**Materials:** computers with VPython and LoggerPro, LabPro, magnetic field probe, coil (from Helmholtz), solenoid, wiring, Heath-kit supply, multimeter, optics track, light-sensor holder

**Objectives**
In this lab you will do the following:
- Computationally model the magnetic field due to a current loop.
- Experimentally measure the magnetic field due to a current loop along its axis and compare with the analytical expression.
- Computationally model the magnetic field due to a solenoid.
- Experimentally measure the field due to a solenoid and compare with the analytical expression.

**Computation: Magnetic field of a Current Loop**

**Overview**

The general process for calculating the Magnetic field due to a continuous current distribution is similar to that you’d learned in chapter 16 calculating the Electric field from continuous charge distribution.

**General Procedure for calculating the magnetic field for a charge distribution**

1. Divide the current-carrying object into small (ideally, infinitesimal) segments and draw $\Delta \vec{B}$, a representative segment’s contribution to the field at an observation location.
2. Use the figure to guide your writing an expression for the magnetic field $\Delta \vec{B}$ due to the representative piece in terms of its location.
3. Add (integrate) up the contributions of all pieces to find the total field at an observation location.
4. Check the results.

Whether or not the sum can be rephrased as an integral with an analytical solution (one that can be done with pencil and paper), it can be approximated computationally. That is what you’ll do in this part of today’s lab.

**I. Visualizing the Problem**

To find the magnetic field due to a current-carrying loop, you must break it into small segments and treat each as a small straight piece of wire. The magnetic field contribution for such a segment of wire carrying a current $I$ is:

$$\Delta \vec{B}(\vec{r}_o) = \frac{\mu_0}{4\pi} \frac{I \Delta \vec{\ell} \times \vec{r}_{oe-s}^2}{r_{oe-s}^3},$$

where $\vec{r}_{oe-s}$ is a vector from the segment of current-carrying wire to the point where the magnetic field is being calculated, and $\Delta \vec{\ell}$ has the length of the segment and is in the direction of the current. An overhead view (looking down the $y$-axis) helps to understand that direction. The net magnetic field is the vector sum of each field due to each piece.
II. Writing / Modifying the Program

This should look very familiar from when you wrote a program to display the electric field due to a ring of uniform charge. So, while you could write a new program from scratch, the following instructions help you modify your old code to handle this new scenario.

- To get started on the program, open VPython from the desktop and then, from WebAssign, you can download a copy of RingE2.py - open it, copy and paste the text into the VPython window, and then save it as RingB.py (remember, you need to type the “.py”). The copy that’s posted here is virtually identical to the one you’d submitted for Lab 3; the only difference is that the ring has been rotated to lie in the x-z plane instead of the y-z plane.
- For the sake of comparing electric and magnetic fields, before modifying this code to model a current ring’s magnetic field, run the program and note the field pattern.
- Add your names in the first comment line of the code.
- Here are the key changes you need to make:
  - Rather than having a charge \( Q \), you have a 1 amp current.
  - To be able to see the magnetic field on the same scale as the loop, you’ll need a much larger scale factor, around 100,000.
  - The differential unit of source is now \( Idl \) rather than \( dQ \), where \( Idl \) is the current \( I \) times the length of ‘infinitesimal’ segments into which we’re dividing the ring; that is, the circumference of the ring divided by \( Ns \) (the number of segments into which you’ve divided the ring) times the unit vector pointing in the direction of the source segment. As you can see in the illustration of an overhead view, that unit vector is \( \hat{l} = (\cos \theta_x,0,-\sin \theta_x) \). Since this depends on the angle to the source location, you’ll need to place the line of code that defines \( Idl \) inside the loop over angles.
  - Instead of \( \frac{1}{4\pi} \), you want \( \frac{\mu_0}{4\pi} = 1e-7 \) (in units of kg m s/C^2, or equivalently, T m s/A.)
  - Obviously, you’re interested in \( \Delta B \), as given on the previous page, rather than \( \Delta E \). Note, the cross product is a defined function in VPython, so \( I\Delta \hat{l} \times \hat{r}_{\theta-s} \) could be coded written cross(I, r).

Run the program, compare with a neighboring group, and upload when you’re satisfied.

Experiment: on-axis Magnetic Field of a Current Loop.

Background

- A small loop of current constitutes a “magnetic dipole.” Recall that an electric dipole consists of equal and opposite electric charges quite close to each other. By analogy, a magnetic dipole is equal and opposite magnetic poles (read “North” and “South”) quite close to each other. That’s exactly what you have for a loop of current (the top face is one pole and the bottom is the other.) As you may recall, the magnitude of an electric dipole’s electric field goes like \( 1/r^3 \). The same should be true for a magnetic dipole’s magnetic field. You’ll investigate that.

- The specific relation for the field on axis is \( B_{axis} = \frac{\mu_0}{4\pi} \frac{2\pi R^2 I}{(r^2 + R^2)^{3/2}} \) where \( R \) is the radius of the loop and \( r \) is the distance along the y-axis from the loop’s center (for a loop oriented as is illustrated on the first page of this handout.)
I. Set Up

(much of this will be done for you, but double check it)

- The magnetic field probe should be plugged into the LabPro interface. *Make sure that the switch on its case is set on the lower scale (0.3mT).*
- Open Logger Pro.
- From within Logger Pro, open the experiment called “31 Mag Field Magnet” (it should be in the “_Physics with Vernier” folder).
- A dialogue box will pop up to complain that the probe’s on the wrong setting; that’s fine. Select “use sensor setting.”
- A picture’s worth a thousand words, so here’s the set up:

- In words:
  - The probe is mounted in a holder near the end of the track which is oriented so the probe is far from the computer and power supply.
  - So the probe can be quite far from the computer and power supply (which can create electrical noise that it will detect), it’s convenient to plug the LabPro into one of the computer’s front USB ports.
  - Plug one of the hoop’s white terminals into the Current port of the multimeter, and then plug its common port into one of the channel A terminals of the power supply. Plug the other terminal of the power supply directly into the other white terminal on the hoop.
  - Dial the multimeter to measure Current so you can monitor the current that will be running through the hoop.
  - Turn the power supply on and dial up channel A’s current to nearly 0.2 Amps (don’t worry about getting it exact.)
  - Make sure things are spaced so you can move the hoop all the way up against the probe holder, with the hoop on edge (as illustrated).
  - Set the hoop aside, near the power supply (away from the probe), and “zero” the probe. That makes it subtract out the background field. In Logger Pro, there’s a button with a blue zero icon – hit that.

    Note: if you move (even just re-angle) the probe, you’ll need to re-zero.

II. Procedure

- Hit “Collect” in Logger Pro.
- Start with the hoop up against the probe holder so the probe is along the hoop’s axis. You’ll move it in 0.01m steps away from the probe, along the track, until the field value levels off. At each step, hit “keep.”
- The field value will automatically be recorded; then you’ll be prompted to manually enter the distance. Note: the actual detector is 1 cm in from the probe’s end and you should use the distance from that to the center of the hoop (which is 2 cm wide).
- When you feel you’ve got enough data, hit “stop.”
III. Analysis

- We expect the magnitude of the field to follow the expression given in the Background section, where \( R = 0.105 \) m and \( \mu_0 = 10^{-7} \text{Tm/Amp} \), so 
  \[ B_{axis} = \frac{I}{4\pi} \frac{\mu_0}{r^2} \left( r^2 + \text{D} \right)^{\frac{3}{2}} \].

- To see how well your data fits this equation, under “Analyze” in LoggerPro select “curve fit...” or hit the button that looks like two hills with a red “U” in the middle. Click on “Define Function.” Type in the above equation using ^ to indicate “raised to the power”, using “A” in place of “I”, and “D” in place of “r” (A just stands for the first fitting parameter, and D stands for “distance.”) Name it something meaningful like “B field of loop”. Save the formula and then hit “try fit.”

  Note: I can help you make Logger Pro do your bidding.

- What we’re really interested in is how well the data fits the theoretical plot. One way to get at that is seeing how well the ‘free parameter’, A, compares with the current you measured going through the hoop. Actually, since the hoop is comprised of 200 loops of wire, the current along the hoop is 200 times the current measured by the multimeter. So, what must be the current through hoop?
  - What’s the current value from the curve fit?
  - What is the percent difference between the measured and fit values? If greater than 10%, check your measurements.

- The ‘root mean square error’ (RMSE) is another measure of how well your curve fits the data. As the name suggests, it’s the square root of the average of the square of the errors between the actual measured field strengths and those that the best-fit curve would predict. Ideally, this much less than the typical current values that you’d measured.
Computation - Magnetic Field of a Solenoid

A solenoid is a spool of current-carrying wire; it conveniently produces a strong and fairly uniform magnetic field inside and a relatively weak field outside. In that way, it’s the magnetic analog to a capacitor (which produces strong and fairly uniform electric field inside and weak field outside.) It can be modeled as a stack of current-carrying loops. So you’ll build on the program you’d written earlier in this lab.

- Make a copy of your program and save it as SolenoidB.py.
- Your solenoid should be composed of 50 loops in an 0.5 m stack along the y-axis and centered on the origin. So you’ll want to define constants \( N\text{loops} \) and \( L \) and give them the appropriate values. You can use the \( \text{linspace} \) function to create a list of all the loops’ y-components, call it \( y\text{list} \), running from \( -L/2 \) to \( L/2 \).
- Change \( B\text{scale} \) to \( 100000/N\text{loops} \) so the field vectors won’t get too large in spite of adding in multiple current loops / field sources.
- To create a visual representation of the stack of loops, you’ll create a list of the objects; you’d done something similar back in Lab 2, so the following should look vaguely familiar. Nestle around your existing \( \text{source} = \text{ring}(… \) line of code the following lines:

```python
solenoid = []
for y in ylist:
    source = ring(pos = (0,y,0),radius = R,…)
    solenoid.append(source)
```

Note: in your \( \text{source} = \text{ring}(… \) line, you initially had \( \text{pos} = (0,0,0) \); changing it to \( \text{pos} = (0,y,0) \) means that as the \( \text{for} \ldots \text{in} \ldots \) loop steps through each y value, it creates new rings at each elevation (y value).

Here’s how these lines work. The \( \text{solenoid} = [] \) tells the program that you’re going to make a list that you’ll name “solenoid” (like writing “groceries” at the top of an empty page that will soon become a grocery list). The \( \text{for} \ldots \text{in} \ldots \) steps through y values from your \( y\text{list} \) and creates rings at each of those; as it does so, \( \text{solenoid.append(source)} \) adds the new ring to your list that makes up the solenoid.

- The above only creates the rings, now down in the \#calculations section, you’ll need to modify the program to loop over all rings while calculating the magnetic field.
  - Nest the \( \text{for theta in soruce\text{esthetas}}: \text{loop inside a for y in ylist: loop}. \)
  - In the \text{Idl = … line change the y-component from 0 to y; do the same in the source\text{loc=… line}}.
- So your display isn’t dominated by a few giant arrows (representing the field right by a source), you should change the \text{if mag(B) >0.0001: condition to if mag(B*B\text{scale})>0.2}. 
- Run your program. With some creative rotating and zooming, you can get a good view of the field inside and outside the solenoid.

Compare your results with those of a neighboring group; when satisfied, save and upload.
Experiment: Magnetic Field inside a Solenoid.

According to your simulation, the field is fairly constant within a solenoid and decays pretty quickly outside. To check that experimentally, connect the solenoid to the power supply as the loop had been and dial up the current to about 0.5amps. Qualitatively, see how constant the field strength is within the solenoid and how quickly it decays outside the solenoid. To do that (without being distracted by fields due to other sources), you’ll want to hold the sensor steady and move the solenoid rather than the other way around.

To get a little more quantitative, use the probe to measure the field strength in near the middle of the solenoid.

- First you’ll want to hold the probe inside the solenoid with the solenoid’s current off, and zero the probe.
- Then dial up the solenoid’s current to about 0.5 amps. Record the current’s strength and the accompanying field strength near the center of the solenoid.

Theory:

According to optional section 18.13 of the text, the field deep within a solenoid is along its axis with a nearly-constant magnitude of approximately

\[ \vec{B} \approx \mu_0 I \frac{N}{L} \]

where \( I \) is the current through the wire, \( N \) is the number of loops (also referred to as the number of “turns”), and \( L \) is the length, so the ratio \( N/L \) is the “turns per unit length.”

Our solenoids have 550 turns (there are a few layers).

Measure the solenoid’s length and, based on the above equation and the current you actually passed through it, determine the field strength you’d expect.