

Mon., 2/16	18.1-3 Micro. View of Electric Circuits	RE14 , Lab Notebook
Tues., 2/17		HW17 :RQ.31, 32, 34; P.49, 51, 52
Wed., 2/18	18.4-7 E. Field of Surface Charges, Transients, Feedback	RE15
Thurs., 2/19	Quiz Ch 17, Lab 6 : E. Field of Surface Charge	RE16
Fri., 2/20	18.8-11 Energy, App's of the Theory, Detecting Surface Q	Exp 18,19,22-24

- **Preparation**
 - Check WebAssign
- Collect Lab Notebooks

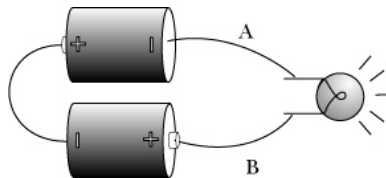
- **Transition.**
 - **So Far – Fundamentals of E&M.** Up to this point in the class, we've been focusing on a fundamental understanding of electric and magnetic interactions; more specifically, what electric fields and voltage differences different charge configurations establish, and what magnetic fields different current configurations produce. In a few chapters, we'll resume this program.
 - **Up Now – Application: Electronics.** Right now, we'll pause and look at an application – electronics. Electrical circuits, with their resistors, capacitors, and even their transformers, are elaborate mazes for charges to maneuver – it's electric and magnetic interactions that make them go.
 - **Microscopic – Understanding Components.** Now we're ready to explore that. First, chapter 18 looks on the microscopic scale, where we can imagine seeing the individual charges moving in response to the fields, thus we'll understand how resistors and capacitors work.
 - **Macroscopic – Understanding Circuits.** Then chapter 19 puts these components together and builds some simple circuits.

Non-Equilibrium Systems.

- **Charge motion through Conductors.**
 - **So Far – Finite Current, Charge build-up & Field Cancellation.** Back in Chapter 14, when we first thought about conductors, we reasoned that applying an electric field to a conductor caused charges to move within it, i.e., drove a “current.” In a finite conductor, this is merely a *transient* situation - it doesn't take very long for enough of the charges to distribute themselves over the surface to establish a *counter* electric field so that there is no net field in the conductor and thus no reason for further motion. Thus **Static Equilibrium** is soon established – no charges moving.
 - **Note: Equilibrium vs. Static Equilibrium.** When learning about forces and potential energies, you've probably heard “equilibrium” used to mean no net force, while “static equilibrium” was used to mean no motion

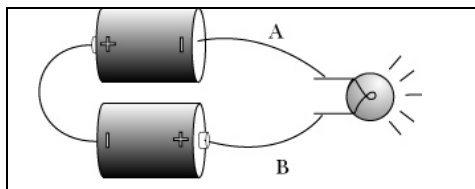
either. With that usage in mind, in this chapter, when the book uses “equilibrium”, there’s an implied “static.”

- **What’s New – Continuous Current.** In an electrical circuit, there is again a *transient* redistribution of charge, but the equilibrium is never established, say the ends of the conductor are attached to a battery and so electrons that *would* build up at one end get siphoned off, and an electron depletion that *would* get established at the other end, gets replenished. The charges quickly settle down, not into static equilibrium but **steady-state** flow – the current goes on and on.
 - **Vocab Reminder – electron and charge currents:** It’s the negatively charged electrons, not the positively charged ions, that are free to move in a solid conductor, but electrically we usually consider just what’s going on with charge (regardless of who’s carrying that charge), so
 - i = electron current, # electrons/s crossing a cross-section.
 - It flows *upstream*: against the E field, “up” voltage differences.
 - I = charge/conventional current, amount of charge /s crossing a cross-section = $|q i|$.
 - It flows *downstream*: with the E field, “down” voltage differences.
- **Current Through Simple Circuit.** Last chapter, we used simple circuits (battery + light-bulb + wire) to be a convenient source of magnetic field, and we mostly focused on the field’s properties. Now we’ll look more closely at the circuit and think about the properties of the current itself.



- **Exp 18.17** – How does the current at points A and B compare?

○ **Predict:**



- 1) $|I_A| > |I_B|$
- 2) $|I_A| = |I_B|$
- 3) $|I_A| < |I_B|$

- **Experiment.** It is the same!

? Why can’t electrons be used up in the light bulb?

- Electrons cannot be destroyed by themselves, because that would violate conservation of charge. Electrons can annihilate with positrons, but there is not much antimatter around.

? Why can't electrons accumulate in the light bulb?

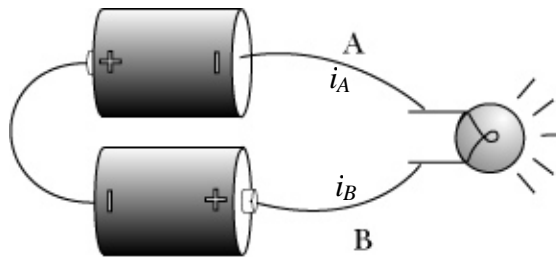
- If electrons accumulated, they would repel other electrons and change the current.

? What does get "used up" in the light bulb?

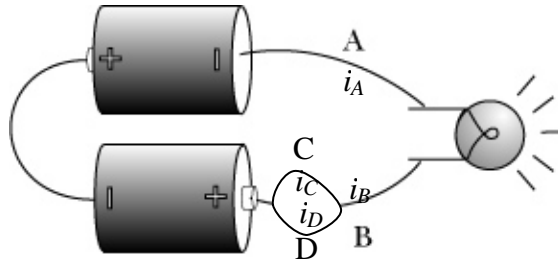
- Nothing, but energy does get converted from a stored form (in the battery) to thermal energy and light. The filament heats up because of some kind of "friction" as the electrons pass through.

- **Steady-state vs. Transient.** When drawing on your intuition, it's important to remember that we're talking the 'steady-state' situation. Right when the circuit is wired up, things will be a bit different. So what's important to remember is that there's nowhere for the charges to go but around and around. That the lightbulb is there *does* slow the flow, but in the steady state, it slows it *everywhere* in the circuit.

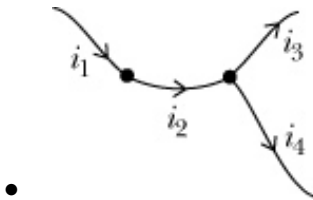
- **Current at a Node**



- Having established that charges don't get 'used up' or otherwise accumulated or depleted in steady-state – we have to say that flow of electrons is uniform around a circuit.
 - $i_A = i_B$. for any two points A and B.
 - If that weren't true, charge would be appearing or disappearing somewhere between the two points – that would be like watching two sides of a highway underpass and seeing more cars coming out the right than going in the left.
- **Node Rule.** Of course, this is a pretty simple circuit. Let's make it minutely more complicated.



- Now we've given current a *choice*. Some can flow along branch C and some can flow along branch D. But one thing is for certain, however many electrons flow past point B per second is however many flow past C and D *combined* per second.
 - $i_B = i_C + i_D$.
- That's the same as saying, however many cars drive *into* an intersection is however many drive *back out* of the intersection. This is the **node rule**.
 - All by itself, it doesn't tell you what fraction of the cars / electrons choose one path and what fraction choose the other, it just says we aren't creating or destroying any charge.
- **Current Node Rule**: In steady state, the current entering a node is equal to the current leaving that node.
 - This applies for electron current or conventional current. The diagram below is a fragment of a larger circuit with two nodes marked.



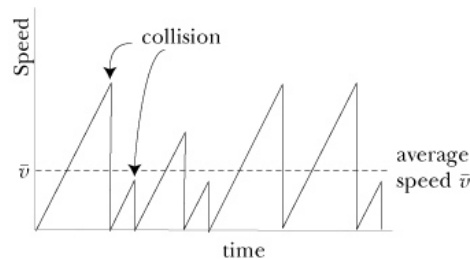
- The node rule implies that $i_1 = i_2$ and $i_2 = i_3 + i_4$.
-
- if a one-way street T's into a two-way street, That is, if 10 electrons pass point B per second, then you're not going to s

18.3 Start-Stop Motion of Electrons in a Wire

Intro. So far, we've just thought about the net flow of electrons. Now we'll zoom in and consider what's going on for individual electrons. We'll develop a very simple model, but that will allow us to understand an awful lot about how current flows.

- **Why we need a field at all**

- **Q.** Why is an electric field needed to keep a current flowing? (Don't Newton's laws say that an object in motion will stay in motion?)
 - **A. Friction/Drag.** The mobile electrons interact with the atomic cores of the metal. Some of their kinetic energy gets converted to thermal energy (and the wire heats up). I.e., there's 'friction' slowing them down.
 - **Electrons can't push each other through the wire**
 - Note: they will refine this to say that electrons accumulated on the surface *do* draw others through the wire. Also, wherever the electron density differs from the ion core density, there would be an electric interaction – but that's quite local and it can be accounted for when we say that jiggling ion cores and impurities disturb electron flow.
- **Drude for drift**
 - A simple model is that the electrons get accelerated until they experience a collision with the atomic ions and lose all of their energy. The speed vs. time of an electron doing that would look like the following.



- We can say something about the average drift speed of an electron. According to the momentum principle (Newton's second law):

$$\Delta \vec{p} = \vec{F}_{\text{net}} \Delta t,$$

- so in a time Δt an electron starting from rest, and accelerated by $F=eE$ undergoes change:

$$\Delta p = p_{\text{max}} - 0 = (eE)\Delta t_{\text{collisions}},$$

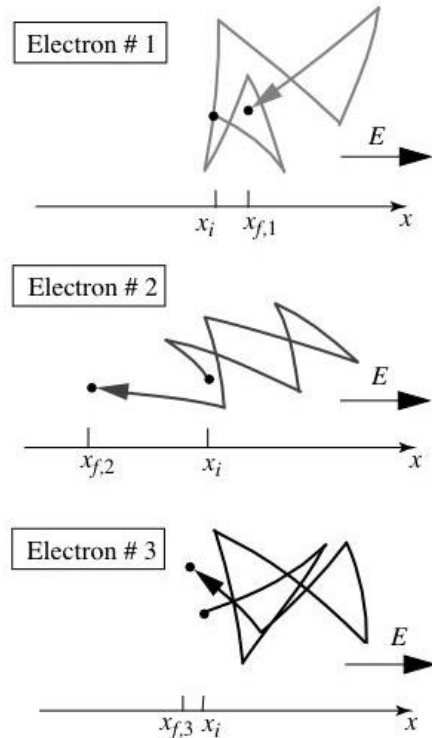
$$v_{\text{max}} = \frac{p_{\text{max}}}{m_e} = \frac{eE\Delta t_{\text{collisions}}}{m_e}.$$

- The average drift speed is:

$$\bar{v}_{ave} = \frac{eE\Delta t_{half}}{m_e}, \text{ where in this simple model } \Delta t_{half} = \frac{1}{2}\Delta t_{collisions}$$

Note: the book doesn't distinguish between Δt_{half} and $\Delta t_{collision}$, but since they don't quantitatively rely on this, it's okay.

- Reasonable?**
 - Stronger force pushing them (eE), get going faster
 - more massive (m), it would take more push to get them going as fast
 - longer time between collisions / decelerations (Δt), the faster they get going.
- Δt 's v_{drift} independence.** To use this later, it's important to note that it's overly simplistic to think of the electron as just moving forward and periodically colliding with fixed defects (thermal or otherwise), and thus the frequency of these collisions depending on its forward speed – instead it's moving *much* faster randomly in all directions and the thermal defects are randomly popping up. The effect of this is that Δt is pretty *independent* of the drift speed.



- Rewriting that a little more suggestively,

$$\circ \bar{v}_{ave} = \left(\frac{e\overline{\Delta t}_{half}}{m_e} \right) E$$

- We see that the drift velocity (how fast, on average, charges are flowing), is proportional to the field that's keeping them going. The term in brackets is called the **electron mobility**.
- We will define the proportionality between the average drift speed and the electric field by:

$$\circ u \equiv \left(\frac{e\overline{\Delta t}_{half}}{m_e} \right)$$

- So,

$$\circ \bar{v} = uE$$

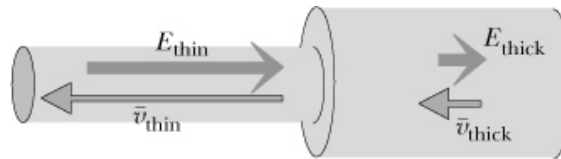
- Recall that $i = \frac{N_e}{t} = \frac{N_e}{Vol} AL/t = nA\bar{v}$, so

$$i = nA(uE)$$

- **Take-away message.** While the above is a kind of sketchy 'derivation' it's only meant as a *motivation* for seeing that the current *should* depend on E and a 'mobility.' Mobilities will differ from material to material and will be temperature dependent (the rate and severity of collisions will increase with temperature as the atomic ions jiggle more.)

- **Implications of constant Current through multi-element circuit**

- Now, we've already argued that, in steady-state, the flow of charge is the same through any bit of a series (no parallel branches) circuit. So, if we mate two wires of different thicknesses in our circuit, the need to maintain *constant flow* in spite of *varying cross-sectional area* means something else has got to vary too.



$$i_{thin} = i_{thick}$$

- **Varying Speeds**

$$nA_{thin}\bar{v}_{thin} = nA_{thick}\bar{v}_{thick}$$

and the drift speeds are related by:

$$\bar{v}_{thin} = \frac{A_{thick}}{A_{thin}} \bar{v}_{thick}$$

The drift speed is larger in the thinner segment of the wire. This is the same as the relation between the speeds of an incompressible fluid in a pipe of varying size. For example, when you put a nozzle on a garden hose – the water comes squirting out that narrow nozzle with a higher speed (shoots much further) than it has in the wide hose.

- **Varying Fields Strengths**

- But, we just argued that the drift speed and field must be proportional to each other.
- The electric field is proportional to the drift speed ($v = uE$), so the electric field is also larger in the thinner wire. Assuming that the wires are of the same material (so same mobility),

$$\bar{v}_{thin} = \frac{A_{thick}}{A_{thin}} \bar{v}_{thick}$$

$$uE_{thin} = \frac{A_{thick}}{A_{thin}} uE_{thick}$$

$$E_{thin} = \frac{A_{thick}}{A_{thin}} E_{thick}$$

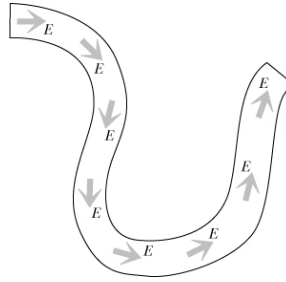
$$E_{thin} > E_{thick}$$

- **Foreshadowing of tomorrow's reading**

- **Q.** Ultimately, what is the source of an electric field?
 - An electric charge
 - So, to have a sudden change in electric field, what must you have? A sudden change in electric charge density - *there must be a some kind of charge build up at the interface of the two wires.*

- **Varying Field Directions**

- Suppose a wire with constant cross sectional area is carries a steady state current. The electric field must be the same magnitude in every part because the current is proportional to it ($i = nAuE$). But the *direction* of the current changes – to always be *along* the wire. So, what can we say about the direction of the electric field? The direction of the electric field must be parallel to the wire.

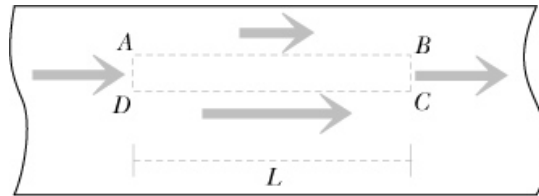


○ **Q.** Again, what does it take to change an electric field (aside from the old $1/r^2$ drop off)?

○ A changing charge density.

Q. Where does the current flow in a uniform (same cross section and material) wire?

A. It is uniform throughout the wire. Suppose the current was not uniform, then the electric field would have different values at different locations as shown below.



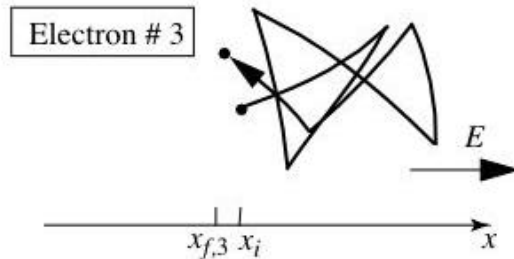
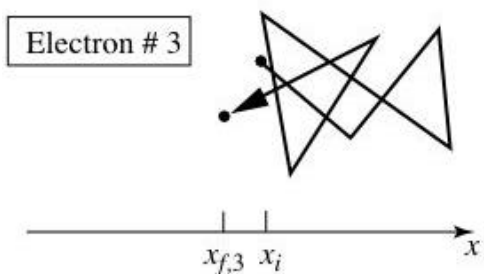
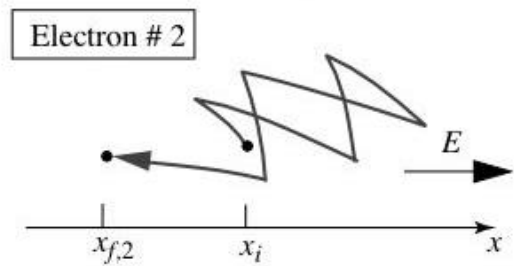
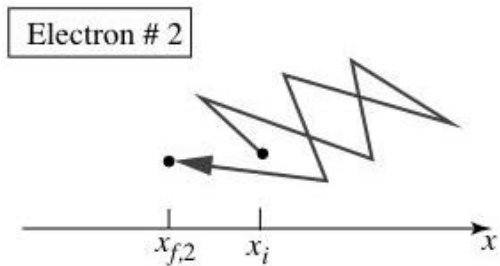
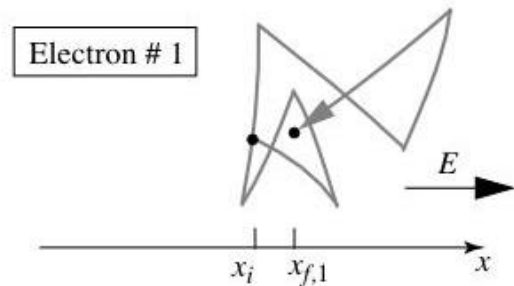
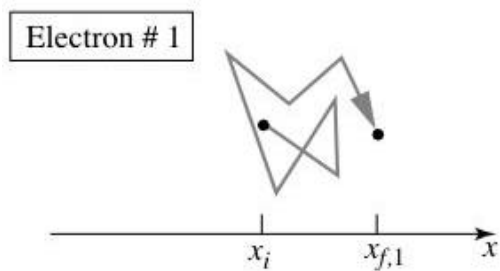
The potential difference for the round-trip path ABCDA would not be zero, so the electric field must be uniform in the wire.

Pinch Effect

Actually, there is a miniscule pinching of the current in from the surfaces. This is because the forward moving charges see the separation between the positive ions as contracted, and therefore the density increased above their own density – they see the wire as net +, and are attracted in a little until their own density increases enough to counter the effect. At typical electron speeds, the “pinching” is far less than the radius of an atom.

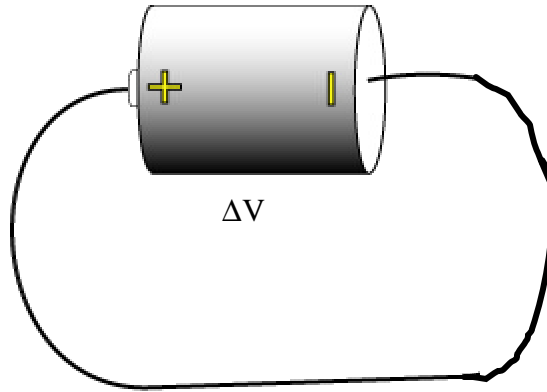
Below here are unused things.

- **Conduction Electron Gas.** So far, we’ve just considered the net flow of electrons. A fairly good model for conduction electrons is a gas – free electrons zipping all around, going nowhere in particular.
- **No Field, no Net Motion.** Just as if you tracked the motion of some individual air molecules, you’d see random motion that, in the end goes nowhere, that’s what you’d see for electrons in a conductor if there is no net electric field. (Left Column).



* x_i = initial position * $x_{f,n}$ = final position of electron n after 7 collisions

- **Net Field, Net Motion.** But if there *is* an electric field (or some other motivator, like a thermal gradient), the motion is ever-so-slightly biased, making for a net motion, as is shown in the Right Column.
- So, while any individual will make it somewhere, averaged over all of them, there's no net motion in the absence of an electric field (or some other motivator ((thermal gradient))).



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- **Voltage Difference, Field, and Electron Flow and Current**
 - Consider this *very* simple circuit. We'll worry about *how* a battery manages it later, but right now, let's accept that the battery establishes a voltage difference across the two ends of the circuit
 - $\Delta V = V_+ - V_-$
 - If this is true, then
 - $\Delta V = - \int_{+terminal}^{-terminal} \vec{E} \cdot \Delta \vec{l}$ tells us that, following any path, say, that of the wire, from the + to the - terminal, there must be an electric field that, on average, points along the path. the electric field's got to point

Review Ch. 17 HW for quiz

Thursday: Quiz 17, VPython programming, & a short lecture