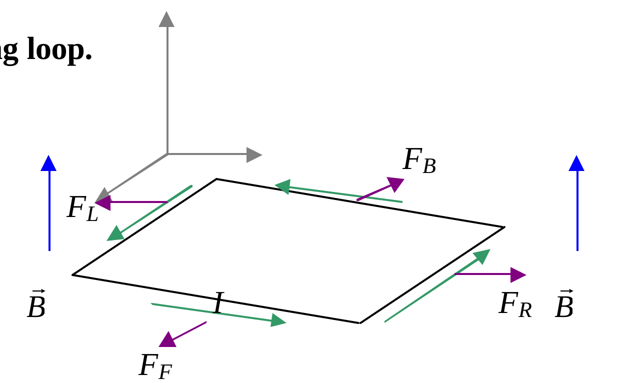
Mon., 3/16 Tues., 3/17 Wed., 3/18 Thurs., 3/19 Fri., 3/20	**20.9-10** Dipole's Potential Energy, Motors & Generators **Review** **Exam 2** (17-20) Magnetic Field and Moving Charges **21.1-3** E Flux and Gauss's Law	**RE22**, **Exp** 26 **HW20:**RQ.28, 34, 36; P.45, 56, **Lab 7** **RE23** , **Lab Notebook**

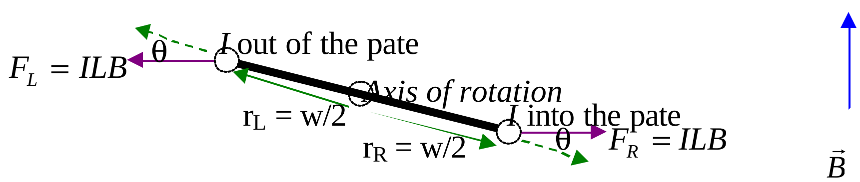
Announcements

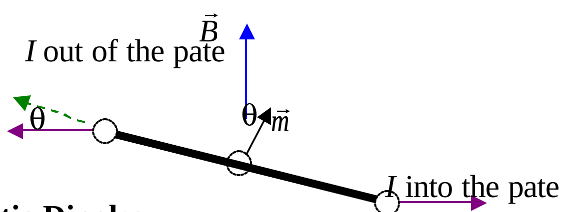
- **Exam 2:** Review material is available on the website. Come to class Wednesday with questions, prompted by that material, by homework,… Those questions will be the basis of the day's discussion.

From Last Time

- **Forces on a current carrying loop.**







Potential Energy for Magnetic Dipoles:

We can phrase the same interaction in the language of work and energy. When the loop rotates a small angle dq , the sides of length w move a distance $(w/2)dq$ in the direction of the perpendicular part of the magnetic force. Suppose the loop starts at an angle q_i and rotates to an angle q_f. The magnetic potential energy is the negative of the work done by the magnetic force. There are forces on both sides of the loop, so:

$$\Delta U_{\mathrm{m}} = -W_{\mathrm{m}} = -\int_{\theta_i}^{\theta_f} 2\vec{F}_{\mathrm{m}} \cdot d\vec{\ell}$$

This gives:

$$\Delta U_{\mathrm{m}} = \int_{\theta_i}^{\theta_f} 2(ILB\sin\theta)\left(\frac{w}{2}d\theta\right) = \int_{\theta_i}^{\theta_f} (ILwB\sin\theta)d\theta = \int_{\theta_i}^{\theta_f} (IAB\sin\theta)d\theta = \int_{\theta_i}^{\theta_f} (mB\sin\theta)d\theta$$

Pause and note that the integrand is the Torque

$$\Delta U_m = mB\int_{\theta_i}^{\theta_f} \sin\theta \; d\theta = mB[-\cos\theta]_{\theta_i}^{\theta_f} = \Delta(-mB\cos\theta)$$

$$\Delta U_{\mathrm{m}} = -ILwB[\cos\theta_f - \cos\theta_i] = \Delta(-ILwB\cos\theta) = \Delta(-mB\cos\theta)$$

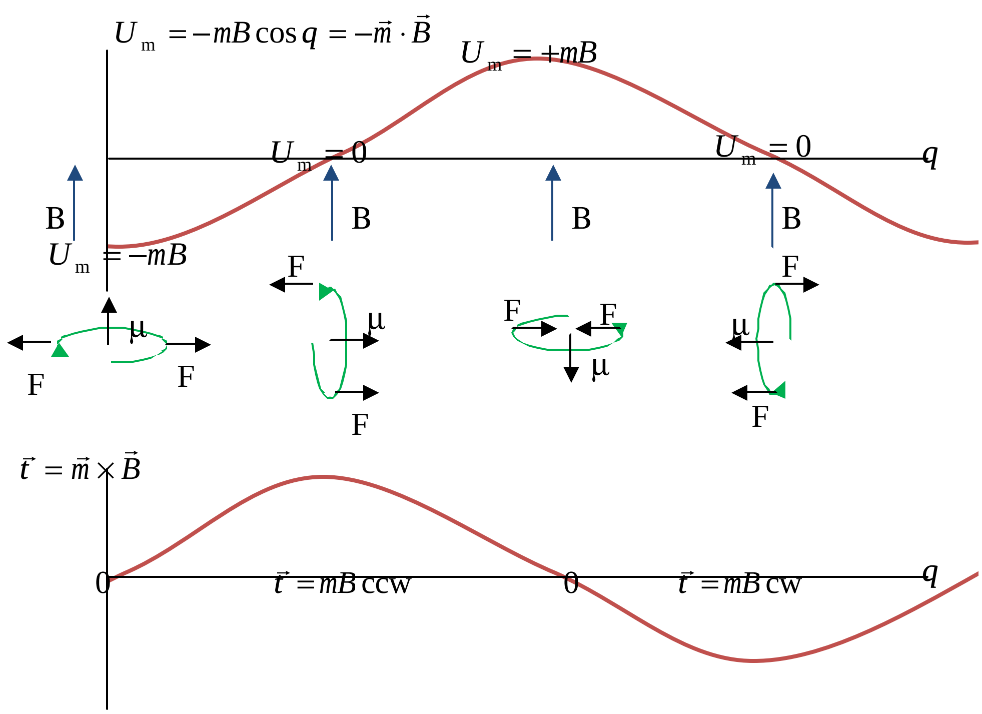
Define $U_{\mathrm{m}} = 0$ at $\theta = 90^\circ$, then:

$$U_{\mathrm{m}} = -mB\cos\theta = -\vec{m}\cdot\vec{B}$$

Energy Maximum and Minimum.

Forces are nice and visualizable – pushes, pulls; torques are a little worse. Energies…not so easy. So it's worth our pausing and relating this expression to our intuition about forces and torques and maybe transfer some of that intuition to our energy expression.

Mapping out the potential for the range of possible orientations:



Notice that, while there is no torque on the loop either at its energetic minimum or maximum, *just* off those points there is some. At theta=0, a minimum, if the loop were twisted a little clockwise, the torque would be counterclockwise (getting it back into alignment), similarly, if it were twisted a little counter clockwise, the torque would be clockwise. We would thus say that this is a "stable" minimum – slight perturbations out of equilibrium lead to torques that push it back into equilibrium.

On the other hand, looking at theta = 90°, a maximum, if the loop were twisted a little counterclockwise, the torque would *also* be counterclockwise, similarly if it were twisted a little clockwise, the torque would also be clockwise. This is an "unstable" maximum – slight perturbations out of equilibrium lead to torques that push it further out of equilibrium.

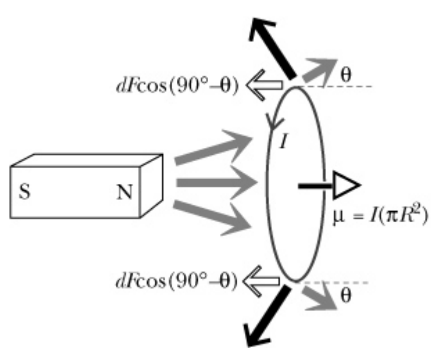
Consider the torques for a moment. Fully aligned or anti-aligned, there's no net torque on the loop

Subtle point of magnetic work and energy

Last time I made the point that, since a magnetic force is always perpendicular to the change in motion that it causes for a *free* particle, it can't do work and it's meaningless to talk about a potential energy for that. Yet here, we *do* have work and potential energy is meaningful. What gives? The obvious difference is that the charged particles that are being interacted with *aren't free*. They're bound (electrically) to the wire. So, the *net* force, the magnetic force, plus the restraining force that holds the current-carrying charges to the wire, dictates the direction of the motion, and that *has* a component in the direction of the magnetic force.

Force on a Magnetic Dipole.

Probably your very first experience with magnets was that they pulled and pushed each other. Yet, we have so far said nothing about the force between two magnets. Now we're ready to. Recall that even a bar magnet is, microscopically, a current loop. So, we'll consider how the field of one bar magnet interacts with the current loop of another. The magnetic field of the bar magnet *diverges* slightly, so the forces on opposite sides of the loop do not quite cancel.



It is difficult to calculate the distance dependence of the force based on the figure above, but we do see that there would not be a net force in a uniform magnetic field.

We will use the potential energy to approximate the force. Suppose a magnetic dipole moment moves a distance Δx along the axis of a bar magnet as shown below. The magnetic field of the bar magnet decreases with distance ($B_1 > B_2$).

The change in potential energy as the dipole move is:

$$\Delta U_{\text{m}} = -W_{\text{m}} = -F_{\text{m}}\Delta x,$$

so the force on the dipole is:

$$F_{\text{m},x} = -\frac{\Delta U_{\text{m}}}{\Delta x} = -\frac{dU_{\text{m}}}{dx} = \mu\frac{dB}{dx}$$

I've used $U_{\text{m}} = -\mu B$ because $\vec{\mu}$ and $\vec{B}$ are in the same direction. If the bar magnet has a magnetic dipole moment μ_1, its field along the x axis is:

$$B(x) \approx \frac{\mu_0}{4\pi}\frac{2\mu_1}{x^3}$$

The force on the dipole is:

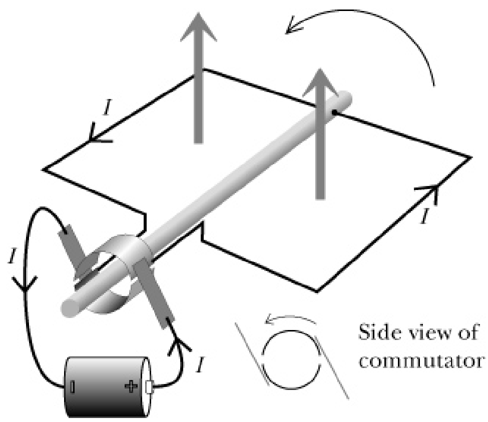
$$F_{\text{m}} \approx 3\mu_2\left(\frac{\mu_0}{4\pi}\frac{2\mu_1}{x^4}\right) \text{ by taking the derivative}$$

where μ_2 is the magnetic dipole moment.

> Example: Suppose $M = 15$ g and $\mu_1 = \mu_2 \approx 1\ \text{A}\cdot\text{m}^2$ (Problem 17.1)
>
> The magnetic force will equal the gravitational force when:
>
> $$Mg = \frac{\mu_0}{4\pi}\frac{6\mu^2}{x^4}$$
>
> $$x = \left(\frac{\mu_0}{4\pi}\frac{6\mu^2}{Mg}\right)^{1/4} = \left(\left(10^{-7}\ \text{T}\cdot\text{m/A}\right)\frac{6\left(1\ \text{A}\cdot\text{m}^2\right)^2}{(0.015\ \text{kg})\left(9.8\ \text{m/s}^2\right)}\right)^{1/4} \approx 0.05\ \text{m}$$
>
> because N=C(m/s)T.

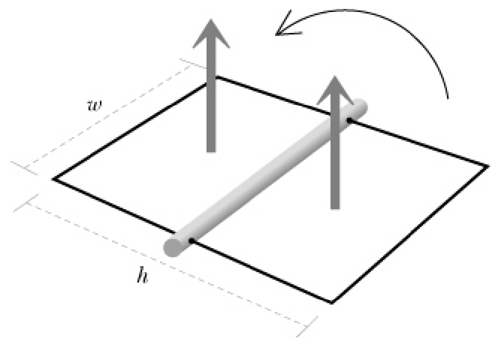
Motors:

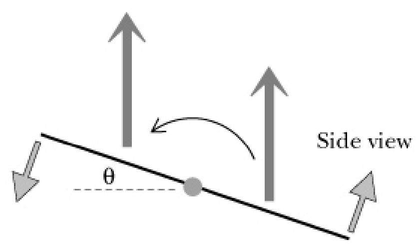
The commutator switches the direction of the current as the loop spins so that it is always moving in the same direction on each side of the axis.



Generators: Suppose a loop rotates at an angular speed *w*.

As the loop spins, there is a motional emf on each side with length *w*, but in opposite directions. That leads to a conventional current around the loop.





The emf is largest when the angle q is 90°, because the wires are moving the fastest in the direction perpendicular to the magnetic field. The size of the emf on the left wire depends on the component of the velocity perpendicular to the magnetic field:

$$\text{emf}_{\text{left}} = B(v\sin q)w.$$

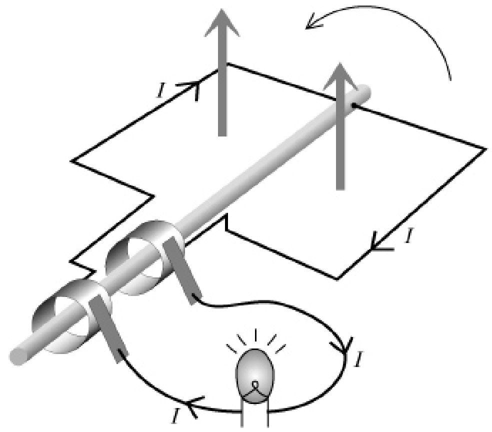
The speed is $v = w(h/2)$, where w is the angular speed. Also, if the angle starts at zero, the angle is $q = wt$, so:

$$\text{emf}_{\text{left}} = B(hw/2)w\sin(wt).$$

Since the left and right sides have the same emf, the total is:

$$\text{emf}_{\text{total}} = wB(hw)\sin(wt).$$

If the connections to the loop are made as shown, the current will alternate directions.



It requires energy to turn the generator because the magnetic force on the loop opposes the motion. A careful calculation shows that the mechanical power input into the generator is the same as the rate energy is transferred to thermal form.