Mon., 4/6	**23.3-4** Accelerating Charges Radiating	**RE29**
Tues, 4/7		**Lab 10 Notes HW22:**RQ.20, 22, 24; P.26, 27, 39
Wed., 4/8	**23.5-6** Effects of Radiation on Matter	
Thurs., 4/9	**Quiz** Ch 23, **Lab 11** Polarization	**RE30**
Fri., 4/10	**23** Conceptual Maxwell's & Applications	
Mon., 4/13	**Review**	**HW23:**RQ.13, 14, 17; P.22, 24, 25 **due beginning of class**
Sat. April. 18 9a.m.	**Final Exam** (mostly conceptual section on 13-20, a separate section on 21-23)	

Maxwell's equations – What is each one good for?

Gauss's law:	$\oint \vec{E} \cdot \hat{n}\, dA = \frac{1}{e_0} \sum Q_{\substack{\text{inside} \\ \text{surface}}}$	find $\vec{E}$ w/ symmetric charges
Gauss's law for $\vec{B}$:	$\oint \vec{B} \cdot \hat{n}\, dA = 0$	recognize bad patterns for $\vec{B}$
Faraday's law:	$\oint \vec{E} \cdot d\vec{\ell} = -\frac{d}{dt}\left[\int \vec{B} \cdot \hat{n}\, dA\right]$	find induce emf / current
Ampere-Maxwell law:	$\oint \vec{B} \cdot d\vec{\ell} = m_0 \left[\sum I_{\substack{\text{inside} \\ \text{path}}} + e_0 \frac{d\Phi_{\text{elec}}}{dt}\right]$	find $\vec{B}$ w/ symmetric current

In the last law, RHR gives the direction that $\vec{B}$ circles around $+d\vec{E}/dt$. The other rule needed to summarize electromagnetism is the Lorentz force rule:

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B},$$

which describes how the fields affect charges.

Last Time

- We demonstrated that these allow for the existence of an electro-magnetic pulse. In particular, we considered a boxy pulse like:

Demo: 23_pulse_sq.py

- **Two points:**
    - **Generalizable.** While this particular pulse looks rather peculiar, and I'd be hard pressed to say what could create this exact pulse, a wide range of geometries could be created by building up from an infinite number of differentially small pulses like this.
    - **Need cause.** Swell, the laws *allow* that such a thing, but they don't tell us how a pulse could be created. That's the main point of today.

Electromagnetic Radiation:

A pattern of electric <u>and</u> magnetic fields (<u>not</u> just one or the other) can propagate through empty space. For this to occur, the following must be true:

- Relative sizes of fields is right: $E = Bv$
- The speed has a particular value: $v = \sqrt{1/m_0 e_0} = 3\times 10^8$ m/s $= c$ (slower in material)
- The velocity ($\vec{v}$) of the pattern is in the same direction as $\vec{E}\times\vec{B}$ (didn't mention Friday)

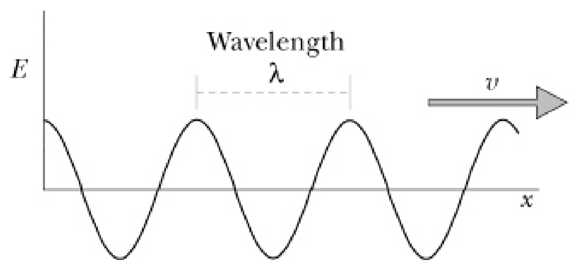
There are other patterns that work. The most important example is an EM "wave" where the pattern of each field (the tips of the vectors) makes a wave. Notice that direction of $\vec{E}\times\vec{B}$ is the same for all parts.

DEMO: **EMWave.py** (show EM wave and how it satisfies Faraday's law and the Ampere-Maxwell law.)

Wave Properties

For a sinusoidal EM wave, we define the following:

- **Wavelength** (l) **–** the distance between peaks of the pattern
- **Period** (T) **–** the time between two peaks arriving a fixed location
- **Frequency** (f) **–** how frequently peaks arrive at a fixed location (#/second)



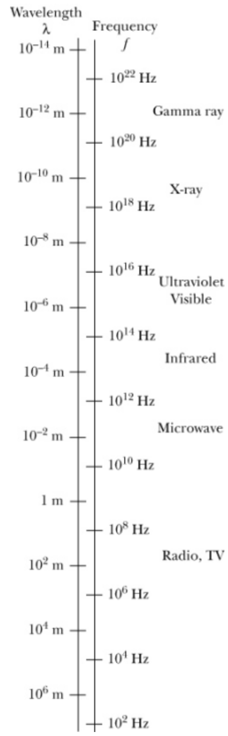
A similar plot could be drawn along a time axis illustrating the period.

The last two quantities are related by: $f = 1/T$

Wave Speed. Two of the quantities can be related to the speed, because the wave moves a wavelength (l) in one period (T):

$$v = c = l/T = lf$$

Different wavelengths or frequencies (they are inversely proportional) make up the *electromagnetic spectrum* (see below). Only a small range of these are visible light.

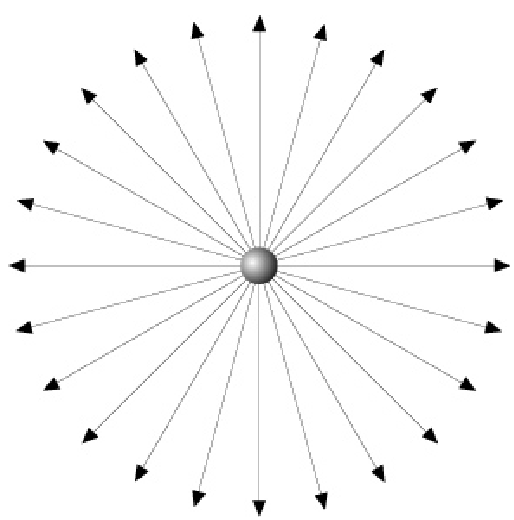


***Accelerated* Charges produce EM Radiation:**

Enough about the character of waves – let's see how we can make them.

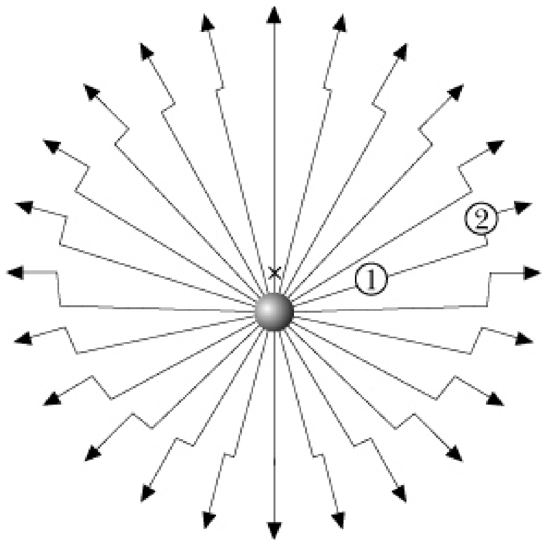
We have been drawing electric field *vectors* (direction and length) at various locations, but there is another way to represent the electric field. We can also draw *electric field lines* which show the direction at each point. The picture for a stationary positive charge is shown below.

23_radiate0_fieldlines.py



What if the charge undergoes a brief acceleration downward? Because of retardation, the change in the direction in the electric field will not be noticed by all observers at the same time. In the first figure below (earlier time), the field points away from the new position at location 1, but away from the old position at location 2. At a later time, the change reaches location 2.

23_radiation2D.py



Earlier time Later time

The “kinks” in the electric field lines propagate outward at the speed of light. It is a sideways or *transverse* electric field at each location (instead of a radial one). Note that the transverse field is biggest perpendicular to the direction of the acceleration and there is none parallel to the acceleration.

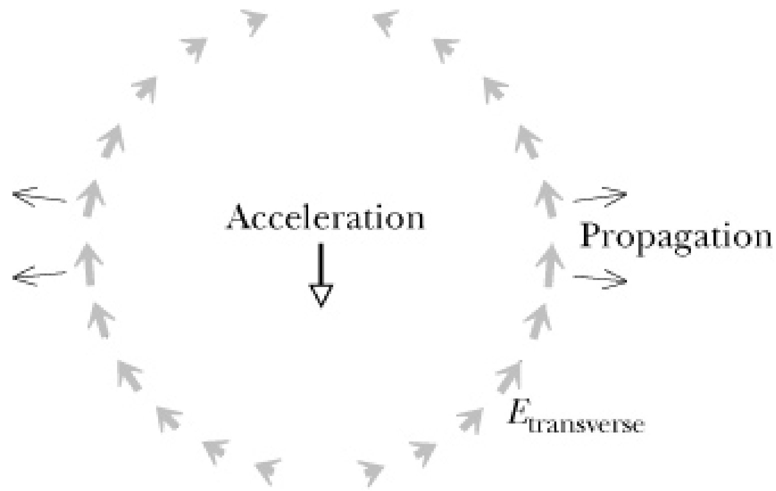
According to Maxwell’s Equations, there can’t be a changing (moving) electric field without a changing magnetic field. At each point, the velocity ($\vec{v}$) of the pattern is in the same direction as $\vec{E}\times\vec{B}$.

DEMO: 23_radiate_2D_show_R.py (this shows E&B and vectors rather than field lines)

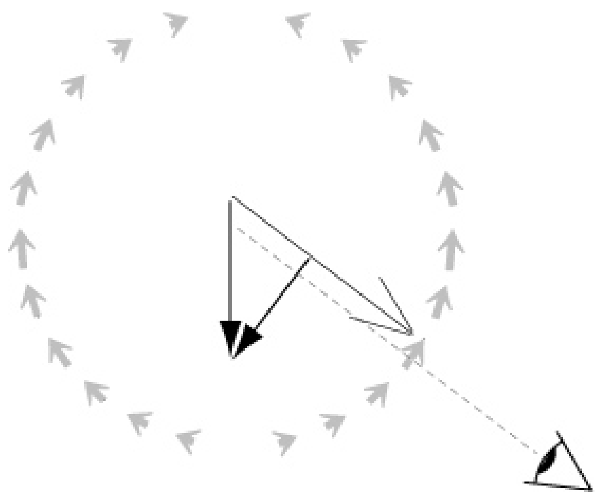
Or in 3-D

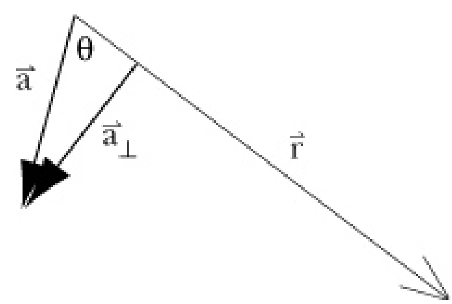
DEMO: 23_Radiation3D.py VPython program showing the expanding spherical pattern away from a positive charge after a brief downward acceleration.

Just the expanding transverse electric field pattern looks like the following for a positive charge that accelerates downward. How can we mathematically describe the size and direction of the transverse (or radiative) electric field?



If $\vec{r}$ is a vector from the charge to the observation point, the radiative field is proportional to $-q\vec{a}_{\perp}$, where $\vec{a}_{\perp}$ is the *projected* acceleration. It is the component of the acceleration if you look back toward the charge. If q is the angle between the acceleration and $\vec{r}$, the size of the projected acceleration is $|\vec{a}_{\perp}| = a \sin q$. For q = 0, there is no projected acceleration ($|\vec{a}_{\perp}| = 0$) and for q = 90°, the entire acceleration is projected ($|\vec{a}_{\perp}| = a$).





If the charge is negative, the radiative electric field will be the same direction at the projected acceleration ($\vec{a}_{\perp}$).

The size of the radiative electric field falls off with distance as $1/r$ – you'll see why in more detail advanced E&M. The full expression is:

$$\vec{E}_{\text{radiative}} = \frac{1}{4pe_0}\frac{-q\vec{a}_{\perp}}{c^2 r}$$

At a large enough distance, this will be larger than the *ordinary* electric field (drops off as $1/r^2$). For this reason, and because the 1/r2 terms are often canceled in neutral mater (equal number of

+ and – charges), this dominates the field of something like, say, the sun, as we experience it here on Earth.

Stability of Atoms:

A simple (Bohr's) model of an atom is that the electrons orbit around the nucleus, which means they are accelerating. Why atoms can be stable is one of the mysteries explained (or at least modeled) by quantum mechanics.

Gravitomagnetism

- **Our $1/r^2$ law: Coulomb's Law.** When introducing magnetism, I made a point of stressing that it could be derived from Coulomb's Law and Special Relativity. In other words, that you could start with Coulomb's Law for a stationary point charge, transform to a frame in which it is seen to move at a constant speed, and get the right electric effects on a charge that is stationary in that frame; transform into a frame in which both the source and test charge are moving (at different velocities) and you'll get the right electric and "magnetic" effects; transform into a frame in which the source is seen accelerating, and you'll get the right electric and magnetic effects. Another way of putting it is that all of Maxwell's Laws are derivable consequences of relativistic transformations of Coulomb's Law – that's just what a simple divergent $1/r^2$ force law looks like in different reference frames! (Mild qualifier: some assumptions go into this process, and the reasonability of an assumption is in the eye of the beholder.)
- **The *other* $1/r^2$ law: Newton's Universal Law of Gravitation.** Now, what *other* force law has the same mathematical form as Coulomb's Law? Newton's Universal Law of Gravitation. So, at first blush, one should expect to get 'Maxwell's Gravitational' laws upon performing the right transformations. Just as Maxwell's laws allow for electro-magnetic radiation, their gravitational analogs allow for gravitational radiation. So, one should also expect to get gravitational waves and radiation of gravitational energy. Strange as that may sound, it's actually an approximation to a reality that is *stranger* still. This approximation is dubbed "Gravitomagnetism." Essentially, one makes the following replacements $q \rightarrow m, \frac{1}{4\pi\epsilon_o} \rightarrow -G$, and $\mu_o \rightarrow \frac{-1}{4\pi G c^2}$, and doubles the effect of the "magnetic" force in the Lorentz force rule (I don't quite follow why this happens, probably has something to do with the lack of a background canceling "charge"). This approximation is good as long as the gravitational fields are relatively weak – to my mind that suggests this is the special relativistic limit for gravitation. If they are strong, things get more complicated because mass has a *very* special property that charge does not – mass warps time and space measurements, in fact, in a full General Relativist treatment, that's all that gravity is – the warping of time and space measurements in response to mass! But, to first order, that looks a lot like Newton's Universal Gravitational Force Law, which transforms just like Coulomb's Law.
- **Gravitational Radiation Energy Scale.** As you might expect, gravitational waves are expected to carry miniscule amounts of energy. For example – the Earth-Sun system should radiate at about 300 Watts (for comparison, I have a 300 Watt light bulb in my

living room lamp – hefty for a household lightbulb, but nothing compared to the 10^{17}Watts of the sun's E&M radiation!)

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