuit is infinite.

Really, there's a resistive path between the two inputs of the Op-Amp, so the Follower will draw some current and the input impedance isn't infinite, but is it close enough?

On the one hand, $V_{out} = A(V_+ - V_-)$, or, in this case,

$$V_{out} = A(V_{in} - V_{out}) \Rightarrow V_{out} = V_{in}\left(\frac{1}{1+1/A}\right)$$

On the other hand, $(V_+ - V_-) = I_{in} Z_{\text{int}}$, or $(V_{in} - V_{out}) = I_{in} Z_{\text{int}}$

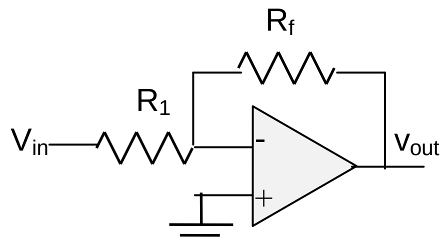
Eliminating V_{out} from these two relations gives

$$V_{in} = I_{in} Z_{\text{int}} + V_{in}\left(\frac{1}{1+1/A}\right)$$
$$V_{in}\left(1 - \frac{1}{1+1/A}\right) = I_{in} Z_{\text{int}}$$
$$V_{in}\left(\frac{1/A}{1+1/A}\right) = I_{in} Z_{\text{int}}$$
$$V_{in}\left(\frac{1}{1+A}\right) = I_{in} Z_{\text{int}}$$
$$V_{in} = I_{in}\left[Z_{\text{int}}(1+A)\right]$$

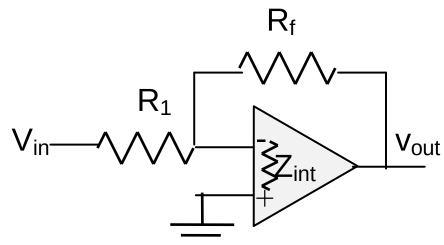
So, the factor in brackets defines the input impedance of the Follower, $Z_{input.Follower} \equiv Z_{\text{int}}(1+A)$. As if Z_{in} weren't big enough, when you multiply it by an A on order of 10^4, you get a Follower input impedance that's just plane huge / close enough to infinite for most purposes! For reasonable input voltages, the input *currents* can easily be down in the pA range.

Inverting Amplifier Input Impedance.

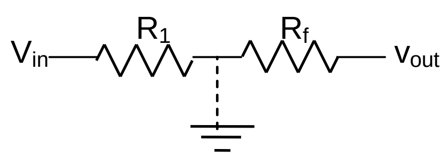
Ideal



Real



Thanks to the Golden Rule, we'd been saying that an *Ideal* Inverting Amplifier's logic is something like



Which would mean

$$V_{in} = IR_1$$

So the *ideal* input impedance is R_1. What's the *real* Inverting Amplifier's Input Impedance?

If you go through the same kind of analysis as for the Follower, you get

$Z_{input.Inv.Amp} \equiv R_1 \dfrac{1}{1+\frac{1}{A}\left(1+\frac{R_f}{Z_{\text{int}}}\right)} \approx R_1\left(1 - \frac{1}{A}\left(1+\frac{R_f}{Z_{\text{int}}}\right)\right)$. With A ~ 10^4, this differs from the ideal circuit's result, R_1, by only about 0.01%.

9-10.4 Input Bias & Offset Current

Going hand-in-hand with an internal impedance is a corresponding current that is drawn into the input terminals. Since how much is drawn will vary from application to application, there's no concise way to for the Op-Amp manufacturer accurately and quantitatively represent in the part's data sheet. So they settle for a couple of measurements made under a specific condition, a measurement that'll give you a ballpark feel for the general behavior.

Input Bias Current, I_B, is the amount of current drawn into the inputs in order to give V_{out}=0. Ideally, that would be zero, but appreciating that there is an internal resistance, you can appreciate that *some* current gets drawn. Depending on the make & model, an Op-Amp's Input Bias Current will be somewhere in the nA to pA range.

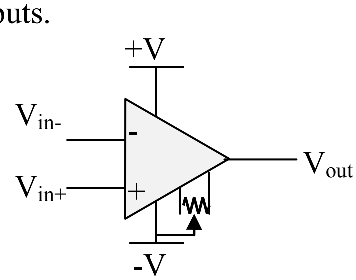
Okay, it's already an unfortunate reality that I_B is not zero, something that we qualitatively model by imagining a (big) resistor connecting the two inputs, but think about what's *really* under an Op-Amp's hood – a mess of transistors. So perhaps we shouldn't be surprised that all the current that's, say, drawn in at the –Input terminal, doesn't make it back out the +Input terminal. So the I_B value quoted is usually the average of the currents at the two inputs.

Input Offset Current, I_{OS}, is then the *difference* between the bias currents at the + and – input terminals. This is usually around 10% of the Input Bias Current.

9-10.5 Input Offset Voltage

It's hard to manufacture an Op-Amp for which the fundamental rule, $V_{out} = A(V_+ - V_-)$, is spot on. Just speaking mathematically, you might imagine the true relationship being more complicated but representable via a Taylor Series expansion as something with non-negligible zero-order correction term and a smaller first-order (and negligible higher order terms.) Physically, the first correction would correspond to an Input Offset Voltage, so that, even when the two inputs are grounded out, there will be an output; we'll deal with that in this section. The second correction would correspond to un-equal gains for the two inputs; we'll deal with that later, in the Common Mode Rejection Ratio section.

So, ideally, when $V_+ = V_- = 0$, we'd expect $V_{out} = 0$, but unfortunately reality's not so nice. There's a slight offset, that is, $V_{out} = 0$ when $V_+ = V_- + V_{OS}$. V_{OS} is the "Input Offset Voltage", the voltage difference you'd need to apply across the inputs to *really* zero the output. This can be in the mV range. For some applications that's as good as 0; for others it's pretty significant. Fortunately, you don't have to just live with it. If there's a consistent offset of a few mV, fine apply that extra voltage and the problem's fixed. Many Op-Amps are designed with a couple of additional inputs to help with that. You've got your two signal inputs, your two power-line inputs, your one output, *and* two more "offset-voltage trim" inputs.



The little arrow means the resistor is a trim pot, i.e. variable resistor, that allows the user to tweak things until they're just right and the Offset Voltage is 0.

9-10.6 Common Mode Rejection Ratio

The flip-side of saying that $V_{out} = 0$ when $V_+ = V_- + V_{OS}$, i.e. that you need to apply an offset voltage to zero the output, is saying that when you *don't* apply the offset voltage, that is, when $V_+ = V_-$, then $V_{out} \neq 0$. It's easy to take care of this at, say $V_+ = V_- = 0$ (or some other single chosen voltage value) by using the offset circuitry of the above picture; however, that doesn't handle the linear correction term previously alluded to: the gains at the two terminals aren't *exactly* the same, that is,

$$V_{out} = (A_+ V_+ - A_- V_-)$$

where $A_+ \approx, \neq A_-$.

Since they are *awfully* close to equal, it may be more telling to write it as

$$V_{out} = A(V_+ - V_-) + a_{CM}(V_+ + V_-)$$

Where $A \equiv \frac{1}{2}(A_+ + A_-)$ and $a_{CM} \equiv (A_+ - A_-)$.

So the first term is what we want, and the second term is the error, which we can say is due to a small but non-zero "common-mode gain", a_{cm}. Sure, the trim-pot can be used to eliminate this at one and only one value of $V_+ = V_-$, but for a different value, it's back. Unfortunately, there's no completely getting rid of this, so the best we can do is characterize it and live with it. The way this is typically characterized is in ratio to the regular gain, A:

Common Mode Rejection Ratio, $CMRR \equiv \frac{A}{a_{cm}}$. This is usually whopping big, so it's more common to quote the "Common Mode Rejection, $CMR \equiv 20dB\log_{10}\left(\frac{A}{a_{cm}}\right)$.

For a good Op-Amp, this might be 120dB, for a so-so Op-Amp this might be 70dB. That means that the desirable gain, A ranges from 6 to 3.5 orders of magnitude larger than the undesirable a_{cm}.

Not that Important (usually). To put this in perspective, qualitatively, the effect of this "common-mode gain" is as if the op-amps gain changed, on order of 0.1 to 0.0001% with the size of the input signal. Since most Op-Amp applications are pretty insensitive to the exact value of the gain anyway (it suffices that it's approximately infinite), these slight changes won't matter at all, not for *most* applications.

9-10.7 Slew Rate

You may remember back when we first met transistors, it takes *time* for them to respond to changing applied voltages, *time* for the junction region to change, *time* for charges formerly happy in donors or conduction band to fall into acceptors. Well then, a mess of transistors, under the hood of an Op-Amp, is going to take time to respond to changing input voltages. Say you suddenly change the input voltages and watch the output voltage as it changes. The rate at which it changes, usually a few V/μs, is the *Slew Rate.*

Say you've got an Inverting Amplifier circuit, given this limitation on how quickly the *output* can change, divide out the circuit's gain, and you've got an upper limit on how quickly a changing *input* it can process correctly. The rate at which the *output* changes is easily related to the rate at which the *input* changes: $\left|\frac{dV_{out}}{dt}\right| = G\left|\frac{dV_{in}}{dt}\right|$. So, if we need to keep $\left|\frac{dV_{out}}{dt}\right| < SlewRate$, that means keeping $\left|\frac{dV_{in}}{dt}\right| < \frac{SlewRate}{G}$. For example, a slew rate of 1 V/μs, and a gain of 10 would mean it couldn't faithfully amplify anything faster than a 1Volt-amplitude sine wave at 16 kHz. Continuing with that example, if, instead, you wanted to have a gain of 20, the limiting frequency would be only 8 kHz. A gain of 40 would imply a limit of 4 kHz, etc.

9-10.8 Frequency Response

So there's a linear relation between the gain and frequency a given Op-Amp can handle, on account of its limited Slew Rate. Of course, the Op-Amp doesn't explode or even shut down if the signal's frequency exceeds the limit for the Gain the Op-Amp circuit is wired for. What does happen, to first order, is the actual gain reduces. Maybe you have things wired for a gain of 80 at 4kHz, but it's going to give you something more like the gain of 40. What's happening is the output just can't keep up with the input signal; the input would have it, say grow 2V in a time interval, but it only makes it 1V. To second order, if you consider that the instantaneous rate of a sine-wave's changing is time dependent

$$\frac{d}{dt}\left(V\sin(\omega t)\right) = V\omega\cos(\omega t)$$

Then you'll recognize that, at some instants the rate of change is slow enough for the op-amp to handle while, at other instants it isn't. That means that, not only does the output's shrink in over-all amplitude, but it also distorts – an input of a perfect sine wave gives a less-amplified and less-perfect almost-sine wave output.

I should point out that you can easily hear a 4kHz or 8 kHz sound (16kHz is pushing it), so a good stereo needs "audio quality" Op-Amps with high slew rates.