Wed., 2/11	**17.7-9** Magnetic Field For Distributions	**RE13, Exp** 20-22
Thurs., 2/12	**Lab 5**: Biot-Savart – B fields of moving charges (no quiz)	
Fri., 2/13	**17.10-11** Permanent Magnets	**Exp** 23-29 (work together for 2 magnets)
Mon., 2/16	**18.1-3** Micro. View of Electric Circuits	**RE14 , Lab Notebook**
Tues., 2/17		**HW17:**RQ.31, 32, 34; P.49, 51, 52
Wed., 2/18	**18.4-6** E. Field of Surface Charges, Transients, Feedback	**RE15**
Thurs., 2/19	**Quiz** Ch 17, **18.7-9**, **Lab 6:** E. Field of Surface Charge	**RE16**
Fri., 2/20	**18.10-11** Applications of the Theory, Detecting Surface Q	**Exp** 18,19,22-24

Preparation

Lab handout

Check WebAssign

Load Vpython

Magnets for those who haven't gotten them yet

(for Thursday – set up, don't forget solenoids too)

Announcements

Candidates Visiting

Next Monday

11 – 12:20 Class

4pm Research Talk – Opportunity to learn a little about what physicists do

Last Time

- **Biot-Savart for wires.** Last time we developed an expression for the bit of magnetic field generated by a bit of current carrying wire.

  o $\Delta \vec{B} = \frac{\boldsymbol{m}_0}{4 \boldsymbol{p}} \frac{I \Delta \vec{\ell} \times \hat{r}}{r^2}$

  o **Right-Hand Rule for wires.** Conveniently, the right-hand-rule gets pretty simple for a current carrying wire. Imagine grabbing the wire with your right hand – such that your thumb points along it in the direction of current flow. Then your hand wraps around the wire just as the magnetic field does.

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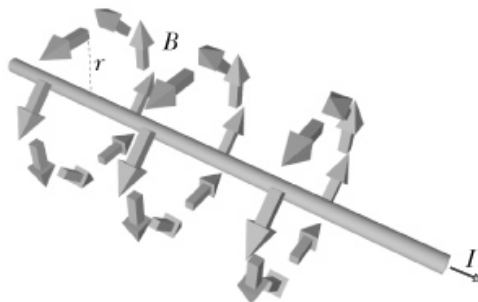
- **Adding Up the Magnetic Field**
    - Just as with finding the net *electric* field due to a *charge* distribution, there's a systematic way to find the net *magnetic* field due to a *current* distribution.
    - **General Process:**
        - **0.** Predict
        - **1.** Divide up current into pieces and draw $\Delta \vec{B}$ for a representative piece
        - **2.** Write an expression for $\Delta \vec{B}$ due to one piece
        - **3.** Add (integrate) up the contributions for all pieces
        - **4.** Check the result
    - Last time, we considered how to *computationally* find the field due to a wire.


This Time: Magnetic Fields for Current Distributions

- Now we'll go on to consider how to find the magnetic field of different current distributions. We'll do it analytically. In lab tomorrow, you'll do it computationally.

- **Straight Wire**
    - **0.** Predict
    - **1.** Divide up current into pieces and draw $\Delta \vec{B}$ for a representative piece
    - **2.** Write an expression for $\Delta \vec{B}$ due to one piece
    - **3.** Add (integrate) up the contributions for all pieces
    - **4.** Check the result

Adding up the Magnetic Field. Straight Line

- Now that you've got the expression for magnetic field due to current in a morsel of wire, you can build up any geometry of current/wire and find the magnetic field due to *it*.
    - **Examples.**
        - **Straight.** We'll start simple, with the field along the center from a straight line.
        - **Loop.** Then we'll venture to something more complicated – the general case of a loop
            - But we'll chicken out at the end, and go for on-axis.

- **General Process:**
- **Magnetic Field for current distributions** – same as with the computer
    - o Divide up current into pieces and draw $\Delta\vec{B}$ for a representative piece
    - o Write an expression for $\Delta\vec{B}$ due to one piece
    - o Add (integrate) up the contributions for all pieces
    - o Check the result
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    - o **1. Divide up current into pieces and draw $\Delta\vec{B}$ for a representative piece**
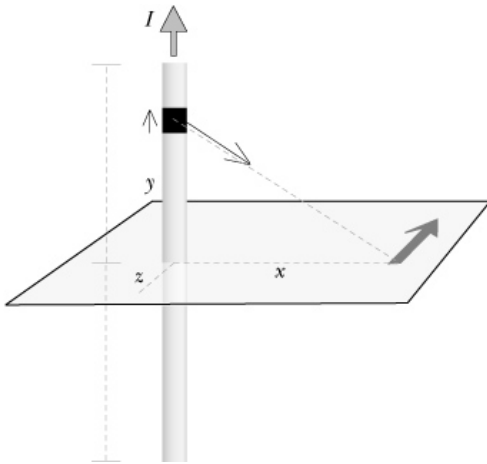
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- **2. Write an expression for $\Delta\vec{B}$ due to one piece**
-
    - $\vec{r} = \rangle$obseration location$\langle - \rangle$source location$\langle = \rangle x_o,0,0\langle - \rangle 0,y,0\langle = \rangle x_o,-y,0\langle$
    - o The length of this vector is $r = \sqrt{x_o^2 + (-y)^2} = \sqrt{x_o^2 + y^2}$, so the unit vector is:
        - $\hat{r} = \dfrac{\vec{r}}{r} = \dfrac{\rangle x_o,-y,0\langle}{\left(x_o^2 + y^2\right)^{1/2}}$
    - o $\Delta\vec{\ell}$ for the segment is $\Delta\vec{\ell} = \langle 0,\Delta y,0 \rangle = \Delta y\langle 0,1,0\rangle$, so
        - $\Delta\vec{B} = \dfrac{m_0}{4p}\dfrac{I\,\Delta\vec{\ell}\times\hat{r}}{r^2} = \dfrac{m_0}{4p}\dfrac{I\,\Delta\vec{\ell}\times\vec{r}}{r^3} = \dfrac{m_0}{4p}\dfrac{I\,\Delta y\langle 0,1,0\rangle\times\langle x_o,-y,0\rangle}{\left(x_o^2 + y^2\right)^{3/2}}$
- 3. Translate this into VPython Code and then nest it in a loop to sum over the length of the wire, from –L/2 to L/2.
    - o **Note:** the cross product can be done with cross(A,B)

$$\Delta\vec{B} = \dfrac{m_0}{4p}\dfrac{I\,\Delta\vec{\ell}\times\hat{r}}{r^2} = \dfrac{m_0}{4p}\dfrac{I\,\Delta y\langle 0,1,0\rangle\times\langle x,-y,0\rangle}{\left(x^2 + y^2\right)^{3/2}}$$

Doing this *analytically* The cross product is (also use RHR to see direction):

$$\langle 0,1,0\rangle \times \langle x,-y,0\rangle = \det\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & 0 \\ x & -y & 0 \end{vmatrix} = -x\hat{k} = \langle 0,0,-x\rangle$$

so:

$$\Delta\vec{B} = -\frac{\pmb{m}_0}{4\pmb{p}}\frac{I\,x_o\Delta y}{\left(x_o^2 + y^2\right)^{3/2}}\langle 0,0,1\rangle \quad \text{or} \quad \Delta B_z = -\frac{\pmb{m}_0}{4\pmb{p}}\frac{I\,x_o\Delta y}{\left(x_o^2 + y^2\right)^{3/2}}$$

3. Add (integrate) up the contributions for all pieces

$$B_z = -\frac{\pmb{m}_0}{4\pmb{p}}I\,x_o\int_{-L/2}^{+L/2}\frac{dy}{\left(x_o^2 + y^2\right)^{3/2}} = -\frac{\pmb{m}_0}{4\pmb{p}}I\,x_o\left[\frac{y}{x_o^2\sqrt{x_o^2 + y^2}}\right]_{-L/2}^{+L/2}$$

$$B_z = -\frac{\pmb{m}_0}{4\pmb{p}}I\,x_o\left[\frac{L/2}{x_o^2\sqrt{x_o^2 + (L/2)^2}} - \frac{-L/2}{x_o^2\sqrt{x_o^2 + (-L/2)^2}}\right] = -\frac{\pmb{m}_0}{4\pmb{p}}\frac{LI}{x_o\sqrt{x_o^2 + (L/2)^2}}$$

or since only the distance from the wire matters (call it r):

$$\left|\vec{B}_{\text{wire}}\right| = \frac{\pmb{m}_0}{4\pmb{p}}\frac{LI}{r\sqrt{r^2 + (L/2)^2}}$$

If $L \gg r$, then:

$$\left|\vec{B}_{\text{wire}}\right| \approx \frac{\pmb{m}_0}{4\pmb{p}}\frac{2I}{r}$$

4. Check the result

The units work out:

$$\left(\frac{\text{T}\cdot\text{m}}{\text{A}}\right)\frac{\text{m}\cdot\text{A}}{\text{m}^2} = \text{T}$$

Very far away ($r \gg L$):

$$\left|\vec{B}_{\text{wire}}\right| \approx \frac{\pmb{m}_0}{4\pmb{p}}\frac{LI}{r^2},$$

which is like a small segment of wire of length L.

Q. Exp 17.20 – You were to have done Experiment 20 for today. What did you find? (If it's clear that they *haven't* done it, have them do it now.)

- Should find that the field strength drops off like 1/r.

- Should find a reasonable current value.

  o At twice the height, the deflection decreases by 1/2 to about 10°. If the deflection is 20°, the magnetic field of the wire is

  $B_{\text{wire}} = B_{\text{Earth}} \tan\theta = \left(2\times10^{-5}\ \text{T}\right) \tan 20° = 7.3\times10^{-6}\ \text{T}$. The current in the wire is found

  using $B_{\text{wire}} \approx \dfrac{\mu_0}{4\pi} \dfrac{2I}{r}$.

Magnetic field of a circular loop – In lab, you'll simulate the Magnetic field due to a circular loop. Now we'll look at it analytically, naturally, the basic set up for the analytical approach will be the same as for the computational one that you'll take tomorrow, so I encourage you to take particularly good notes on the set up.

Now we can look at the loop centered on z axis, find field at an arbitrary location.

See PowerPoint

- Check the result

- The units are correct (you can confirm that).

- At the center of the loop, all points on the wire are the same distance, so:

  - $B_z(z=0) = \dfrac{\mu_0}{4\pi} \dfrac{I(2\pi R)}{R^2}$

- which makes sense because $2\pi R$ is the length (circumference).

Along the z axis, the field is largest at the center and decreases farther away.
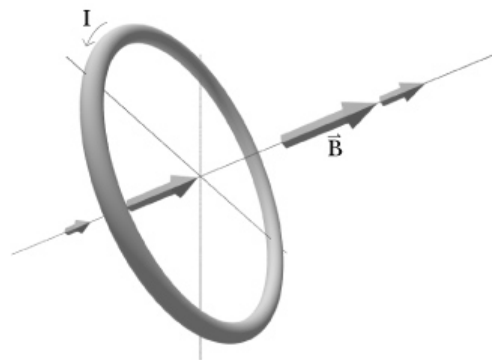
Very far from the loop ($z \gg R$):

$$\left|\vec{B}_{\text{loop}}\right| \approx \dfrac{\mu_0}{4\pi} \dfrac{2I\pi R^2}{z^3}$$

Demo. Let's look at this thing. 17_B_loop_xy_xz.py

- **Q.** Look familiar? Where have you seen a field pattern like that?

    o **A.** The *electric dipole.*

- **Current Loop =** *Magnetic* **Dipole.**
- **Magnetic dipole moment**
- Recall that the electric field along the axis of an *electric* dipole was

    o $E_{dipole.axis} \approx \dfrac{1}{4\pi\varepsilon_o}\dfrac{2p}{z^3}$ where $p=qs$ defined the *electric* dipole moment.

- Now, the *magnetic* dipole field looks

    o $\left|\vec{B}_{loop}\right| \approx \dfrac{\mu_0}{4\pi}\dfrac{2IpR^2}{z^3}$ on axis.

    o Define $m \equiv IA$, where $A = \pi R^2$.

    o So $\left|\vec{B}_{dipole}\right| \approx \dfrac{\mu_0}{4\pi}\dfrac{2m}{r^3}$

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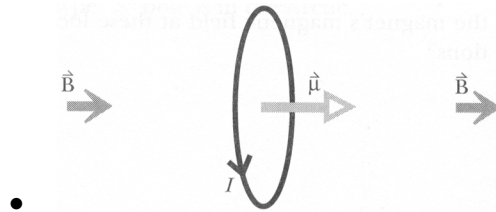
- A magnetic dipole will tend to line up with an external magnetic field if it is free to rotate.

- **Exp 17.21** – Find the magnetic field of the coils using $B_{coils}=B_{Earth}\tan\theta$. Find the current with $B_{coils}=\dfrac{\mu_0}{4\pi}\dfrac{2NIpR^2}{\left(z^2+R^2\right)^{3/2}}$, where R is the radius of a loop and z is the distance from the coil's center.

Exercises

1. A very long wire carrying a conventional current *I* in the direction shown is straight except for a circular loop of radius *R*. Calculate the magnitude and direction of the magnetic field at the center of the loop.

The magnetic field at the center of the ring is the sum of the magnetic field of the straight wire and the magnetic field of the loop. Both the wire and the loop make B out of the page, so the net field points out of the page.
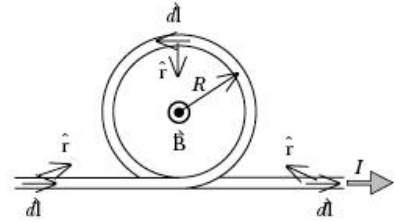

$$\vec{B}_{loop} = \int \frac{\mu_0}{4\pi} \frac{I d\vec{l} \times \hat{r}}{r^2} \text{ and } d\vec{l} \perp \hat{r} \text{ everywhere on the loop, so}$$

$$B_{loop} = \frac{\mu_0}{4\pi} \frac{I}{r^2} \int dl \sin 90° = \frac{\mu_0}{4\pi} \frac{I}{r^2}(2\pi r) = \frac{\mu_0}{4\pi} \frac{2\pi I}{r} \text{ out of the page}$$
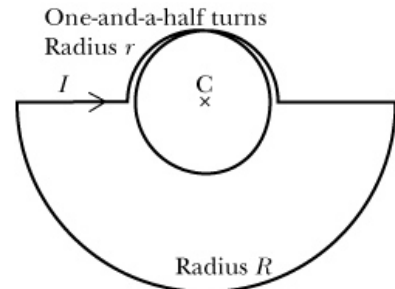
(special case of on-axis field of loop)

$$B_{straight\,wire} = \frac{\mu_0}{4\pi} \frac{2I}{r} \text{ out of the page}$$

$$B_{net} = \frac{\mu_0}{4\pi} \frac{2I}{r}(1 + \pi) \text{ out of the page}$$

2. A conventional current I flows in the direction shown below. Determine the magnitude and direction of the magnetic field at point C.

One-and-a-half turns
Radius r

Radius R

By the RHR, all of the curved segments produce magnetic fields into the page at C. The straight segments produce no (zero) magnetic field at C since $d\vec{\ell} \parallel \hat{r}$ for those.

The outer arc (radius R) produces half the magnetic field of a loop ($z=0$):

$$\left| \vec{B}_{outer} \right| = \frac{1}{2} \left(\frac{\mu_0}{4\pi} \frac{2\pi I}{R} \right) = \frac{\mu_0}{4\pi} \frac{\pi I}{R}$$

The inner turns (radius r) produces 1.5 times the magnetic field of a loop ($z=0$):

$$\left| \vec{B}_{inner} \right| = \frac{3}{2} \left(\frac{\mu_0}{4\pi} \frac{2\pi I}{r} \right) = \frac{\mu_0}{4\pi} \frac{3\pi I}{r}$$

Since the magnetic fields from the two parts are in the same direction (they are vectors!), the net magnetic field into the page is:

$$\left| \vec{B}_{net} \right| = \left| \vec{B}_{outer} \right| + \left| \vec{B}_{inner} \right| = \frac{\mu_0}{4\pi} \frac{\pi I}{R} + \frac{\mu_0}{4\pi} \frac{3\pi I}{r} = \frac{\mu_0}{4\pi} \pi I \left(\frac{1}{R} + \frac{3}{r} \right)$$

3. In a house, the maximum current carried by most wires is 15 amperes (circuit breakers flip off the power above that). Suppose two wires in a home power cord are about 3 mm apart as shown below and carry current in opposite directions at any instant (the directions alternate 60 times per second).

(a) Calculate the maximum magnitude of the magnetic field 50 cm away from the center of the power cord.

(a) At any instant the wires make magnetic fields in opposite directions. Since I_2 is closer, this results in a small net field. Assuming the wires are very long, the maximum magnitude of the time-varying magnetic field is this:
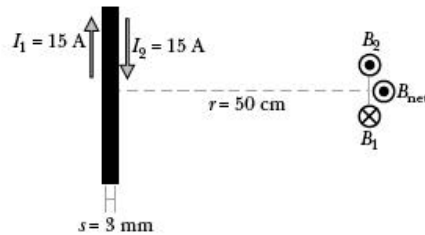
$$B_{net} = \frac{\mu_0}{4\pi}\frac{2I_2}{(r-s/2)} - \frac{\mu_0}{4\pi}\frac{2I_1}{(r+s/2)} = \frac{\mu_0}{4\pi}2I\left[\frac{1}{(r-s/2)} - \frac{1}{(r+s/2)}\right]$$

$$B_{net} = \frac{\mu_0}{4\pi}2I\left[\frac{s}{r^2-(s/2)^2}\right] \approx \frac{\mu_0}{4\pi}\frac{2Is}{r^2} = \left(10^{-7}\frac{\text{T·m}}{\text{A}}\right)\frac{2(15\text{ A})(3\times10^{-3}\text{ m})}{(0.5\text{ m})^2} = 3.6\times10^{-8}\text{ T}$$

(b) Explain briefly why twisting the pair of wires into a braid as shown would minimize the magnetic field.

(b) If the wires are twisted then their fields should very nearly cancel, since the magnetic field contributions by regions where wire 1 is closer are nearly canceled by regions where wire 2 is closer.

Monday: Permanent Magnets