Mon., 3/9 Tues., 3/10 Wed., 3/11 Thurs., 3/12 Fri., 3/ 13	**20.1,3-4** Magnetic Force **20.2,5** Current and Motional Emf **Quiz** Ch 19, **Lab 8** Cyclotron & Electron Mass Lab **20.6-8** Reference Frames and Relativity, Torque **Bonus: Phys. Sr. Thesis Presentations @ 4pm**	**RE19** **HW19:**RQ.42, 49, 52; P.61, 66, 69 **RE20**, **Exp** 23-25 new **RE21**
Mon., 3/16 Tues., 3/17 Wed., 3/18 Thurs., 3/19 Fri., 3/20	**20.9-10** Dipole's Potential Energy, Motors & Generators **Review** **Exam 2** (17-20) Magnetic Field and Moving Charges **21.1-3** E Flux and Gauss's Law	**RE22**, **Exp** 26 **HW20:**RQ.28, 34, 36; P.45, 56, **Lab 7** **RE23** , **Lab Notebook**

Load VPython

Check – Missing a few people's homework

Who's planning on coming to the Sr. Presentations?

Magnetic Force on a Moving Charge:

Back to Magnetism. Now that we have a good feel for currents and circuits, we're ready to get back to that interaction that relies on currents / moving charges: magnetism.

Recall that when we first started talking about Magnetism, I brought in a plank with two wires and we saw that when anti-parallel currents ran through them, they repelled. I suggested that parallel currents attracted, … And this, I said, was the heart of magnetism. I also flashed up a rather ugly expression for the magnetic force between two moving charges:

$$\vec{F}_{mag_{2\leftarrow 1}} \approx \frac{\mu_o}{4\pi}\frac{q_2\vec{v}_2\times\left(q_1\vec{v}_1\times\hat{r}_{2\leftarrow 1}\right)}{\left|r_{2\leftarrow 1}\right|^2} \text{ (neglecting terms on order of } (v/c)^2\text{)}$$

Particularly considering all that those two cross products entail, this is a rather ugly expression. So, aside from its other merits, defining a magnetic field,

$$\vec{B}_{mag_1}(\vec{r}_2) \equiv \frac{\mu_o}{4\pi}\frac{\left(q_1\vec{v}_1\times\hat{r}_{2\leftarrow 1}\right)}{\left|r_{2\leftarrow 1}\right|^2}$$

allowed us to conceptually and mathematically break up this interaction and treat it piecewise – What field does moving charge 1 create at location 2, and *then* what force does that exert on charge 2. We'd spent some time mapping out the magnetic fields due to moving charges. Now it's time to look at the resulting forces.

The force on a moving charged particle in a magnetic field is:

$$\vec{F}_{mag_{2\leftarrow1}} = q_2\vec{v}_2 \times \vec{B}_{mag_1}(\vec{r}_2)$$

That cross-product means that the force is perpendicular to both the velocity and the magnetic field (use RHR to find the direction). The size of the magnetic force is:

$$\left|\vec{F}_{\text{mag}}\right| = F_{\text{mag}} = qvB\sin q\,,$$

where q is the angle between the velocity and magnetic field vectors.

As you are quite familiar by now, don't forget that the sign of the charge effects the direction.

The combined electric and magnetic forces or the "Lorentz force" is:

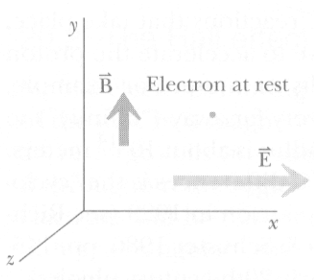
$$\vec{F} = q\vec{E} + q\vec{v}\times\vec{B}$$

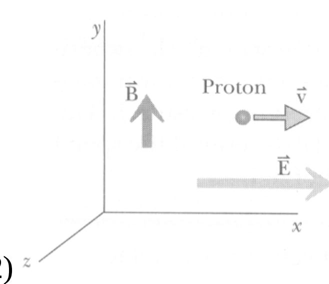
Magnetic Force does not change a particle's speed (just direction of motion):

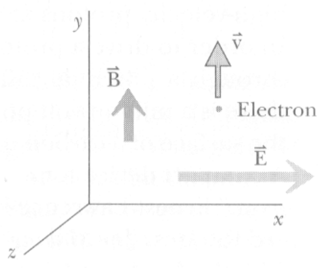
- Given all the cross-producting, one unique thing about the magnetic force (and therefore the resulting acceleration) is that, since it's always *perpendicular* to a particle's velocity because $\vec{v}\times\vec{B}$ is perpendicular to both $\vec{v}$ and $\vec{B}$, it alone can't effect the *speed*, just the direction.
- **VPython: 20_helix_in_B_Private (Case = 1)**
- **Work and Energy.**
    - That means that it can't effect the *kinetic energy* (which depends on speed), and, for that matter, it can't do *work*.
    - In a very short time, the particle will undergo a displacement $d\vec{\ell}$ in the direction of its velocity. Therefore, the work done on the particle in that time is $dW = \vec{F}_{\text{mag}}\cdot d\vec{\ell}$ which is zero. That means that the magnetic force cannot change the particle's kinetic energy or speed. The magnetic force can change the direction of the particles motion and its velocity.

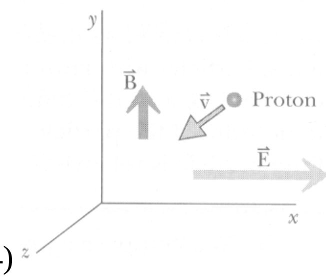
Exercises: (from Ex. 20.6-7) Draw the net force on the particle in each of the figures

Julie's 1st two slides

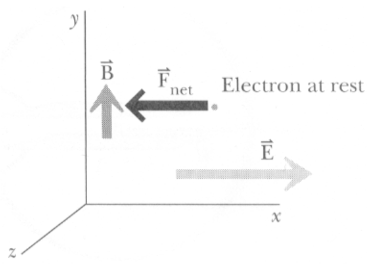
(1) 

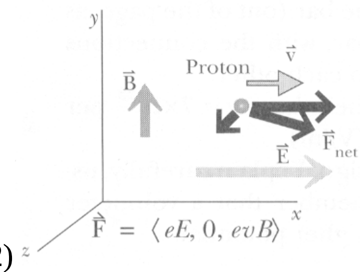
(2) 

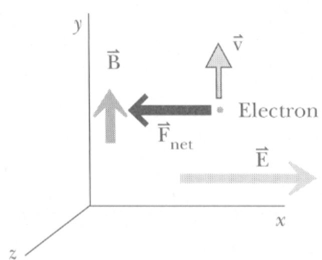
(3) 

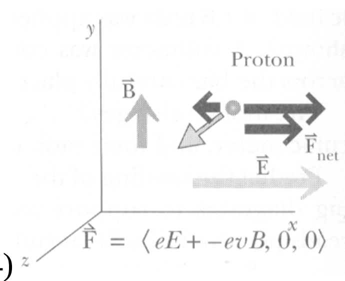
(4) 

Answers:

(1) 

(2) 

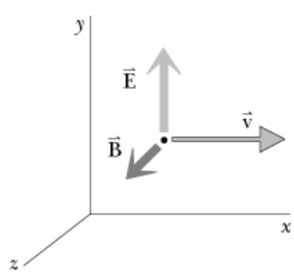
(3) 

(4) 

Julie's 4th slide

Example: Velocity Selector

Suppose a proton is moving relative to electric and magnetic fields as shown below.



The electric and magnetic forces are in opposite directions, so they will cancel if they are the same size. This requires:

$$eE = evB \sin 90^\circ$$

$$E = vB$$

If a proton is traveling at a different speed it will be deflected.

Circular motion:

Suppose a charged particle is moving perpendicular to a uniform magnetic field. The force will be perpendicular to the velocity, so it will change the particles direction but not its speed. Therefore, the sized of the force will not change either. The particle will travel in a circle.

VPython 20_helix_in_B

The moment principle (Newton's 2nd law) is:

$$\frac{d\vec{p}}{dt} = \vec{F}_{\text{net}}\,.$$

For circular motion, the change in momentum is

$$|\vec{p}|\left|\frac{d\hat{p}}{dt}\right| = |\vec{p}|\left|\frac{dq}{dt}\right| = |\vec{p}|w = \vec{F}_{\text{net}}$$

$$w = \tfrac{2p}{T} = \tfrac{|v|}{R}$$

$$|\vec{p}|\frac{|v|}{R} = \vec{F}_{\text{net}}$$

But the net force is the magnetic force which is $\left|\vec{F}_{\text{mag}}\right| = F_{\text{mag}} = qvB\sin q = qvB$ (for 90°).

Then,

$$|\vec{p}|\frac{|v|}{R} = q|v|B \text{ so } |\vec{p}| = qRB$$

Application: Particle Identification

Field Trip: Omega^{-} picture. Knowing the strength of the field and magnitude of charge, usually e, the radius of the path and its bend (clockwise vs. counterclockwise) tells the particle's momentum and sign of charge.

The charged particle moves in a circle of radius:

$$R = \frac{p}{|q|B}\,.$$

The time T (the period) to go around a circle (at constant speed, since magnetic forces don't change the speed) is found using:

$$v = \frac{2pR}{T},$$

so the period is:

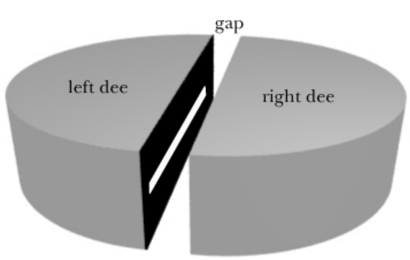
$$T = \frac{2pR}{v} = \frac{2pm}{|q|B}. \text{ (assuming non-relativistic speeds)}$$

Note that the time to go around a circle doesn't depend on the speed, because a faster particle will make a larger circle (for the same magnetic field).

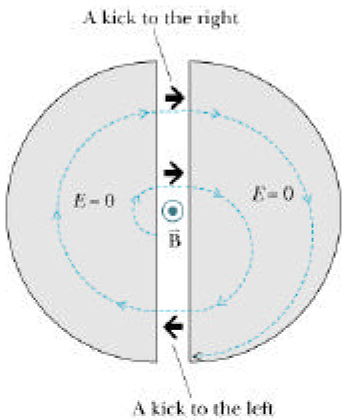
Application: the Cyclotron

We can take advantage of this circular motion and constant period to accelerate particles to high energies. In the areas of Particle and Nuclear Physics, one of the best ways we have to experimentally probe nuclei and produce exotic particles is with very high-energy particle collisions. For example, you could collide high energy protons. As you may recall from scattering problems from last semester, the faster their going / the higher their initial kinetic energies, the closer they can get to each other before deflecting. Now, for accelerating charged particles just a little, it's easy enough to use big capacitors – insert the particle between two plates, and it gets swept up by the field. However, if you want to get up to kinetic energies of, say 25 MeV, that would be one *huge* capacitor.

Here's an elegant alternative that could use much, much lower accelerating voltages. As we've just seen, a charged particle will circle around a constant magnetic field, and the period of that orbit is, for v<<c, independent of the speed. In particular, $T = \frac{2\pi m}{|q|B}$. Say you have an 0.1 Tesla magnetic field, then a proton would cycle at $T = \frac{2\pi \cdot 1.67\times10^{-27}\,kg}{1.6\times10^{-19}\,C\cdot 0.1T} = 6.6\times10^{-7}\,s$. A cyclotron takes advantage of this to accelerate a stream of particles to fairly high velocities. A radioactive source that emits protons is placed in the gap between two hollowed out "dees" as shown. They are wired to an oscillating voltage source so that the voltage difference ΔV_{L-R} oscillates between positive and negative. On top of that, the dees are sandwhiched between two magnets, say, a S pole just above and a N pole just below so a fairly uniform magnetic field points up through the apparatus.



So, what happens. When a proton is emitted in the middle of the gap, it's swept off to the left or right according to the direction of the electric field at that instant, let's say left. When it enters the left dee, it's in a region of nearly zero electric field, but it still feels the magnetic field, so it arcs. Half a period later, ($T/2 = 3.3\times10^{-7}\,s$, in this particular example), the proton crosses back into the gap, this time headed right. Now, if, in the meantime, the voltage difference between the two dees has been flipped so the electric field now points *right*, so if it oscillates with period T/2, then the proton gets accelerated some *more*, until it enters the right dee. This can go on, and on, with the proton spiraling out further and further.

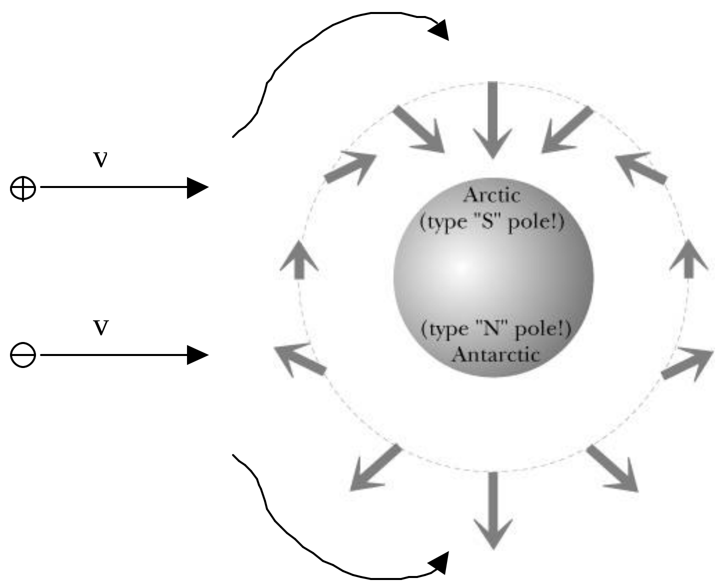


One beautiful thing is, though the proton is getting accelerated / its speed is growing, so is its radius *just* so that it maintains a constant period. So the voltage difference of the two dees just needs to oscillate with a constant period of T/2 to, each time the proton enters the gap, provide the correct electric field to accelerate it some more. Another consequence of this constancy is that a whole stream of protons, all at different speeds / radii can be accelerated – they all have the same orbital period. Now, rather than needing one 25 MV capacitor, we need, say, a 2.5 kV gap, through which the proton passes 100 times.

VPython: Cyclotron

Deflection of Charged particles by Earth's magnetic field:

Charged particles from the Sun are deflected by the Earth's magnetic field. Positive particles tend to spiral along the magnetic field and negative particles spiral the other way. To see this, you have to look at what happens after the initial deflection. The positive particles are initially deflected out of the page, then somewhat upward.

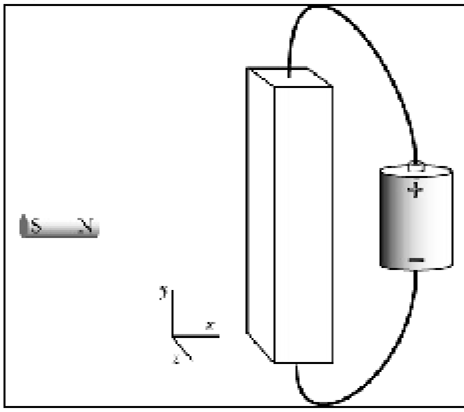


These particles striking the atmosphere near the poles case the Aurora Borealis (northern lights) and Aurora Australialis (southern lights).

The Hall effect:

Julie's last slide

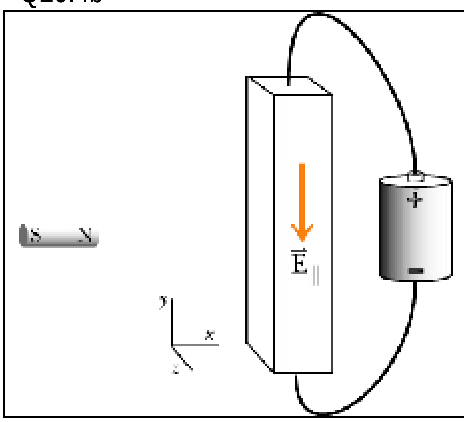
Q20.4a



Direction of $\vec{E}_{\parallel}$ inside bar?

1. +x
2. –x
3. +y
4. –y
5. +z
6. –z
7. none (zero magnitude)

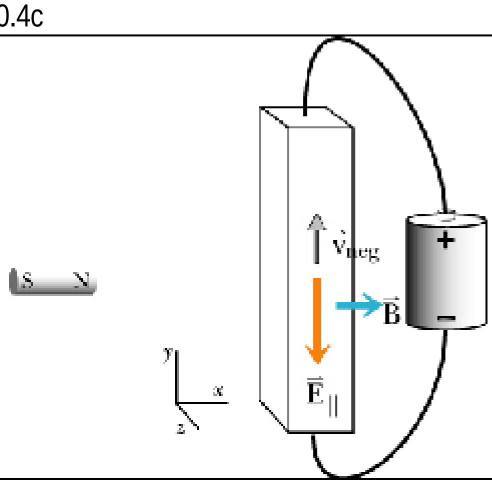
Q20.4b



If mobile charges are negative, direction of motion inside bar?

1. +x
2. –x
3. +y
4. –y
5. +z
6. –z
7. none (zero magnitude)

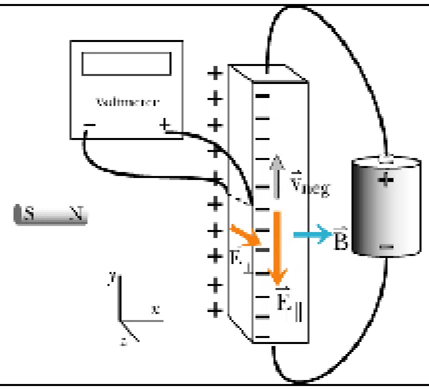
Q20.4c



Direction of $\vec{F}_{mag}$ on moving negative charge?

1. +x
2. –x
3. +y
4. –y
5. +z
6. –z
7. none (zero magnitude)

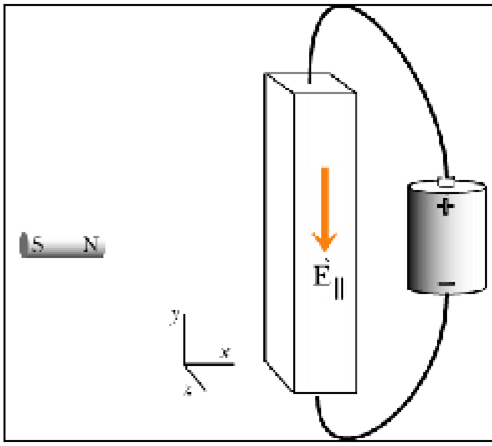
Q20.4d



If mobile charges are negative, sign of voltmeter reading?
(Voltmeter reads positive if + lead is connected to higher potential location)

1. positive
2. negative
3. zero

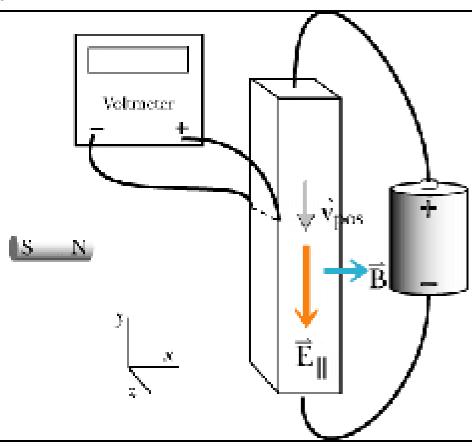
Q20.4e



If mobile charges are positive, direction of motion inside bar?

1. +x
2. –x
3. +y
4. –y
5. +z
6. –z
7. none (zero magnitude)

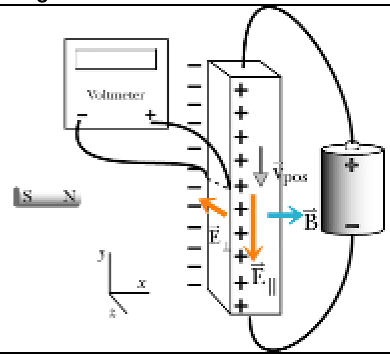
Q20.4f



If mobile charges are positive, direction of magnetic force?

1. +x
2. –x
3. +y
4. –y
5. +z
6. –z
7. none (zero magnitude)

Q20.4g



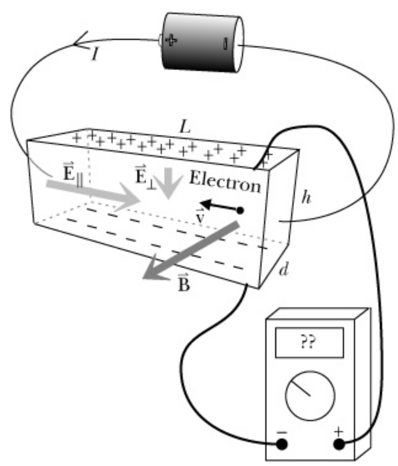
If mobile charges are positive, sign of voltmeter reading?
(Voltmeter reads positive if + lead is connected to higher potential location)

1. positive
2. negative
3. zero

Clicker Questions 20.4a-g

The magnetic force can be used to determine the sign of the charge carriers in a conductor. Typically, a wide conducting strip is used. A magnetic field is applied perpendicular to the direction of the current (and electric field in the metal).

Suppose negative electrons are moving in the metal. They would be deflected downward in the diagram below. The bottom of the strip will become negatively charged and the top positively charged. Eventually, the "transverse" electric field $\vec{E}_\perp$ due so the charges on top and bottom will result in an electric force that will balance the magnetic force



Eventually, the "transverse" electric field $\vec{E}_\perp$ due so the charges on top and bottom will result in an electric force that will balance the magnetic force. This occurs when:

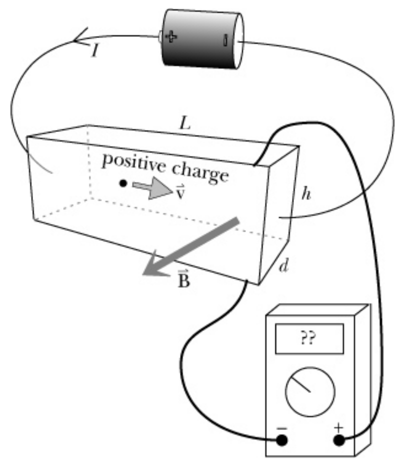
$$eE_\perp = e\bar{v}B,$$

which means that the size of the potential difference between the top and bottom is:

$$|\Delta V| = E_\perp h,$$

where h is the height. This can be measured with a voltmeter. For a meter connected as shown above, the meter will give a positive reading because the potential at the "+" socket is higher.

Suppose positive protons were moving in the opposite direction so that the conventional current was the same as above. How would the situation change?



The direction of the magnetic force on the protons would also be downward because both the sign of the charge and the direction of the velocity are switched. The top and bottom surfaces would become charged in the opposite way. The condition for the transverse electric field would be the same, so the size of the potential difference would be the same. However, a voltmeter connected in the same way will give a negative reading because the potential is lower at the "+" socket.

It is the sign of the potential difference that tells what the sign of the charge carriers is.

The charge carrier density can also be determined:

$$\Delta V = E_{\perp} h = vBh = \left(\frac{I}{|q|nA} \right) Bh \Rightarrow n = \left(\frac{I}{|q|\Delta VA} \right) Bh$$

For next time: remind myself about Quantum and Fractional Quantum Hall Effects.

DEMO: Bring the Hall effect apparatus to class!