Physics 310
2 – Circuit Transients and Oscilloscopes

Fri. 1/20	**Ch 2:** Just the Capacitor bits (exclude 2.4 and subsections about inductors)	**Lab 1 Notebook**
Mon. 1/23 Wed. 1/25 Thurs. 1/26 Fri. 1/27	**Ch 2** (the rest)**, & 6.6-7, App B-2:** Capacitors, Inductors, & Oscilloscopes **Quiz** Ch 2 & 6, **Lab 2:** Oscilloscopes & Circuit Transients more of the same **Ch 3, 4.5, 6.5:** AC Circuits	**HW2**: Ch2 Pr 1,2,3,6* ,8,10 **Lab 2 Notebook**

Materials:

- Lab #2 w/ Capacitor supplement
- Set up computer for http://www.falstad.com/circuit/ - from circuit menu, select basics / capacitor

Study List for Quiz #2:

1. Capacitors (series/parallel combinations).
2. Inductors
3. RC Circuit transients (charging and discharging)
4. RL Circuit transients

Equation List: [units are in square brackets]

$$1/C_S = 1/C_1 + 1/C_2 + 1/C_3 + \dots \qquad C_p = C_1 + C_2 + C_3 + \dots$$

$$v_C = Q/C\ [1\ \text{V} = 1\ \text{C/F}] \qquad dv_C/dt = i/C \qquad \tau_{RC} = RC\ [1\ \text{s} = 1\ \Omega \cdot F]$$

$$v_L = L(di/dt)\ [1\ \text{V} = 1\ \text{H}\cdot\text{A/s}] \qquad \tau_{RL} = L/R\ [1\ \text{s} = 1\ \text{H}/\Omega]$$

<u>Topics</u>:

- Symbols for capacitors (also polarized) and inductors.
- Voltage relation for capacitors and series & parallel combinations
- Voltage relation for inductors
- RC circuits (transient behavior – turning voltage on or off)
- RL circuits (transient behavior – turning current on or off)
- RC combinations as differentiators or integrators (see pp. 25-7 of H&H for math)
- RLC transient behavior - "ringing"

Lect. Prep Requests

-
- 2. Why would they call a pico farad a micro-micro farad? Pico Farad is shorter and easier to say. Is this just because people might not remember the multiplier for pico?

    Yup, it may be convenient for someone, but there's no other significance to the choice.

3) what is the significance of an EMF. I understand that a varying current is required to produce one, but what does it do?

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EMF is an awkward bit of notation that isn't completely consistently used. In general, it represents an electric potential difference that is *not* produced by simple columbic electric fields. Sometimes you'll see a battery's voltage labled with "emf" because it's established by chemical processes; the voltage of produced by an inductor is similarly labeled "emf". It's mostly due to the tangential (rather than radial) component of the electric field which is produced by an accelerating charge.

Chapter 2: Capacitors and Inductors

2-1 Introduction

- This chapter introduces two circuit elements that bring *time dependence* to circuits. Battery – resistor circuits very quickly (in a few nanoseconds) reach their steady state. After that, a constant current flows through each element, and a constant voltage drop exists across each element. But, capacitors and inductors are a different matter.
    - A capacitor is a circuit element that has been designed to optimize its *capacity* to store charge. But, it takes time to fill or empty a capacitor of charge – thus the *time* dependence of its behavior.
    - An inductor is a circuit element that has been designed to optimize its *induced electric field* that opposes changing currents (recall from Phys 232 or Phys 221 that a time varying current creates both time varying magnetic and electric fields.) Thus, again, the *time* dependence (as long as the current changes, the induced field is created).
- In point of fact, *every* circuit element has all three basic behaviors – resist but pass current, accumulate charge, and radiate electric fields when charges accelerate, but the three basic circuit elements have been optimized differently to emphasize the different behaviors: resistance, capacitance, and inductance.
- **Notational Note:** The book uses I and V to represent constant current and voltages; it uses i and v to represent time varying ones. I'll *try* to employ this convention.

Student Q: Differences between RC and RL circuits. I know one has a capacitor and one has an inductor. But how does current act in each. And how do capacitors and inductors differ?
A: Both a capacitor and an inductor oppose current flow; however, they respond to different aspects of that flow.

Capacitor. The electric field that a *capacitor* 'pushes back against' on-coming charges grows stronger and stronger as it accumulates more and more charge – right when you turn on a power supply, the capacitor it initially has very little effect, but over time becomes more effective until, for constant applied voltage, it would completely stop current flow along its branch of the circuit.

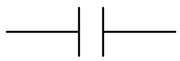
Mechanical analogy. As we'll discuss more later, the effect a capacitor has on current is very similar to the effect that a spring has on speed. – Imagine a person with a slinky hooked to his/her belt – as the person walks away – initially, it doesn't pull back at all / has no effect on the person's speed, but further they walk / the more it stretches, the harder it pulls back until they eventually can't go any more.

Inductor. The electric field that an *inductor* 'pushes back against' on-coming charges is actually proportional to the *change* in current flow – the more rapidly the current is trying to *change* the harder the inductor pushes back and so slows the current. So right when you turn on a power supply, the inductor will have its largest effect since the current's suddenly jumping from 0.

Mechanical analogy. An inductor's effect on current is similar to a *mass's* effect on speed. The more massive something is, the harder it is to *change* the speed, but otherwise, it's got no effect.

Today – Capacitors. Today we'll focus on capacitors and circuits with them; Monday we'll consider inductors and circuits with them as well as circuits with both capacitors and inductors.

2-2 Capacitance

 Non-polar capacitor

 Polar capacitor (can blow up if wired backwards!!)

- Where you've got a charge separation, you've got an electric field; where you've got an electric field, you've got an electric potential difference. You may recall from Phys 232 how you found the electric field between the oppositely charged plates of a capacitor using Gauss's Law.
  - $E = \dfrac{Q/A}{\varepsilon}$ where A is the surface area of a plate and ε is the electric permittivity: dialectic constant, K, times ε_o; $\varepsilon = K\varepsilon_o$
- Since this field is uniform, and points from the + plate to the – plate, it's quite simple to come up with the voltage drop between the two plates:
  - $\Delta V_c = -\int \vec{E} \cdot d\vec{x} = -\dfrac{Qd}{\varepsilon A}$ where d is the distance between the two plates.
- The constant parameters are clumped together and called the Capacitance
  - $Q = -C\Delta V$ where $C = \dfrac{\varepsilon A}{d}$ [units: Farad] {note: just as with Ohm's law, most texts are very sloppy about the sign, i.e., just consider the magnitudes}
- **Qualitatively**, the bigger the Capacitance, the more charge is stored for the same voltage difference; so, as the name suggests, it's a measure of the devices capacity to store charge.

- What are some common uses of capacitors in the real world?

  Here's another example of the two distinct uses of electronics – used for its ability to transmit and manipulate *energy* or its ability to transmit and manipulate *information.* For *energy,* having a charge separation means having a bit of potential energy. In that way, a capacitor can be used very much like a battery. In general, you can't store *as much* energy in a capacitor as in a battery, however, capacitors can be made that hold their energy better and they can definitely be made to release their energy faster. In electric cars, I believe that capacitors get used to help provide sudden bursts of energy when you need to quickly accelerate. Today though we'll mostly focus on their ability to manipulate information – I'll touch on some of the uses.

- . In the book the author mentioned the case of two capacitor plates being able to rotate relative to each other. My question has to do with the geometry of these plates. It seems that these would be square capacitor plates. Are there ever any instances of capacitor plates not being square or circular?

  - Square or circular capacitors are the easiest for us to visualize, but in reality, most look more like if you'd made an aluminum foil – seranwrap – aluminum foil sandwich and then rolled it up.

- 2) No capacitor is perfect. How can we determine how much energy is lost? How do we set up a real capacitor? / What is the significance of the leakage time of a capacitor?
-

  The ideal capacitor just *holds* its charge; however, a real one will slowly let charge leak from one plate to the other, so you can think of a *real* capacitor as being a combination of an *ideal* capacitor in parallel with a (very large) resistor. To test the capacitor's ability to hold charge, you can check the voltage across it periodically. Of course, you don't want the voltmeter to be contacting the capacitor for very long because the voltmeter itself has an internal resistance and the charges on the capacitor could flow through *it* and thus discharge the capacitor. Qualitatively then the "leakage time" is a measure of how quickly a real capacitor would discharge. We can get more quantitative about that in a moment.

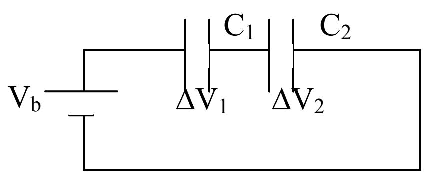
2-2-1 Example 2.1: Capacitance Calculation

Two 1cm^2 metal sheets with a 0.01 mm thick polystyrene sheet (dielectric constant of 2) sandwiched in between have a capacitance of

$$C = \frac{\varepsilon A}{d} = \frac{2\cdot\left(8.85\times10^{-12}(F/m)\right)\left(cm^2\times\left(10^{-4}m^2/cm^2\right)\right)}{0.01mm\times10^{-3}mm/m} = 18\times10^{-12}F = 18\,pF$$

2-3 Capacitors in Series and in Parallel

Series



$$V_b + \Delta V_1 + \Delta V_2 = 0$$

- Loop rule: $V_b - Q_1/C_1 - Q_2/C_2 = 0$

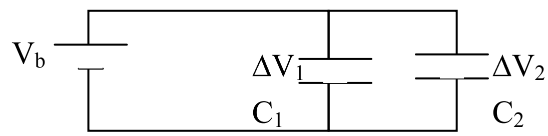
  $$V_b = Q_1/C_1 + Q_2/C_2$$

- In series, it's clear that whatever charge leaves capacitor 1 ends up on capacitor 2, so $Q_1 = Q_2 = Q$
- $V_b = Q\left(1/C_1 + 1/C_2\right)$
- So, the equivalent circuit would have just one capacitor whose capacitance satisfies
- $1/C_s = 1/C_1 + 1/C_2$
- The book points out that this makes geometric / electric sense: the two plates linked to each other in the middle are necessarily at the same voltage, so the total voltage drop

between the far left and far right plates should be proportional to the sum of their gaps:

$$\Delta V = -E_1 d_1 - E_2 d_2 = -\frac{Q}{\varepsilon_1 A_1} d_1 - \frac{Q}{\varepsilon_2 A_2} d_2 = -Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

Parallel



$$V_b = -\Delta V_1 = -\Delta V_2$$

$$V_b = \frac{Q_1}{C_1} = \frac{Q_2}{C_2}$$

If we call the total charge stored $Q = Q_1 + Q_2$ then

$$Q = -C_1 \Delta V_1 - C_2 \Delta V_2$$

But

$$\Delta V_1 = \Delta V_2 = \Delta V$$

So,

$$Q = -(C_1 + C_2)\Delta V$$

Thus the equivalent circuit would have just one capacitor of capacitance

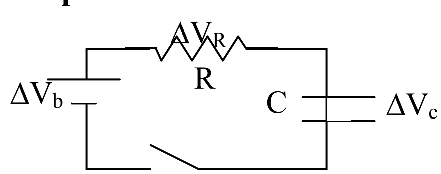
$$C_p = C_1 + C_2$$

The book points out that this too makes geometric / electric sense. Having two capacitors in parallel is kind of like having one capacitor whose area is the sum of the two.

2-5 RC (and RL0 Circuit Transients

- Before getting into the details, consider for a moment that the voltage drop across a resistor depends on the *current,* and the voltage drop across a capacitor depends on the *charge, i.e. the time integral of the current.* Any circuit that has more than one of these three types of elements is bound to have some interesting time dependence.

2-5-1 Resistor-Capacitor Circuits



Qualitatively

- When the switch is closed, charge rushes off the battery terminals and through the resistor, as if the capacitor weren't in the way. When the first charge gets to one plate of the capacitor, it dominoes a like charge off the other plate, so current continues on as if the capacitor weren't even there. But when the second charge approaches the plate, it feels the fringe field of the charge already on the capacitor, so it's a little less enthusiastic. Ditto for each subsequent charge. So, as time goes by, the capacitor charges up, and its fringe field gets larger and larger, more and more opposing continued current, until, ultimately, the current dies completely (after all, the capacitor does represent a physical break in the circuit.)

- **Demo -** http://www.falstad.com/circuit/ - from circuit menu, select basics / capacitor

Quantitatively

- Loop Rule:
  - $\Delta V_b + \Delta v_R + \Delta v_C = 0$
    $\Delta V_b - iR - q/C = 0$
  - $\Delta V_b = iR + q/C$
  - **Note on signs.** Going around the circuit in a clockwise manner, there is a voltage *drop* across the resistor, of magnitude iR, and another *drop* across the capacitor, of magnitude q/C. Thus, both enter with *negative* signs in the equation.
- Take the derivative of both sides to get it purely in terms of I.
  - $0 = \frac{di}{dt}R + i/C \Rightarrow \frac{di}{dt} = -\frac{1}{RC}i$
- Guess a solution, looks exponential $i = i_o e^{-t/\tau}$ and plug in and cancel like factors to get $\tau = RC$.
- If the initial case is that the capacitor has no effect, then $i_o = \Delta V_b / R$
- So, it exponentially decays with a time constant of RC (a product whose unit is seconds).

Quantitative – Qualitative

- Qualitatively we said that, as more and more charge accumulates on the capacitor plates it becomes harder and harder for more charge to get on. Quantitatively we see that's the case – the longer current flows (charging up the plates) the weaker the current gets (due to the repulsion from charges already on the plates.)
- **Analog**
  - While all the math works out right, it might still be a little unintuitive to think that two 'electronic' parameters (R and C) lead to a 'time' parameter. Here's a nice, physical analogy that might help it to feel a little more natural: an Hour Glass. An Hour Glass has two key properties: a pair of reservoirs for sand (analogous to the two plates of the capacitor) and a narrow throat that restricts the sand's flow (analogous to the resistor) – the bigger the reservoirs (Capacitance) and the tighter the throat (Resistance), the longer time the sand will run – varying these two parameters will change the thing from a "minute egg timer" to a full "hour glass." Okay, the circuit uses a *battery* to drive *charge* while an hour glass uses *gravity* to drive *sand*, but they're otherwise very similar timers.

Corresponding Voltages.

- If this function gives the current flowing through the circuit at time *t,* then we can use Ohm's law to find the corresponding voltage across the resistor:
- $\Delta v_R = -iR = -\frac{\Delta V_b}{R}e^{-t/\tau}R = -\Delta V_b e^{-t/\tau}$

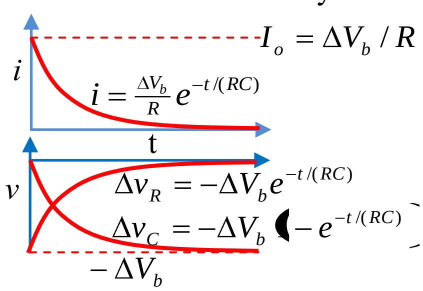
        - (Note: electronics books tend to be negligent about the deltas and the signs of voltage changes across components)
    - So the voltage drop across the resistor slowly decays away since the current through it is slowly decaying away.
- While we're at it, we can use Kirchhoff's Loop Rule to figure out the voltage drop across the capacitor:

$$\Delta V_b + \Delta v_R + \Delta v_C = 0 \Rightarrow \Delta v_C = -(\Delta V_b + \Delta v_R)$$

- $$\Delta v_C = -(\Delta V_b + (-\Delta V_b e^{-t/\tau}))$$

$$\Delta v_C = -\Delta V_b (1 - e^{-t/\tau})$$

- Note that the way we derived this was to say that whatever the voltage drop is across the resistor, the drop across the capacitor must be the rest to add up the battery's voltage. So while the voltage across the resistor slowly dies away, that across the capacitor slowly grows toward that of the battery.
- **Demo –** Notice the exponential curve in the simulation. Vary the R value, vary the C value; see how the curve changes.
- Visualizing all these things,



2-5-1-1Example 2.2 Voltage as a function of time

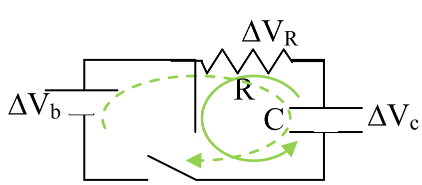
A. If we've got a 1.5V battery, a 10K resistor, and a 3μF capacitor, then 0.04s after the switch is closed, what should be the voltages across the resistor and capacitor.
B. How long after closing the switch will the voltage be 0.5 V?

Got this far.

Group Problem 1

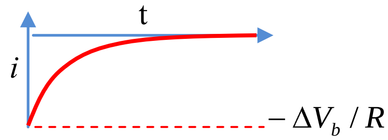
Discharging a Capacitor

Running with the Group Problem, say we now leave the switch in its new position for quite a long time so that the capacitor gets fully charged. Now, flipping the switch *back* allows the charge on one terminal of the capacitor to run around to the *other* one – the capacitor gets discharged. If we've really qualitatively and quantitatively *get* what happens when the capacitor charges, then we should be able to say qualitatively what happens in the discharge and deduce a) the plots of current and voltage and b) the equations.



Qualitatively, the moment we flip the switch into discharge position, the battery's just out there dangling – it can't drive the current, and the + charges that have been piled up on one side of the capacitor now have a straight shot to recombining with the negative charges on the other terminal, so off they go flowing *back* they way they'd come. Initially, the fringe field (those charges' mutual repulsion) is quite large since they're packed on the capacitor pretty tightly, so initially there's a big current flowing backwards. As it flows, the charge density on the plates decreases, and so the fringe field does too, and so the current gets weaker and weaker.

So, the plot we expect for the current is and the expression that goes with it is

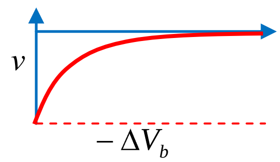


$$i = \frac{\Delta V_b}{R} e^{-t/(RC)}$$

The current is *negative* to indicate that it's now flowing in the *opposite direction.*

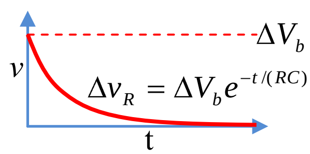
As for the voltage across the capacitor, it started out all charged – equal and opposite to that across the battery, and it will slowly decay to 0 as the charges rearrange themselves.

So, the plot for the voltage across the capacitor is and the expression that goes with it is



$$\Delta v_C = -\Delta V_b \left(1 - e^{-t/(RC)}\right)$$

Finally, either applying Ohm's law across the resistor or appealing to Kirchhoff's loop rule tells us that the voltage across the resistor must be equal and opposite to that across the capacitor,

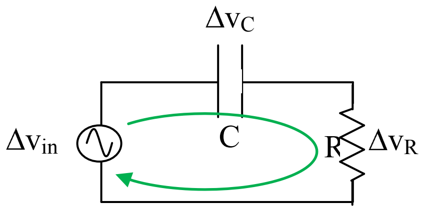


$$\Delta v_R + \Delta v_C = 0 \Rightarrow \Delta v_r = -\Delta v_C$$
$$\Delta v_R = \Delta V_b e^{-t/\tau}$$

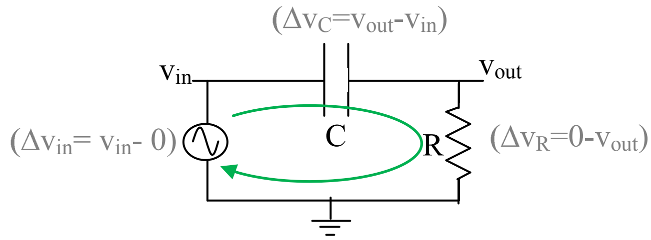
Leakage Time. Returning to the subject of leakage time, recall that a *real* capacitor can be modeled as an ideal one in parallel with a large resistor, call it R_{cap}. So say we charged up a capacitor and then pulled it from the circuit, it would be like having a little circuit of a capacitor & resistor, with a leakage time constant of $R_{cap}C$.

Alternative Representations

- You will encounter three different schematic representations of circuits; it's worth getting familiar with them and recognizing their equivalence. The representation we've used thus far is particularly useful for helping us think about what the *current* is doing, since we explicitly draw the path the current takes and the corresponding voltage changes across different elements. However, as we go on in this course, our focus will shift to looking more at the voltage values at specific locations in the circuit; also, we'll look at more and more complex/cluttered circuits. For both of these reasons, its nice to eliminate from our schematics representations of the rather boring lines along which the current travels, unimpeded, back to the source's negative terminal.
- As for switching from focusing so much on the voltage *across* an element to the voltage *at* a point, the representation shifts like this

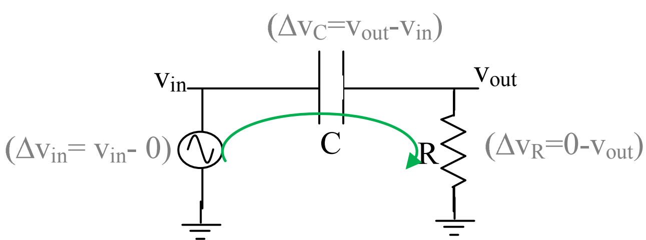


To



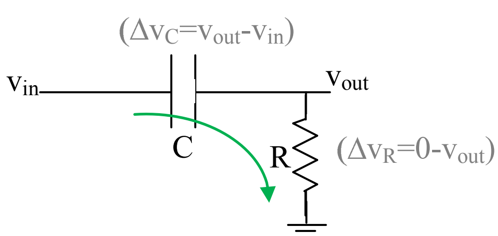
Now the voltage values being labeled are those at particular locations, and are implicitly measured relative to "ground." The part of the circuit that's at the reference value has a tied to it a symbolic spade that would be stuck in the ground. I've put in parentheses how the voltages *across* the circuit elements are related to the voltages at the different points.

The next step in the evolution is to ditch the really boring wire along the bottom, and individually label the important "grounded" points in the circuit



Sure, if both the resistor and the supply have an end stabbed in the ground, then they are electrically connected, so we just aren't explicitly drawing that connection

Now, a very slight variation on this theme is that the schematic might not bother representing the voltage source with a component; you don't need to know what physical entity generated v_{in} to know how v_{out} will be related to v_{in}.



Evolving our Circuitry Concepts toward Signal Processing. Remember that the first day of class I said modern electronics is mostly about first encoding information (say, the words you speak) in an electronics signal, usually the voltage, and then operating on that information. This kind of schematic representation lends itself to considering a circuit that way. For example, say v_{in} comes from a microphone and v_{out} goes to a loudspeaker. So, the information is what you're saying; that's written into time variations in a voltage (rather than spatial variations ink) and then manipulated by this little circuit before being read back out. How does this little circuit 'process' your voice? As we'll see next chapter, it filters out the lower frequency components, so maybe it's selecting which part of the 'signal' is appropriate to send to a tweeter.

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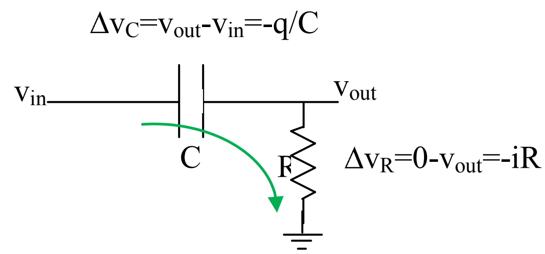
Differentiators

This bit of circuitry actually performs a useful function in the context of a larger, more complex circuit. The output voltage is *approximately* the derivative of the input voltage.

$$\Delta V_C = v_{out} - v_{in}$$
$$-q/C = v_{out} - v_{in}$$
$$-i/C = \tfrac{d}{dt}V_{out} - \tfrac{d}{dt}V_{in}$$
$$-\frac{V_{out}}{R}/C = \tfrac{d}{dt}V_{out} - \tfrac{d}{dt}V_{in}$$
$$V_{out} = RC\left(\tfrac{d}{dt}V_{in} - \tfrac{d}{dt}V_{out}\right)$$

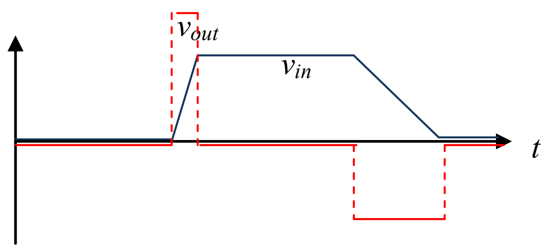


If V_{out} changes much more slowly than V_{in}, (say, RC is quite large) then

$$V_{out} = RC\tfrac{d}{dt}\left(V_{in} - V_{out}\right) \approx RC\tfrac{d}{dt}\left(V_{in}\right)$$

In a few chapters, we'll se how incorporating an operational amplifier (op-amp, for short) into this kind of circuit makes that approximation, for most practical purposes, an equality.

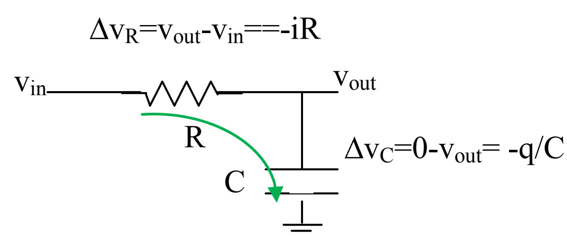
For an *ideal* differentiator,



Integration

Of course, the inverse of differentiation is integration, and flipped around a little, this circuit's output voltage is *approximately* the time integral of the input voltage.

$$\Delta v_R = -iR = v_{out} - v_{in}$$
$$iR = v_{in} - v_{out}$$
$$\int (iR)\,dt = \int (v_{in} - v_{out})\,dt$$
$$qR = \int (v_{in})\,dt - \int (v_{out})\,dt$$
$$(-C\Delta v_C)R = \int (v_{in})\,dt - \int (v_{out})\,dt$$
$$RCv_{out} = \int (v_{in})\,dt - \int (v_{out})\,dt$$
$$v_{out} = \frac{1}{RC}\left(\int (v_{in})\,dt - \int (v_{out})\,dt\right)$$



If the integral of V_{out} is much less than that of V_{in}, then

$$V_{out} \approx \frac{1}{RC}\int V_{in}\,dt$$

Again, in a later chapter we'll use an operational amplifier to make both the Differentiator and Integrator much more exact.

2-5-1-2Example 2.3 (optional)
2-5-1-3Example 2.4 (optional)