Physics 310
Lecture 4 – Transformers, Diodes, and Power Supplies

Fri. 2/3	**Ch 4.2-.3, Ch 5.1-.2:** Transformers, Power Supplies, & Semiconductors	**Lab 3 Notebook**
Mon. 2/6 Wed. 2/8	**Ch 5 – the rest**: Diodes, & Diode-based Devices **Lab 4:** Transformers, Diodes, & Power Supplies	**HW4**: A* & Ch4 pr 1,2,5; Ch5 pr 3*,12

Equipment

Function Generator, O'scope, and nested coils for simple transformer (warning: since these aren't really designed to be good transformers, and there is not significant load on the secondary, their self inductance is significant, and so you see a frequency dependence in the voltage induced on the secondary coil.)

Ball-spring crystal model.
Sample Transformers and Transformer^{tm}
Ppts for p & n types

Chapter 4

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1)  What is the practical purpose of a transformer.  Why would one be used in a
circuit?
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Maybe the second day of class, someone had wanted to know why we choose AC vs. DC. Part of my answer was that we choose AC for transmitting power because we can transmit it across the desert at high voltage / low current and then *transform* it to low voltage and high current for when we use it in our homes. 4.2 & .3 are about the devices that do this for us.

4.2 Transformers

A transformer is essentially two entwined inductors (either physically entwined, or sharing an iron core). So, the principles that underlie their function are the same as underlie the function of a single inductor: Faraday's Law. We'll start there.

Faraday's Law and Coils

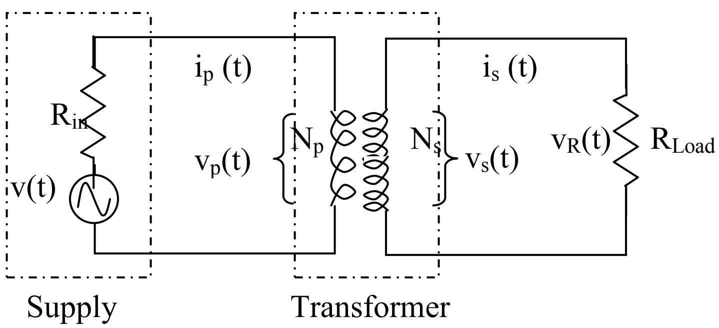
Faraday's Law is often phrased in terms of magnetic flux (the magnetic field piercing an area integrated over that area), $\Phi_B = \int \vec{B} \cdot d\vec{A}$. It relates the change in flux through a loop to the voltage induced around the loop: $V_{Loop} = \frac{d\Phi_B}{dt}$. Since we're going to be using this relationship backwards and forwards, I should stress that this is a correlation, not causation. You can read it as 'if, for whatever reason, there's a voltage around the loop, then there will be a changing magnetic flux through it / if, for whatever reason, there's a changing magnetic flux through the loop, there will be a voltage around it.' The "whatever reason" is, invariably, a time-changing current density somewhere.

Your typical inductor is a coil, i.e., a string of loops. If it has N loops, each with the same time changing flux through it, then the total voltage, from end to end of the whole coil, is $V_{coil} = N_{loops} V_{loop} = N_{loops} \frac{d\Phi_B}{dt}$.

As an aside, we can go ahead and close the gap between this expression and the one we've used for inductors. As you recall, a magnetic field is generated by a current. Mathematically, the field and current are proportional, so the flux is proportional to the current, $\Phi_{\mathbf{B}} \propto I$, and the rate of change of the flux is proportional to the rate of change of current, $\frac{d\Phi_{\mathbf{B}}}{dt} \propto \frac{dI}{dt}$. In a stand-alone inductor, the current that causes this magnetic field is the self-same one running along the coil. In that case, the proportionality constant times the N_{coil} is the "self inductance", L. So we have the familiar expression $V_{coil} = L\frac{dI}{dt}$.

Faraday's Law and Transformers

To focus our application of Faraday's Law to a transformer (and to see what transformers are good for), we'll imagine a concrete example. Let's say we have it hooked up in a simple circuit with a real, AC power supply attached at on end, and a "load" at the other. As already mentioned, a transformer is two entwined coils. The symbol is two inductor symbols side-by-side (actually drawing them entwined would be messy). So, the circuit's schematic looks like this:



We'll call the transformer's coil that's attached to the power supply the "primary" coil, and it's composed of N_p loops. The other coil in the transformer is called the "secondary" coil, and it's composed of N_s loops.

Thanks largely to the power supply, there will be a time-varying voltage along the primary coil, v_p(t). Thus, there will be a corresponding time-varying magnetic flux through each loop of the primary coil,

$$v_{loop}(t) = \frac{v_p(t)}{N_p} = \frac{d\Phi_B(t)}{dt}.$$

Now, if the secondary coil is entwined with the primary coil, then it enwraps the exact same time varying flux, so, along it there must be a corresponding voltage,

$$\frac{d\Phi_B(t)}{dt} = \frac{v_s(t)}{N_s} = v_{loop}(t).$$

(Note that, if the transformer were constructed so the two coils were not entwined, or had different cross-sectional areas, then they would not have the exact same flux, but for convenience, transformers *are* constructed to approach this ideal).

Setting these two expressions equal,

$$\frac{v_p(t)}{N_p} = v_{loop}(t) = \frac{v_s(t)}{N_s}$$

or

$$\frac{v_s(t)}{v_p(t)} = \frac{N_s}{N_p}$$

This is one of the important uses of transformers; depending on the ratio of turns, $\frac{N_s}{N_p}$, the voltage can be "stepped up" or "stepped down" across the transformer. For example, in lab you used a transformer that took a 110 V rms v_p and gave a 6.3 V rms v_s.

Demo: with o'scope, function generator, and nested coils, show that the ratio of the number of coils enclosing the changing flux determines the ratio of the voltages.

For completeness, it's worth noting that the magnetic flux shared by the two coils is caused by the currents flowing through *both* coils: $\frac{d\Phi_B(t)}{dt} = \frac{L_p}{N_p}\frac{dI_p(t)}{dt} + \frac{L_s}{N_s}\frac{dI_s(t)}{dt}$. Perhaps you first plug the transformer into the power supply, so a current starts flowing through it, but as that current ramps up, it generates a time varying flux and induces a voltage in the second coil, through which a current ramps up that, generates an additional contribution to the time varying flux.

Power and Impedance.

In a qualitative way, the current running through the primary is responsible for driving another current through the secondary, and, ultimately, through the load. One implication is that, as the load impedes the flow of the second current which the first current is driving, so in a second-hand way, this load actually hampers the first current too. Another way to say this is that whatever energy is dissipated as the secondary current goes through the load must have been imparted to the secondary current by the primary current to begin with, and the rates, i.e., powers, of those two energy transfers (from primary into secondary and from secondary into load) must be the same. Making this point quantitative will actually get us two things: the relation between the *currents* through the two sets of coils and the impedance of the transformer.

Physics 310

Lecture 4 – Transformers, Diodes, and Power Supplies

Stepping up/down currents

$$
\begin{array}{ccc}
P_p(t) & = & P_s(t) \\
\Downarrow & & \Downarrow \\
i_p(t)v_p(t) & = & i_s(t)v_s(t) \\
 & \Downarrow & \\
\dfrac{i_s(t)}{i_p(t)} = & \dfrac{v_p(t)}{v_s(t)} = & \dfrac{N_p}{N_s}
\end{array}
$$

This is our first result: while the ratio of the *voltages* is proportional to the ratio of the windings (a.k.a. number of coils), the ration of the *currents* is *inversely* proportional to that ratio. So, when a transformer "steps *down*" a voltage from maybe 120kV on the primary to 120V on the secondary, it "steps *up*" the current from maybe 1 A to maybe 1,000 A! Again, this is why AC is used for transmitting electrical power over great distances – it can be transmitted at a very high voltage / comparatively low current that doesn't heat the wires too much and so doesn't lose too much energy in the process, , and then *transformed* to a lower voltage and much higher current for use running your home appliances.

The perfect transfer of energy from primary to secondary, and thus the equality of the powers, is an idealization. In actuality, some energy is transferred into the magnetic field and returned to the primary later in the cycle, only $i_p \left(v_p(t) - L_p \dfrac{di_p}{dt} \right)$ is transferred to the second coil. For that matter, the primary coil has *some* resistance, and thus some of the energy is lost to resistive heating. As with all basic electrical devices, real transformers are designed to optimize their desired behavior and minimize the other – compared with the energy transferred to the secondary coil, that lost resistively or invested in self-inductance is negligible for most considerations. If needed, a more realistic model of a "real" transformer has a stand-alone "ideal" inductor and an "ideal" resistor in series with the "ideal" transformer's primary coil. Ignoring the stand-alone inductor may be akin to saying that the "magnetization current" (the current running through the primary if there were no secondary), $i_{p_{open}} = \dfrac{v_p}{\omega L_p}$ is quite small. For more on a "real" transformer, see Sprott, p86 – 89.

Physics 310
Lecture 4 – Transformers, Diodes, and Power Supplies
Transformer Impedance

Now we're ready to ask (and answer) the question 'what does the transformer look like to the power supply?' In other words, what's the equivalent impedance of the transformer's primary coil? So we'll try to get an expression for the *voltage* across the primary in terms of the *current* through the primary, that is, something in of the form of Ohm's Law.

$$v_p(t) = v_s(t)\frac{N_p}{N_s}$$

$$v_p(t) = v_L(t)\frac{N_p}{N_s}$$

$$v_p(t) = i_s(t)R_L\frac{N_p}{N_s}$$

$$v_p(t) = i_p(t)\frac{N_p}{N_s}R_L\frac{N_p}{N_s}$$

$$v_p(t) = i_p(t)\left(R_L\left(\frac{N_p}{N_s}\right)^2\right)$$

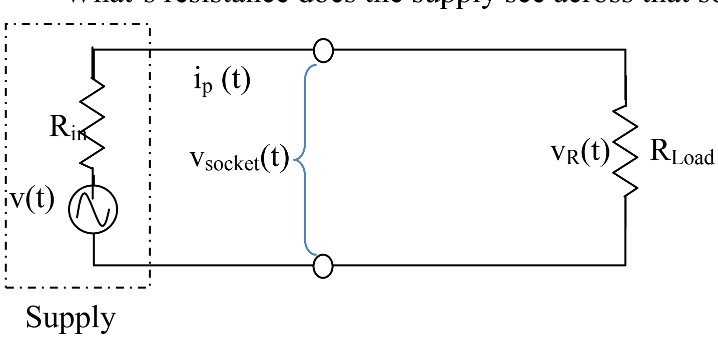
There we have it; the power supply knows that it's pumping out i_p of current and that there's a voltage drop of v_p across that slot where the primary coil is plugged in; thus, the effective resistance must be their ratio,

$$R_p \equiv R_L\left(\frac{N_p}{N_s}\right)^2.$$

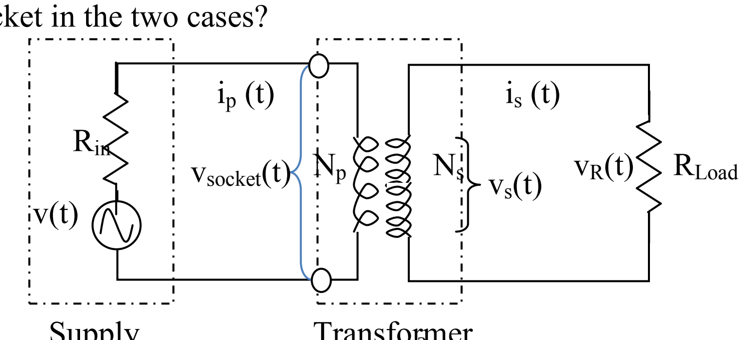
Thinking in terms of the tools of the previous chapter, maybe the "load" that's plugged into the transformer's secondary coil has inductance, maybe it has capacitance. So the *Impedance* of the primary coil (assuming its own inductive impedance is much smaller than the Impedance of the load) is

$$\vec{Z}_{primary} \equiv \vec{Z}_{Load}\left(\frac{N_p}{N_s}\right)^2$$

This relation highlights a second big use of transformers. Compare these two circuits, one has a power supply with the load resistor plugged right into its output socket and the other has the load resistor plugged into a *transformer* which is then plugged into the output socket. What's resistance does the supply see across that socket in the two cases?



$$R_{socket} = R_L$$



$$R_{socket} = R_L\left(\frac{N_p}{N_s}\right)^2$$

Regardless of what the load's real resistance is, you can insert a transformer of whatever turn ratio you want and make the load *appear to the supply* as if it has any value you want !

Okay, while that may be kind of nifty in and of itself, there's real reason to want to be able to change the apparent resistance; that has to do with maximizing energy transfer.

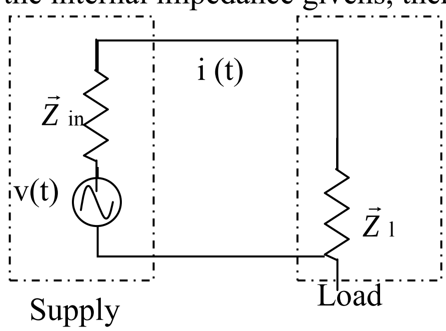
4.3 Impedance Matching and Power Transformers

Impedance Matching, General

In many applications, one cares not about the current or the voltage being provided by a supply, but the *power* , i.e., the rate at which *energy* is being provided. Think of a fan or a speaker. The fan's motion can be quantified in terms of energy, and that energy had to come from somewhere, an electric circuit; the strength of the speaker's sound can be quantified in terms of power, and that has to come from somewhere, an electric circuit. Of course, when transferring energy, you want to do it as efficiently as possible – you don't want to waste more than is necessary on, say, resistively heating the power supply itself. So the question arises, how do I *maximize* the power delivery to the load? That's where Impedance Matching comes in.

Qualitiatively, since *P=iv*, you know that the smaller the impedance, the more current can flow (good for power) but it also means a smaller the voltage drop across the element (bad for power), and of course the flip is true: more resistance means less current (bad for power) but greater voltage drop (good for power). Clearly, you don't want the impedance to be too big or too small, but some magic "just right."

To get quantitative, consider the following vague circuit in which we'll consider the source voltage and the internal impedance givens, then we want to know what's the best load impedance:



Let's assume that, as is often the case, the voltage of the supply is a given, so the current is what would change if we changed the load impedance. To handle the general case, where the impedances can be due to any combo of resistors, inductors, and capacitors, I'll work with the complex vector of the amplitudes and phases:

$$\vec{P}_{load} = \vec{i}\,\vec{v}_{load}$$

Where $\vec{v}_{load} = \vec{i}\,\vec{Z}_{load}$ and $\vec{i} = \dfrac{\vec{v}}{\vec{Z}_{in} + \vec{Z}_{load}}$

So,

$$\vec{P}_{load} = \vec{i}\,\vec{i}\,\vec{Z}_{load} = \frac{\vec{v}^{\,2}}{\left(\vec{Z}_{in} + \vec{Z}_{load}\right)^2}\vec{Z}_{load}$$

Physics 310

Lecture 4 – Transformers, Diodes, and Power Supplies

To find the load impedances for which the load's power is maximized (or minimized) I'll play the old trick of taking the derivative and setting it equal to zero, then solving for the impedance that is implied.

$$\frac{\partial \vec{P}_l}{\partial \vec{Z}_l}=\frac{\vec{v}^2}{\left(\vec{Z}_{in}+\vec{Z}_l\right)^2}\left(1-\frac{2}{\left(\vec{Z}_{in}+\vec{Z}_l\right)}\vec{Z}_l\right)=\frac{\vec{v}^2}{\left(\vec{Z}_{in}+\vec{Z}_l\right)^2}\left(1-\frac{2}{\left(\frac{\vec{Z}_{in}}{\vec{Z}_l}+1\right)}\right)=0$$

This can go to zero in two cases, if the first factor is zero, i.e., if $\vec{Z}_l=\infty$ or if the second factor is zero, i.e., $\vec{Z}_l=\vec{Z}_{in}$. In the fist case, *no* current passes, so no power is delivered (you can also see this by simply taking the limit of the power expression as the load impedance goes to infinity.) The rate of energy delivery, i.e., the power to the load is maximized when the load and internal impedances are equal, or "balanced,"

Maximize Power transfer to load if

$$\vec{Z}_l=\vec{Z}_{in}.$$

This is why stereos often specify the impedance of the speakers that you should use with them (if I recall, usually around 4 to 8Ω) – so you a) get the most power out of the speakers and b) heat the stereo itself the least.

Of course, this begs the question "now that we know how the impedances should be related to most efficiently transfer energy, how efficient is it?"

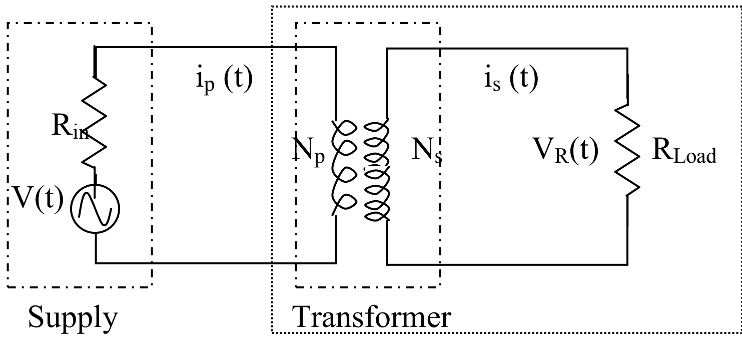
Plugging this relation back into our expression for the power delivered to the load,

$$\vec{P}_{load}=\frac{\vec{v}^2}{\left(\vec{Z}_{in}+\vec{Z}_{in}\right)^2}\vec{Z}_{in}=\frac{\vec{v}^2}{4\vec{Z}_{in}}\ .$$

Of course, the same expression applies to the power dissipated by the internal resistance; thus, this most efficient configuration still has only half the total power being delivered to the load resistor while the other half is wasted on the internal resistance.

Impedance Matching and "power transformers."

Okay, so if you want to maximize the energy delivery to the load, you need its impedance to match the supply's internal impedance. Perhaps you can imagine having a given power supply and a given load that you want to drive with it, and their impedances are *nothing* like each other's. The solution is you can pick a transformer to insert between them. Pick the right transformer, and, from the supply's perspective, the load will *look* like it's got the right impedance.



From the supply's perspective, all this is the "load"

Physics 310
Lecture 4 – Transformers, Diodes, and Power Supplies

From the supply's perspective, the load it has to deal with is that Primary coil, with its impedance of

$$"Z_{load}" = R_p = R_L \left(\frac{N_p}{N_s}\right)^2.$$

Say you're tasked with selecting a transformer to most efficiently mate a given supply to a given load. If the impedance of the load is a given, and the supply's internal impedance is a given, then the one thing you can tweak is the "turn ratio" of the transformer, in order to match the impedances and thus maximize power to the load.

Maximize Power transfer / match impedances:

$$R_p = R_{in}$$

$$R_L \left(\frac{N_p}{N_s}\right)^2 = R_{in} \Rightarrow \frac{N_p}{N_s} = \sqrt{\frac{R_{in}}{R_L}}$$

Gives the turn ratio that will match the impedances.

To actually find the load's power in terms of this, its impedance, and the source's voltage,

$$P_{load} = \frac{v^2}{4Z_{in}} = \frac{v^2}{4R_{in}} = \frac{1}{4R_L}\left(v\frac{N_s}{N_p}\right)^2$$

Chapter 5: Diodes et al.

2) What are different types of crystal lattice structures and how are they formed? – What crystal lattice forms is dictated by the electronic structure of the atoms that its made of, that is, what are the outer-most occupied electronic states, how fully occupied they are, and what geometry they have (are the spherical like a ball, elongated like a cigar, lobed like the ears on a Mickey-mouse hat,…) As for different structures, imagine taking a bunch of sugar cubes and stacking them up to form a giant sugar cube; with that picture in mind, now imagine dots at each corner where each cube meets its neighbors – Okay, now forget about the sugar cubes and just imagine those regularly spaced dots – atoms that are thus arranged form a "simple cubic" crystal. Another common crystal has atoms at all those dots *and* atoms in what would have been the centers of each sugar cube – the "body centered cubic". Another crystal has atoms at all the corners and, instead of in the center of each cube, in the center of each *face* where cubes meet – the "face centered cubic." There are others, but those are the easy ones to describe.

What purpose do conductors have in a circuit? – all the metal wires are classified as "conductors" – electrical current easily passes along them.

3) Forward and reverse bias and p vs n types? What is the significance? Next time.

Physics 310

Lecture 4 – Transformers, Diodes, and Power Supplies

5-1 Semiconductors: Physical Model

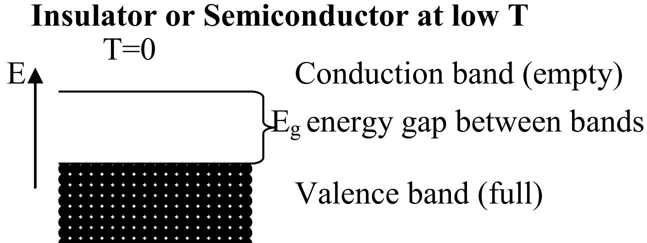
The book gives a pretty good qualitative description and pretty good illustrations of what's going on spatially. Another useful perspective to use is energy. This is in no way intended to be a substitute for what the book did, you can get a qualitative feel for how semiconductors work only if you look at it both ways – spatially and energetically.

Energy Bands

The first step is building up a picture of the energy levels in a solid. You're familiar with the ladder of energy levels in an atom. We'll start there, and conceptually build our solid atom-by-atom. When two atoms bond, the inner energy levels are pretty much unperturbed (the electrons down there are much closer to their own nucleus than to the new neighbor's), but the outer, or valance level, and all those empty ones above, merge from one atom to the next. The resulting energy ladder looks a little more complicated because the electrons now have some choices: hang-out mostly in the gap between the two atoms, hang out mostly right on the two atoms. The different choices have *slightly* different energies. The more atoms are linked up, the more choices the valence electrons get, the more *very near* energy levels arise. When you've got a bone-a-fide solid, with gobs of atoms bonded up, rather than having a discrete atomic valence level, you have a whole "band" of *very near by* energy levels. Much like the energy levels for an atom in a gas, these energy levels are so near by that, in many respects, it's like the "valance band" is continuous. Something similar happens to the unoccupied atomic levels above the valance level – virtually continuous bands are formed there too.

The following are common cartoons that are based on energy level diagrams that you may be familiar with for atoms. This in particular is the cartoon for an insulator or a semiconductor at low temperatures.

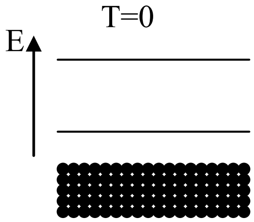
Insulator or Semiconductor at low T



At zero temperature, the valance band is completely full and the next band up, the "conduction band" is completely empty. From an electronics perspective, the problem with an empty band is obvious – there aren't any electrons there, much less any moving around to constitute an electrical current. The problem with a full band is actually just the same. It's completely full, so there's no where for an electron to go! With each electron state available full, there are just as many electrons running left as right, as up as down,… through the material – so no net current. A good analogy is a cup that is completely full with a lid on top. Tip the cup as you may, you get now sloshing, no flow of water across the cup because it's completely full, there's no where for the water to go. On the other hand, if the cup's only half full, then you can very readily get water to flow back and forth as you tip it. The electrical analog of the 'half-full' cup is a metal which typically has a half-full valence band.

Physics 310
Lecture 4 – Transformers, Diodes, and Power Supplies

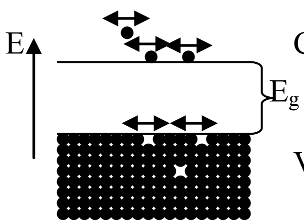
Conductor at low T



Valence band = Conduction band
(half full / half empty)

So, a semiconductor, at low T, *is* an insulator. The reason that temperature maters is that the hotter something is, the more violently the atoms are jiggling and the more violently they collide, and the more likely that they'll knock their electrons free into the conduction band. That leave's sloshing room behind in the Valance band and puts mobile electrons up in the conduction band – now the semiconductor *can* carry an electrical current if a voltage is applied.

Insulator or Semiconductor at high T



Conduction band (empty)
E_g energy gap between bands
Valence band (full)

(image interpretation note: the different elevations of the electrons (dots) and holes (absence of dots) conveys that they have slightly different energies. The horizontal displacement conveys that they're at different locations in the solid. But don't take either horizontal or vertical placement too literally.)

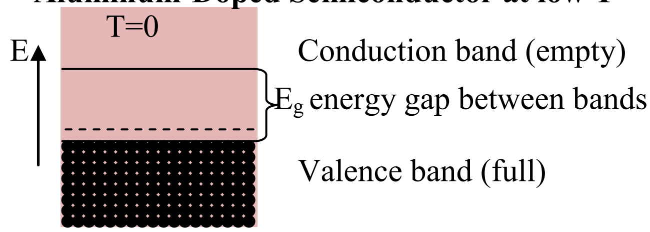
Still, it takes an awful lot of energy to knock an electron all the way from the Valance band to the conduction band. In silicon, it takes about 0.6 eV. Thermal banging about is this energetic only when the silicon is around the temperature of the sun (at which point the material may have other issues).

It's time for performance enhancing doping. As the book discusses, there are two kinds, those that have one more atomic valance electron than the semiconductor atoms and ones that have one less. To make it a little more specific, let's say that the material's made of Silicon, which, when bonded up, shares four of its electrons with its neighbors. If every so often there's an aluminum atom where a silicon atom should be, then it's in a funny situation. On the one hand, it only has three electrons to share, on the other hand, all of its silicon neighbors have silicon neighbors of their own and have arranged their electrons to make four bonds. So the Aluminum atom ends up time-sharing its three electrons in four bonds with its neighbors. While it doesn't have enough protons in its nucleus to firmly hold an additional electron, it's made space for one. The result is the aluminum atom has an energy level just above the top of the valance band. To represent that in the cartoon, we use a dashed line: the level exists, but only at the locations of the aluminum atoms that are scattered about.

Physics 310
Lecture 4 – Transformers, Diodes, and Power Supplies

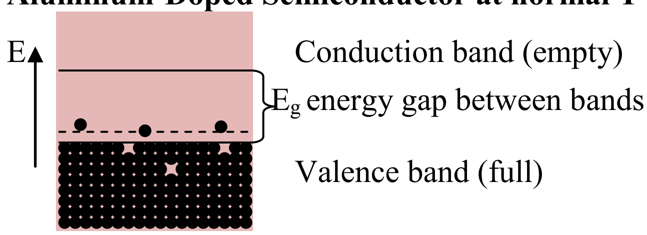
P-type (doped) semiconductor

Aluminum-Doped Semiconductor at low T



At regular temperatures, the conduction band is out of reach of the valence electrons – thermal jostlings just aren't enough to kick them up that high. However, these states at the aluminum atoms are accessible. We call these states "acceptor" states since they can "accept" valance electrons. Their energy level is then called the "acceptor band." At modest temperatures, some number of electrons will be in this "acceptor band."

Aluminum-Doped Semiconductor at normal T

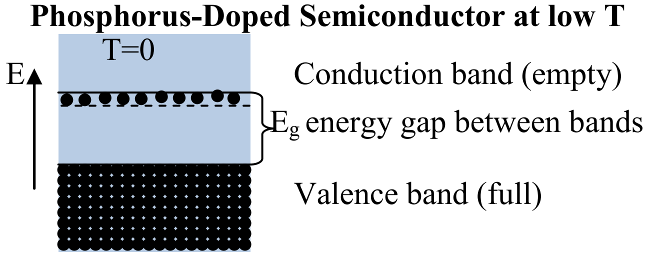


By accepting the electrons, the acceptor band has left "holes" behind in the valance band. Other electrons in the valance band are free to move into the holes, and thus leave holes behind them. In this way the electron holes can wander through the material. A mobile lack of electrons is electrically the same as a mobile positive charge. Thus, such material is called "p-type" for the mobile **p**ositive charges. So, if you apply a voltage across this chuck of doped semiconductor, holes will flow / current will flow, not as well as in a metal (there are far more mobile charge carriers in a metal than in a doped semiconductor) but they'll flow.

N-type (doped) semiconductor

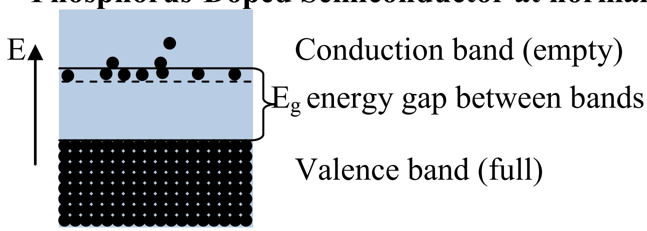
Alternatively, say we "doped" our semiconductor with Phosphorus. Phosphorus atoms have one more proton and one more electron than do silicon atoms. However, when surrounded by silicon in a solid, phosphorus atoms reach out to form just four bonds. So a silicon atom has one more outer electron than it can incorporate in the solid's valance band. Of course, it's got the protons to hold it, but just barely. Each Phosphorus atom has an additional energy level just below the bottom of the conduction band.

Phosphorus-Doped Semiconductor at low T



At very low temperatures, all the phosphorus atoms would hold onto their extra electrons; however, at normal temperatures, random thermal jostelings would have kicked a fair number of these extra electrons into to the conduction band. Since the phosphorus acts to provide or “donate” electrons, they’re known as “donors”, and the energy level is called the “donor band.”

Phosphorus-Doped Semiconductor at normal T



Because this kind of doping produces mobile electrons in the conduction band, and because electrons are negatively charged, this is called “n-type” semiconductor for the **n**egative electrons. Just like the “p-type” semiconductor, “n-type” semiconductor will conduct electricity.