Fri. 2/17	**Ch 9.1-.5, .9, App B-3:** Operational Amplifiers	**HW5**: * ; **Lab 5 Notebook**
Mon. 2/20	**Ch 9** (the rest)**:** Operational Amplifiers	
Wed. 2/22	**Quiz** Ch 9, **Lab 6:** Operational Amplifiers	**HW 6:** Ch9 Pr 1,3*, 4*, 5, 21

Equipment

      Electronics portion of Chris's Thesis to demonstrate use of Op-Amps

      Lab #6

Announcements

- Midterm just after break.

I do Non-Inverting Amp
They do Differential Amp (we set up equivalent together)

For Thursday:
      Input Impedance / output impedance
      One example of reasoning out op-amp operation
      They then work group problem on at least one.

Symbol & connections

Stand-Alone Behavior

- comparator

Feedback – positive & negative

Schmitt trigger as example of positive feedback

Golden Rules for operation with negative feedback

1. inverting and non-inverting inputs are kept at the same voltage

2. no current flows into the inputs

Applications:

- Follower

- Inverting Amp

- Non-inverting Amp

- Current-to-Voltage Converter

- Summer / Adder

- Difference Amp

- Integrator & Differentiator

- Active Filters?

Characteristics of <u>Real</u> Op Amps

- Input impedance

- Input offset voltage

- Common mode rejection ratio

- Slew rate

- Frequency response

Introduction

Up to this point, we've only dealt with discrete components – individual resistors, capacitors, diodes, transistors,… Any interesting operation we want performed calls for some combination of these. Now we'll meet our first Integrated Circuit, an Operational Amplifier. To, momentarily, peak under its hood, look at page D-70. It's a whole circuit of resistors (~11) and transistors (~24) and one capacitor, all are already wired up for you and sealed in a case. We are *not* going to try to reason through its detailed inner workings, rather we'll adopt a few rules of thumb and consider its applications. There is a world of applications for this handy circuit-in-a-case. The book points out that it was originally designed with analog mathematics (adding, subtracting, differentiating, integrating,…) in mind. It certainly makes these operations easy, but you can dream up all sorts of other uses.

The Ideal Operational Amplifier
Initially, let's not worry about the details of how it's achieved (or nearly achieved) yet, let's just think about how we'd want it to behave. Of course, the ideal can't actually be met, but here's the wish list, and, in many applications, real op-amp behavior is essentially ideal.

- Infinite Input impedance $Z_{in} = \infty$ (so it doesn't effect the input signal or create other loading problems)
- Zero output impedance $Z_{out} = 0$ (so it can maintain its output signal independent of the load it bears)
- Infinite Amplification, $A = \infty$ (want an amplifier – go all the way!)

Here's a symbol for the Op-Amp.
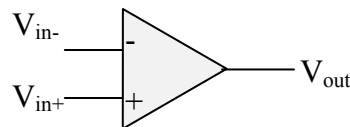
Where $V_{out} = A(V_{in+} - V_{in-})$.

9-9 Comparator
With the fundamental rule that $V_{out} = A(V_{in+} - V_{in-})$, the most basic thing that an Op-Amp does is *compare* its two inputs: if $V_{in+} > V_{in-}$, then the output is large and *positive;* if $V_{in+} < V_{in-}$, then the output is large and *negative.* So, perhaps the most natural use of an Op-Amp is as a Comparator – one input may take a *reference* voltage while the other takes your signal; the comparator tells you, in a big way, if the signal is greater or less than the reference.

For you to really get how comparators work, I should slightly refine our picture of an Op-Amp. In most circuit-logic diagrams, we don't bother showing it, but in addition to having two input and one output lead, an op-amp has two power leads, just like any other electrical device (think the power cord for a stereo, toaster, vacuum cleaner, computer,…) When we *do* bother showing it, the drawing looks like this.

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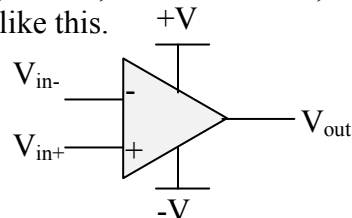
These power lines are often on the order of +/- 15 Volts. Under that smooth triangular hood of the Op-Amp is a mess of transistors (and a few resistors), and these power rails act kind of like the V_{cc} and V_{EE} of the whole mess.

With that in mind you may not be surprised when I say that there's a very practical limitation on the relation

$$V_{out} = A(V_{in+} - V_{in-}) \quad \text{where} \quad A \gg 1.$$

The output *can't* exceed these "rails." An Op-Amp plugged into a +/- 15 Volt supply can't generate a signal outside that range: higher than +15 V or lower than -15V.

Now, since A is generally *huge,* it doesn't take a very big difference between V_{in+} and V_{in-} to send V_{out} to one rail or the other. In fact, for most practical purposes,

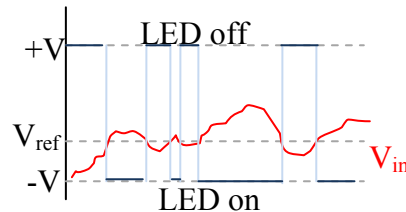|			
If	$V_{in+} > V_{in-}$	then	$V_{out} \rightarrow$ +V
If	$V_{in+} < V_{in-}$	then	$V_{out} \rightarrow$ -V

Now, let's consider this bit of circuitry. It's designed so that an indicator light turns on whenever the input signal, V_{in}, exceeds a threshold reference value, V_{ref}.
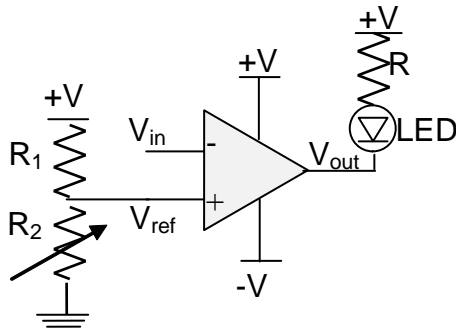
On the left we have a voltage divider that uses a variable resistor in order to fine-tune the desired reference voltage, V_{ref}. On the right we have an LED which, like any diode, only passes current / lights up, when its 'head' is lower voltage than its 'tail.' That means

If	$V_{in} < V_{ref}$	then	$V_{out} \rightarrow$ +V and the light is OFF
If	$V_{in} > V_{ref}$	then	$V_{out} \rightarrow$ -V and the light is ON

To get a better feel for this circuit, see what output a hypothetical (and a bit messy) input signal prompts.

A comparator with output like this can be an awfully handy thing. Maybe, it's got an LED which acts as an "indicator" light. Perhaps the V_{in} comes from a thermocouple and represents a temperature, then the light turns on to alert you when the temperature gets too low. Or maybe, rather than outputting to an LED, the comparator's output would then be used to tell a heater to turn on. That's useful!

9-9.1 Schmitt Trigger

Sometimes though, such a comparator is a little *too* sensitive. Think of that heater situation, if the heater turned on *every* time the signal dropped below the reference and turned off *every* time it crossed above the reference, then the darn thing would be *incessantly* turning on /off/on/off…

The way my thermostat works is the heater is turned on if the room drops below, say 70°F and keeps going until the temperature climbs above 72°F. That 2° difference buys some time – after the heater turns off, it'll take a while before the temperature drops all the way to 70°F again, and the heater's turned on again. All this, no doubt, is controlled by a comparator that has *two* reference points, instead of just one. When the input signal (tracking the temperature) drops *below* the *lower* reference value, then the output goes low (and the heater is turned on). As the input signal (temperature) climbs, the output *stays low* until the upper reference value is crossed, at which point the output goes high (and the heater is turned off). Similarly, as the input signal (temperature) falls again, the output *stays high* until the lower reference is crossed again.

You might imagine that some new fangled component is needed to achieve this, or at least a peculiar combination of multiple Op-Amps. In fact, it just takes one Op-Amp simply, though cleverly, wired up.
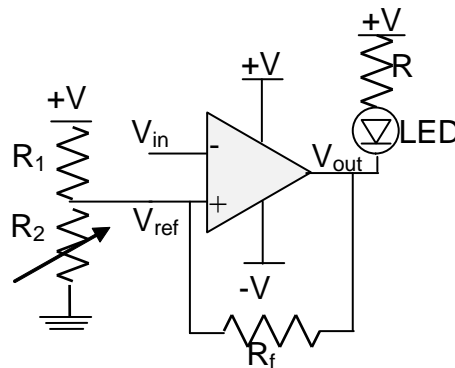

We've simply added a "feedback resistor", R_f, connecting the output to the positive input. In general, this kind of configuration (output connected to + input) is known as **positive feedback.** Qualitatively, it's easy to see that the reference voltage, V_{ref}, now isn't determined by *just* the voltage divider; it also depends on the output, V_{out}. Seeing *how* it depends on the output takes only a little bit of work.

If we maintain that there are essentially only two possible output values, $V_{out} = +V$ or $V_{out} = -V$, then we have only two cases to consider (in principle, V_{out} can assume any value between the two, but that would require V_{in} and V_{ref} being so close to exactly equal that you'll never see it).
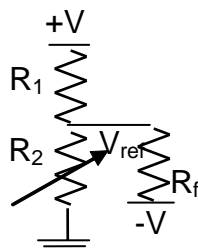
When $V_{in}<V_{ref}$ then $V_{out}=+V$
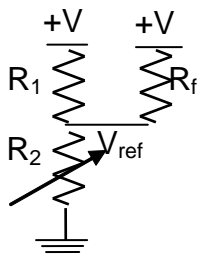
This looks like a voltage divider but with R1 replaced by R_1 & R_f in parallel, i.e., $\dfrac{R_1 R_f}{R_1 + R_f}$.

Then V_{ref} is set at $V_{ref+} = +V \dfrac{R_2}{\frac{R_1 R_f}{R_1 + R_f} + R_2}$

When $V_{in}>V_{ref}$ then $V_{out}= -V$

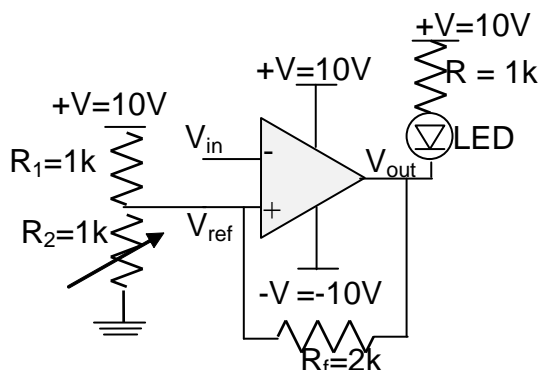This is a little more complicated, but the analysis is still quite straightforward.

$I_1 = I_2 + I_f$

$$\frac{^+V - V_{ref}}{R_1} = \frac{V_{ref}}{R_2} + \frac{V_{ref} - {}^-V}{R_f} \Rightarrow \frac{^+V}{R_1} + \frac{^-V}{R_f} = V_{ref}\left(\frac{1}{R_2} + \frac{1}{R_f} + \frac{1}{R_1}\right)$$

$$V_{ref} = \frac{\dfrac{^+V}{R_1} + \dfrac{^-V}{R_f}}{\left(\dfrac{1}{R_2} + \dfrac{1}{R_f} + \dfrac{1}{R_1}\right)} = \frac{^+VR_f + {}^-VR_1}{\left(\dfrac{R_1 R_f}{R_2} + R_1 + R_f\right)}$$

Example.

To make it a little more concrete, let's say we have

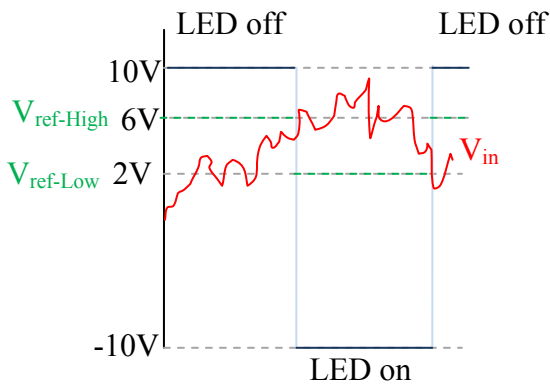

Then, if the output is at the +10V rail ($V_{in}<V_{ref}$), the reference voltage is

$$V_{ref} = 10V \frac{1k}{\frac{1k\,2k}{1k+2k} + 1k} = 6V$$

If the output is at the -10V rail ($V_{in}>V_{ref}$), the reference voltage is $V_{ref} = \dfrac{10V\,2k - 10V\,1k}{\left(\dfrac{1k\,2k}{1k} + 1k + 2k\right)} = 2V$

So, here's how input and output signals would be related:

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LED off   LED off

10V

$V_{ref\text{-}High}$ 6V

$V_{ref\text{-}Low}$ 2V

V_{in}

-10V

LED on

$$V_{in} < V_{ref} \Rightarrow V_{out} = +10V, V_{ref} = 6V$$
$$V_{in} > V_{ref} \Rightarrow V_{out} = -10V, V_{ref} = 2V$$

With this particularly messy input, you can see a second effect of the Schmitt Trigger circuit – not only does the Schmitt trigger give a margin between swinging the output high and low, but that means that noisy oscillations within that margin have no effect on the output.

9-2 Voltage Follower
Negative Feedback & the Golden Rule. While there are uses for op-amps in "positive feedback" configurations, the far more common is a "negative feedback" configuration. That's one in which the output is connected back to the negative input (often via a resistor or a capacitor.) How an op-amp behaves in "negative feedback" can easily be summed up in what's known as the *Golden Rule* of negative feedback: V_{out} becomes *whatever it needs to be* to make $V_{in+} = V_{in\text{-}}$.

Let's see why that is and what use we can make of it in the simplest imaginable negative feedback circuit: the Follower.
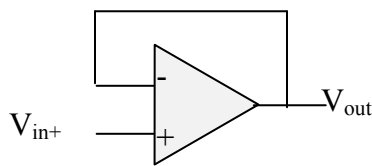
V_{in+} V_{out}

Now, if $V_{out} = A(V_{in+} - V_{in\text{-}})$

and $V_{in\text{-}} = V_{out}$,
then $V_{out} = A(V_{in+} - V_{out})$,
   or
$$V_{out} = \frac{A}{A+1} V_{in+} .$$
Ideally $A = \infty$, at any rate $A >> 1$, so $V_{out} \approx V_{in+}$.

Why bother?, couldn't we get $V_{out} = V_{in+}$ by just removing the Op-Amp all together? Sure. The virtue of the follower is that it acts like a buffer: whatever source is generating the V_{in+} signal sees infinite impedance up the line, so it has no problem maintaining the desired V_{in+} value;

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meanwhile whatever's going to go on and use V_{out} sees zero output impedance, so no matter how small a load it provides, it will not effect the V_{out} value it's getting fed.
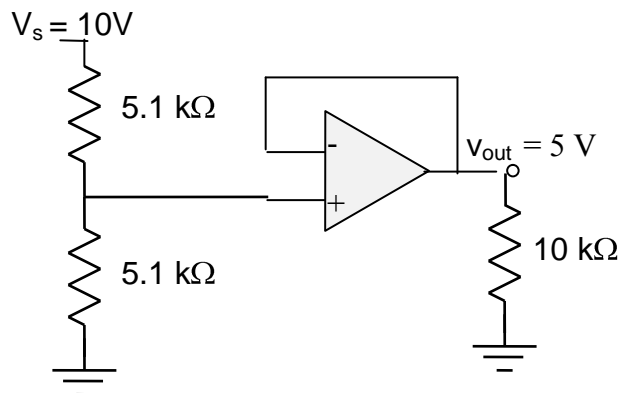
Putting a follower to use should drive home its worth. Say you've got a simple voltage divider:

$V_s = 10V$

5.1 kΩ

$V_{out} = 5V$

5.1 kΩ
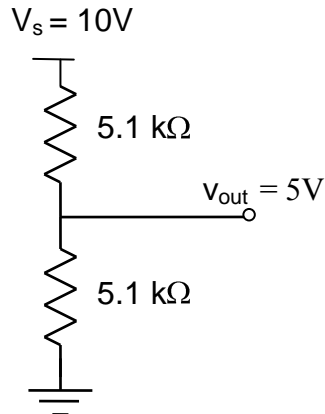
Then you wire it up to some other bit of circuitry that has an input impedance of 10 k.

$V_s = 10V$

5.1 kΩ

$V_{out} = 3.99V$

5.1 kΩ   10 kΩ
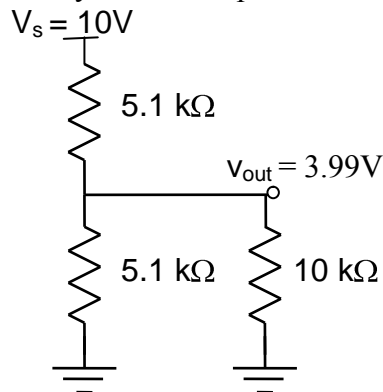
But, if you really *want* that 5Vs across your load, then insert a follower.

$V_s = 10V$

5.1 kΩ

$V_{out} = 5$ V

10 kΩ

5.1 kΩ

9-3 Inverting Amplifier
More frequently, you don't just want the output = input, you want the op-amp to, well, *amplify* the voltage, and not infinitely, but by some nice, reasonable factor. A quirk of the op-amp's design is that it's quite easy to do that, but only if you're willing to *invert* the signal while you're

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at it. We'll first analyze the inverting amplifier without assuming the golden rule, and then we'll see the 'golden rule of op-amps' short cut.
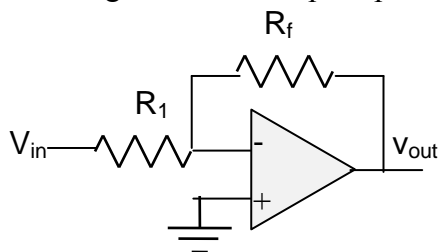
Again, what' special about an Op-Amp is that
$$V_{out} = A(V_+ - V_-).$$
And its input impedance is nearly infinite (so it draws no current) and its output impedance is nearly zero. Let's analyze this circuit making only the approximation that the op-amp draws no current / has infinite input impedance. Then

$$i_f = i_1$$

$$\frac{V_{out} - V_-}{R_f} = \frac{V_- - V_{in}}{R_1}$$

But since $V_{out} = A(V_+ - V_-)$, or $V_- = V_+ - \frac{1}{A}V_{out}$, and here, $V_+ = 0$, so

$$V_- = -\frac{1}{A}V_{out}$$

$$\frac{V_{out} + \frac{1}{A}V_{out}}{R_f} = \frac{-\frac{1}{A}V_{out} - V_{in}}{R_1} \Rightarrow V_{out} = \frac{-V_{in}}{\frac{R_1}{R_f}\left(1 + \frac{1}{A}\right)\frac{1}{A}}$$

Now, if $A \gg 1$, then the denominator is approximately just $\frac{R_1}{R_f}$, i.e.,

$$V_{out} \approx -V_{in}\frac{R_f}{R_1}.$$

How good an approximation is this? Well, the book gives a table that shows, if $\frac{R_f}{R_1} = 10$, then A can be as small as 10^3 and the approximation will still be 99% right. If $\frac{R_f}{R_1} = 100$, then A has to be at least 10^4 to make this approximation 99% good. That's pretty good.

Now, if you're concerned about the effect of our initial approximation, that is approximating that $R_{int} \approx \infty$, I won't go through all the work here (you could if you like), but not making that approximation gives

$$V_{out} = \frac{-V_{in}}{\frac{R_1}{R_f}\left(1 + \frac{1}{A}\right)\frac{1}{A}\left(1 + \frac{R_1}{R_{int}}\right)}$$ which, for large A and large $R_{internal}$ approximates to the same thing.

$$V_{out} = A(V_+ - V_-) = -AV_-$$

$$V_- = V_{in} - i_1 R_1$$

$$i_1 = i_{int} + i_f$$

$$i_{int} = \frac{V_- - V_+}{R_{int}} = \frac{V_-}{R_{int}}, \quad i_f = \frac{V_- - V_{out}}{R_f}$$

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Making these substitutions and solving for V_{out} gives

$$V_{out} = \frac{-V_{in}}{\frac{R_1}{R_f}\left(1+\frac{1}{A}\right)+\frac{1}{A}\left(1+\frac{R_1}{R_{int}}\right)}$$

Golden Rule for negative feedback
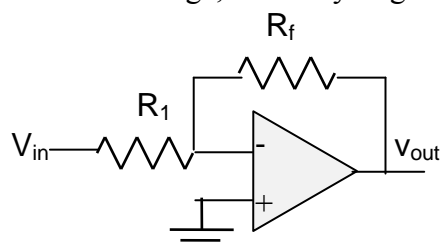It's enlightening to plug this expression back into $V_{out} = A(V_+ - V_-)$. That gives

$$\frac{-V_{in}}{\frac{R_1}{R_f}(A+1)+1} = \left(V_+ - V_-\right) \text{ if A is rather large, then } 0 \approx \left(V_+ - V_-\right), \text{ i.e.,}$$

$$\boxed{V_+ \approx V_-.}$$

That's the "golden rule" for op-amps in negative feedback (that is, where the output is connected back into the negative input): the output will do what it's got to do to make the two inputs equal.

Here, we've shown that that's a good approximation (since A is supposed to be woppin' big). For future circuit analysis, as long as there's negative feedback, we can assume this to be the case. That makes our analysis much quicker. Just to show its use, I'll reanalyze the inverting amplifier, taking $V_+ \approx V_-$ as a given.

Now, since the amplifier draws negligible current, and the two input terminals are (essentially) at the same voltage, the analysis goes

$$\underbrace{\frac{i_f}{V_{out}-V_-}}_{} = \underbrace{\frac{i_1}{V_- - V_{in}}}_{} \text{ where } V_- \approx V_+ = 0 \text{ so,}$$
$$\frac{V_{out}-V_-}{R_f} = \frac{V_- - V_{in}}{R_1}$$

$$\frac{V_{out}-0}{R_f} \approx \frac{0-V_{in}}{R_1} \Rightarrow V_{out} \approx -V_{in}\frac{R_f}{R_1} \text{ and we're done!}$$

A useful way to *look* at such circuits is to think of their op-amp-less equivalents. The sole function of the op-amp here is to nail V₋ at ground *without* diverting any current to ground. So, the equivalent circuit is
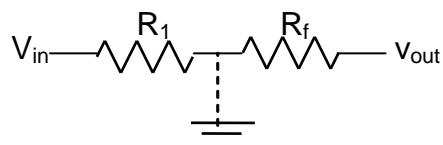
The dashed line is to remind us that the midpoint happens to have V=0, *but* no current is drawn down there.

In general, op-amps are often used to do this, to force a point in a circuit to have zero voltage (because we all love zeros in our mathematical equations) without having to divert any current.

Input impedance.

While the ideal *op-amp* may have infinite input impedance, this chunk of circuitry that involves two resistors and an op-amp ideally has an input impedance equal to $R_{in} \equiv \dfrac{V_{in}}{i_{in}} = R_1$. So one typically wants this to be fairly large so it doesn't draw too much current from whatever's upstream of it in your larger circuit.

Note: later we'll look a little closer at the effects of real op-amps *not* having infinite input impedance and infinite gain. For the moment, I'll borrowing a result from that later work to show how good an approximation $R_{in} \approx R_1$ is: $R_{in} = R_1 + \dfrac{R_f}{1 + A + \dfrac{R_f}{R_{int}}}$. If both R$_{internal}$ and A are quite large, then this whole second term is negligible.

$V_{in} = i_{in} R_1 - V_-$

$V_{out} = A(V_+ - V_-) = -AV_- \Rightarrow V_- = -\frac{1}{A} V_{out}$

$V_{out} = V_- - i_f R_f$

$i_f = i_{in} - i_{int}$

$i_{int} = \dfrac{V_- - V_+}{R_{int}} = \dfrac{V_-}{R_{int}}$

Making these substitutions (with an intermediate step of isolating V.) yields

$V_{in} = i_{in}\left(R_1 + \dfrac{R_f}{1 + A + \dfrac{R_f}{R_{int}}} \right)$ so $R_{in} \equiv \dfrac{v_{in}}{i_{in}} = R_1 + \dfrac{R_f}{1 + A + \dfrac{R_f}{R_{int}}}$

9-4 Non-Inverting Amplifier

We've used the specific occasion of analyzing the Inverting Amplifier circuit to learn some general things about analyzing Op-Amp circuits. Now we're going to make use of some of those lessons. You'll see that analyzing these circuits can actually be rather simple. Often, you identify the equivalent non-op-amp circuit and use our old-fashioned, ohm's-law kind of reasoning to analyze it.
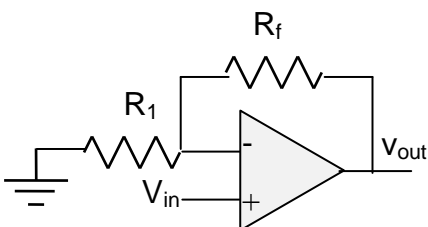
The output is connected to the negative input, so we're in "negative feedback" and we can assume that $V_+ = V_-$. Additionally, the op-amp draws virtually no current from its inputs. That gives us an equivalent circuit of
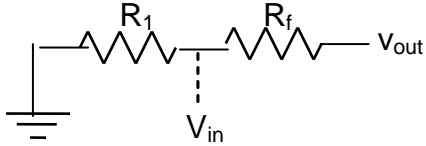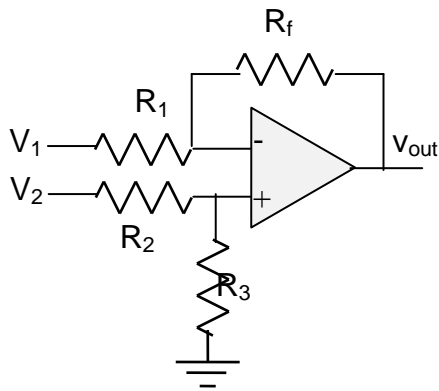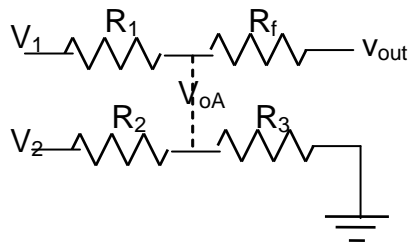

$$i_f = i_1$$

$$\frac{V_{out} - V_{in}}{R_f} = \frac{V_{in} - 0}{R_1} \Rightarrow V_{out} = V_{in}\left(1 + \frac{R_f}{R_1}\right)$$

9-5 Differential Amplifier

Now you try. What's the equivalent no-op-amp circuit of this?

Again, we've got negative feed back, so we can take for granted that $V_+ = V_-$, and, of course, no current is drawn by the amplifier. So, the equivalent circuit is

What can you say about the currents through R_1 and R_f? So if you apply Ohm's law, what do you get?

$$i_f = i_1$$

$$\frac{V_{oA} - V_1}{R_1} = \frac{V_{out} - V_{oA}}{R_f}$$

How about the currents through R2 and R3?

$$i_2 = i_3$$

and $\dfrac{V_{oA} - V_2}{R_2} = \dfrac{0 - V_{oA}}{R_3}$

Solving both for V_{oA} and setting them equal through that gives

$$\frac{\frac{R_1}{R_f} V_{out} + V_1}{\left(1 + \frac{R_1}{R_f}\right)} = V_{oA} = \frac{V_2}{\left(1 + \frac{R_2}{R_3}\right)}$$

So, $V_{out} = V_2 \dfrac{\left(\frac{R_f}{R_1} + 1\right)}{\left(1 + \frac{R_2}{R_3}\right)} - \dfrac{R_f}{R_1} V_1$

The book notes that *if* you build this circuit with perfectly balanced the resistors, $R_1 = R_2$ and $R_3 = R_f$, *then* the output is simply $V_{out} = \frac{R_f}{R_1}\left(V_2 - V_1\right)$.

9-5.1 Instrumentation Amplifier

It's a hassle to perfectly balance resistors, but a single – on chip circuit can do that better. While we're at it, we can slap some followers on the inputs so we don't need to worry about the circuit's input impedances affecting the input signal. The resulting chip (roughly two followers and a differential amplifier, all on one chip) is called an *instrumentation amplifier*.

I'll run through the logic of a simple Instrumentation Amplifier circuit, it should *make sense*, but don't feel like you need to be able to master it – for the most part, it suffices to merely *appreciate* how it works and *know* how to make it work for you.
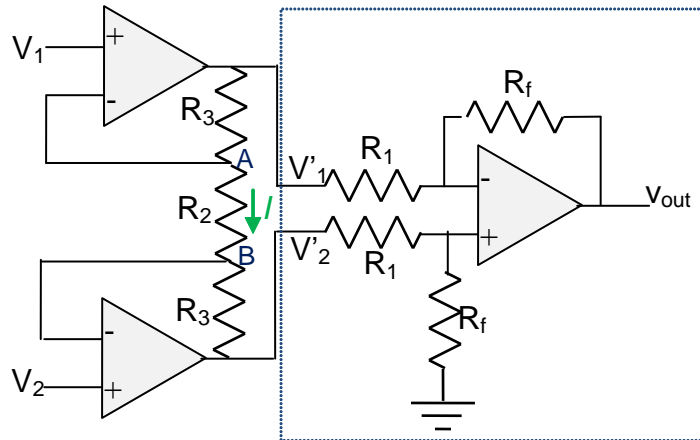
This is the exact same circuit as shown in the book's Fig 9.10; however, I've bent the wires and named the resistors to emphasize it's similarity to the circuit we've just seen (that in the book's Fig. 9.8). It's essentially the circuit we'd just analyzed (in the blue box, with a couple of Op-Amps tacked on to the inputs.

Quoting the result we'd just gotten,

$$V_{out} = \frac{R_f}{R_1}\left(V_2' - V_1'\right) \qquad \text{(A)}$$

So now we just need to relate V_1' and V_2' to V_1 and V_2. Here we go.

With the top left Op-Amp in negative feedback, it's two inputs must have equal voltage, so

$$V_A = V_1$$

But Ohm's law across the top R_3 says $V_1' - V_A = IR_3$

So

$$V_1' - V_1 = IR_3 \qquad\qquad (B)$$

Similarly, looking at the bottom left Op-Amp,

$$V_2 - V_2' = IR_3 \qquad\qquad (C)$$

(the current is the same since the Op-Amps themselves drain off negligible current.)

Looking at R_2, Ohm's Law says

$$V_1 - V_2 = IR_2 \qquad\qquad (D)$$

Putting these three equations together, B+C, and D eliminating I,

$$V_1' - V_1 + V_2 - V_2' = 2IR_3 = 2\left(V_1 - V_2\right)\frac{R_3}{R_2}$$

$$V_1' - V_2' = \left(V_1 - V_2\right)\left(1 + 2\frac{R_3}{R_2}\right)$$

There's our relationship between V_1', V_2' and V_1, V_2. Plugging that into A tells us how the output depends on the two inputs.

$$V_{out} = \frac{R_f}{R_1}\left(V_1 - V_2\right)\left(1 + 2\frac{R_3}{R_2}\right)$$

This is the book's Eq'n 9-20, but tidied a little and using *my* names for the resistors. If we design our circuit so that $2\frac{R_3}{R_2} = 1$, i.e., $R_2 = 2R_3$, then this has the very simple form of

$$V_{out} = \frac{R_f}{R_1} 2\left(V_1 - V_2\right)$$

Tight Requirements. This relationship is just as simple as the one for the simpler circuit, that with just one Op-Amp, but the two followers give it more desirable, high input impedance. Unfortunately, this relationship is only as true as are $R_2 = 2R_3$ and that the two R_f resistors actually have the same resistance, and ditto for the two R_3 resistors. That's not easy to get in reality, outside of a single chip. So chip manufacturers make Instrumentation Amplifier chips that are essentially three (or more) Op-Amps + some resistors all made on one chip.