Mon., 2/9 Tues., 2/10 Wed., 2/11 Thurs., 2/12 Fri., 2/13	**17.4-.6** Biot-Savart law for Currents **17.7-9** Magnetic Field For Distributions **Lab 5**: Biot-Savart – B fields of moving charges **17.10-11** Permanent Magnets	**RE12** **RE13, Exp** 20-22 **Exp** 23-29 (work together for 2 magnets)
Mon., 2/16 Tues., 2/17 Wed., 2/18 Thurs., 2/19 Fri., 2/20	**18.1-3** Micro. View of Electric Circuits **18.4-6** E. Field of Surface Charges, Transients, Feedback **Quiz** Ch 17, **18.7-9**, **Lab 6:** E. Field of Surface Charge **18.10-11** Applications of the Theory, Detecting Surface Q	**RE14** **, Lab Notebook** **HW17:**RQ.31, 32, 34; P.49, 51, 52 **RE15** **RE16** **Exp** 18,19,22-24

Preparation

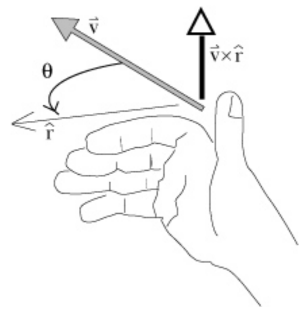
Check WebAssign

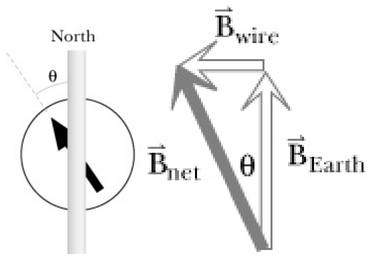
Put a Vpython illustration of an accelerated charge in the folder

Load Vpython

Last Time Highlights

- **Ask them.**
- **Refresher Questions.**
    - **PowerPoint Slides 1-3**
- **Introduced Magnetic Interaction.** When charges are in motion, the world looks rather different to them, thanks to relativistic effects. This gives rise to a velocity-dependent force, which, from our perspective seems quite distinct from the electric force; we dub it the magnetic force.
    - $\vec{F}_{M.1\leftarrow 2} \approx \frac{m_o}{4p} \frac{q_1\vec{v}_1 \times (q_2\vec{v}_1 \times \hat{r}_{1-2})}{r_{1-2}^2}$
- **Magnetic Field.** As with the electric interaction, it's convenient to separate this expression to speak of a magnetic field.
    - $\vec{B}_{1\leftarrow 2} \approx \frac{m_o}{4p} \frac{(q_2\vec{v}_1 \times \hat{r}_{1-2})}{r_{1-2}^2}$, so $\vec{F}_{M.1\leftarrow 2} = q_1\vec{v}_1 \times \vec{B}_{1\leftarrow 2}$
- **Cross Product**
    - You can figure it out by either using
        - $\vec{A}\times\vec{B} \equiv \langle (A_yB_z - A_zB_y), (A_zB_x - A_xB_z), (A_xB_y - A_yB_x) \rangle$
        - Or
            - **Magnitude**: $|\vec{A}\times\vec{B}| = AB\sin q$
            - **Direction**: Right Hand Rule

- 

- **Compass.** Points in the direction of the magnetic field at its location.
  - 

  - $\tan\theta = B_{wire}/B_{Earth}$

This Time

Ask them – what do they want to go over?

17.4 Relativistic Effects

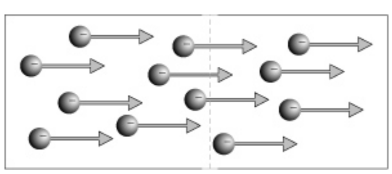
- **Magnetic Field Depends on your frame of reference**
  - **E & M Relativistically Related.** As we discussed (in far greater detail) last Monday, and recapped last Friday, Electric and Magnetic interactions are inextricably intertwined. The example we looked at was that of a charged particle moving alongside a current carrying wire. To the particle, who thinks of itself as stationary and the wire as charged, the interaction appears to be "electric", while, to us, who see the wire as neutral and the charge as moving, we'd call the interaction "magnetic." We used the results of Special Relativity to figure this out.
  - **Regardless of Perspective – You still get Theory to match Experiment.** It may be disturbing that we have drastically different interpretations of the interaction depending on our reference frame; but really, these differences are superficial. At the end of the day, either perspective will give the right answer. If someone were playing some kind of electro-magnetic pool game while riding past you in a train car, both you and he would correctly predict whether or not the electron went in the 'right corner pocket;' your theoretical work would agree with your experimental observations.
- **Retardation**
  - It was in the reading, but we didn't really spend any time on it in Chapter 13. Let's pause and consider the electric field first, since it's simpler to visualize. Consider a point charge, picture the electric field it generates. I've talked about the field as being analogous to a breeze or streams in which positive test charges

would get swept up. One nice analogue is the water shooting out of a spray nozzle on a hose. It's radiating out. Say you're spraying your friend Bob (it's a hot day, he's greatful), and you suddenly move it to spray someone else; water doesn't stop hitting Bob immediately, it's a fraction of a second later, when the last of the water that had been emitted while you were still pointing at him finally reaches him. There's a similar delay for electric and magnetic fields. The radiate at speed c. A test charge a distance r away gets the field that was emitted a time $Dt=r/c$ ago. If things have changed, charges have moved, since then, it doesn't immediately know. Another way to put it is that what the test charge feels at time t depends on what the sources were doing previously, at time t- $r/c = t_r$ this is known as the "retarded" time.

- Here's a visual for when a stationary charge suddenly moves somewhere else (maybe not too physical a scenario, but it gives you the idea.)
- **Demo: 23_radiate0_fieldlines.py** then **23_radiate2D.py**
- **Biot-Savart law for Point Charges**. In principle, the Biot-Savart law tells you the field at location r and time t only if you evaluate the right hand side for time t- $r/c = t_r.$ Fortunately, if distances aren't huge and speeds aren't near c, then $B(t) \approx B(t + r/c)$, so we don't need to bother ourselves with this detail.

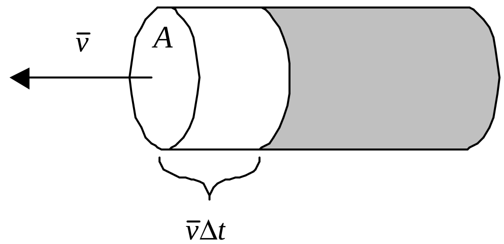
17.5 Electron Current & Conventional Current

- **Electron Current.**
  - **Intro.** The Biot-Savart law for point charges is all good and well for theorizing (non-relativistic approximation not withstanding), but in practice, it's much simpler to maintain a *current* and measure its field than it is to deal with individual point charges. We already experimented a little with them on Friday. Now, we'll get a little more quantitative about what we mean by an electrical current.
  - Because electrons are a little more concrete than "charges", and they're quite often the mobile ones anyway, we'll first consider the flow or "current" of electrons before moving on to the more abstract follow or "current" of charge.
  - **Electron Current.** $i = N_e / \Delta t$ (electrons per second)
    - Defined as the number of electrons per second passing through a cross section of a conductor.



    - But what is this number of electrons N_e passing down the wire?
      - Suppose a metal wire has a cross-sectional area of *A* and there are density of mobile electrons is *n* (number of mobile electrons per

volume). If the average “drift speed” of the mobile electrons is $\bar{v}$ (the “bar” means average), they move a distance $\bar{v}\Delta t$ in a time Δt. All of the electrons in a volume $A\bar{v}\Delta t$ will move past a point in the wire in this time (see the diagram below).



- That means that there are $N_e = n\left(A\bar{v}\Delta t\right)$ electrons passing a point in the wire and the electron current is:
  - $i = \frac{nA\bar{v}\Delta t}{\Delta t} = nA\bar{v}$.

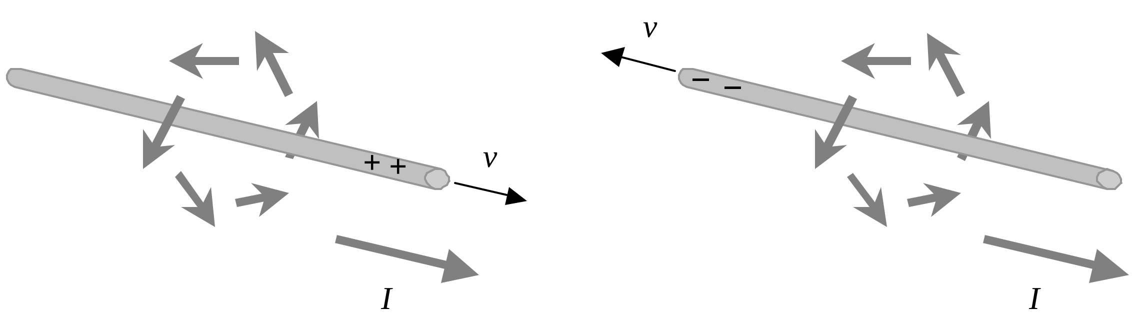
WebAssign Problem 1.

WebAssign Problem 2.

WebAssign Problem 3.

- **Conventional Current**
  - **Negative left or Positive right.** The magnetic field due to positive charges moving in one direction is the same as the magnetic field due to negative charges moving in the opposite direction (see diagrams below) In other words, changing the signs of both q and $\vec{v}$ does not change:

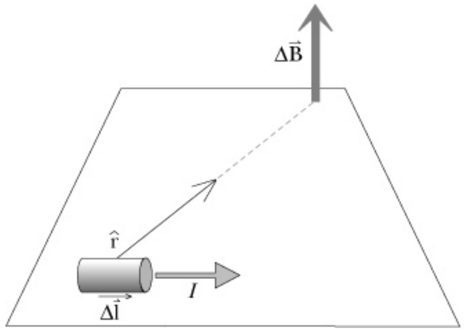
$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\,\vec{v}\times\hat{r}}{r^2} .$$



  - **Electrons and Holes.** In some materials (e.g. most metals), negative electrons move. In others (e.g. semiconductors), the current behaves more like “holes” in the sea of electrons move and act like positive charges. In Ch. 20, you will learn how the sign of the charge carriers is determined (Hall effect).
  - **Conventional Current**

- The *size* of *conventional current* is defined as the amount of charge (with no sign) passing a point per second:
  - I = $|q|i$.
    - or
  - $I = |q|nA\bar{v}$, (note the speed here is just that, speed, not velocity)
- which is in coulombs per second or amperes (amps or A). The direction of the conventional current is the direction positive holes are moving or the opposite of the direction electrons are moving (see diagrams above).
- **Vector.** This book mostly sticks with current as a magnitude, but more generally, it's a vector: $\vec{I} = qnA\vec{\bar{v}}$

- **Biot-Savart law for Conventional Current**
  - The magnetic field due to a point charge is
    - $\vec{B} = \frac{m_0}{4p}\frac{q\,\vec{v}\times\hat{r}}{r^2}$
  - If you have a whole stream / current of point charges running along, the field due to N_{Dl} of them in a short stretch of wire, Δl is
    - $\Delta\vec{B} = \frac{m_0}{4p}\frac{Nq\,\vec{v}\times\hat{r}}{r^2}$
  - But if i is the number passing through a cross-section per time, then $N_{\Delta l} = i\Delta l$.
    - $\Delta\vec{B} = \frac{m_0}{4p}\frac{q\mathrm{i}\,\Delta\vec{\ell}\times\hat{r}}{r^2}$
  - But qi is just I, the conventional current (rate of charge flow down the wire).
    - ***The* Biot-Savart Law**
      - $\Delta\vec{B} = \frac{m_0}{4p}\frac{I\,\Delta\vec{\ell}\times\hat{r}}{r^2}$,
  - where $\Delta\vec{\ell}$ is a vector of length $\Delta\ell$ pointing in the direction of the conventional current (see the figure below).

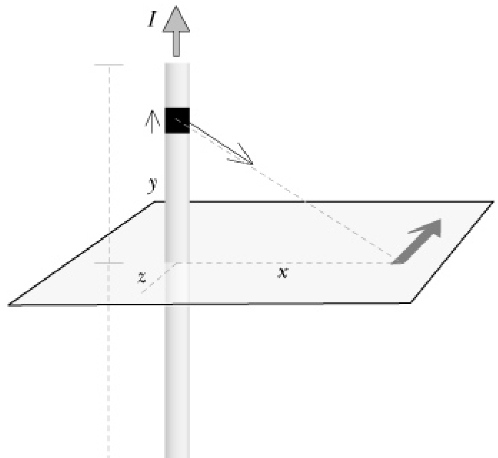
- 

- **Relativistically Correct.**
    - For a steady-state current, we do <u>not</u> have to worry about retardation! Even if the charge that produced the magnetic field at a certain instance has moved on, there is the same amount of charge passing through later.
    - Furthermore, the correction factor that I showed you on Friday (the ugly one in the curly brackets) goes away when a proper derivation of this relation is done.
    - So this is absolutely right, as long as charges aren't accelerating.

Adding up the Magnetic Field. Straight Line

- **How we did it for Electric fields.** You recall that when we wanted to find the *electric* field due to a charge configuration, we treated the source like a distribution of point charges, and we summed over all the point charges' contributions to the field in order to find the net field. We used $\Delta\vec{E} = \frac{1}{4\pi\varepsilon_o}\frac{\Delta Q}{r^2}\hat{r}$ as our basic building block.
- **How we'll do it for Magnetic Fields.** For the magnetic field due to a current configuration, we'll do something quite similar. Now, the "basic building block" is the bit of field due to a morsel of current carrying wire: $\Delta\vec{B} = \frac{\mu_0}{4\pi}\frac{I\Delta\vec{\ell}\times\hat{r}}{r^2}$.
- **General Process:**
    - **Magnetic Field for current distributions**
        - Divide up current into pieces and draw $\Delta\vec{B}$ for a representative piece
        - Write an expression for $\Delta\vec{B}$ due to one piece
        - Add (integrate) up the contributions for all pieces
        - Check the result
- **Example: B-Field of Wire – Computational.**

- **Intro.** We'll start simple: with the Magnetic field due to a wire. To make things particularly simple, we'll just consider doing this computationally – how to set up the code. Tomorrow, we'll look at the analytical approach.
  - **Note:** In lab you'll do this for a ring. While the geometry will be different, there will be a lot of similarity.
- **1. Divide up current into pieces and draw $\Delta\vec{B}$ for a representative piece**



- **2. Write an expression for $\Delta\vec{B}$ due to one piece**
  - On White Board: Write down the expression for the magnetic field at our observation location due to a morsel of current.
    - $\Delta\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,\Delta\vec{\ell}\times\hat{r}}{r^2}$ where
    - $\vec{r} = \langle \text{observation location} \rangle - \langle \text{source location} \rangle$
      $\vec{r} = (x_o, y_o, 0) - (0, y, 0)$
    - and
    - $\Delta\vec{\ell}$ for the segment is $\Delta\vec{\ell} = \langle 0, \Delta y, 0\rangle = \Delta y\langle 0,1,0\rangle$
  - **2.b Translate into VPython Code**
    - **Note:** the cross product can be done with cross(A,B)

      $obsloc = vector(x_o, y_o, 0)$
    - $source = vector(0, y, 0)$

      $r = obsloc - source$
    - $deltal = vector(0, dy, 0)$
      - **Note:** $\Delta\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,\Delta\vec{\ell}\times\hat{r}}{r^2} = \frac{\mu_0}{4\pi}\frac{I\,\Delta\vec{\ell}\times\vec{r}}{r^3}$
    - *deltaB=k*I*cross(deltal,r)/mag(r)**3*

-
    - **3. Add (integrate) up the contributions for all pieces**
        - *y = 0*
        - *B=0*
        - *While y < L*
            - $source = vector(0, y, 0)$
$r = obsloc - source$
            - *deltaB=k*I*cross(deltal,r)/mag(r)**3*
            - *B=B+deltaB*
            - *y=y+deltay*

Demo: 17_Bwire_with_r.py

Next time, we'll look at this analytically, and consider sources of other geometries.