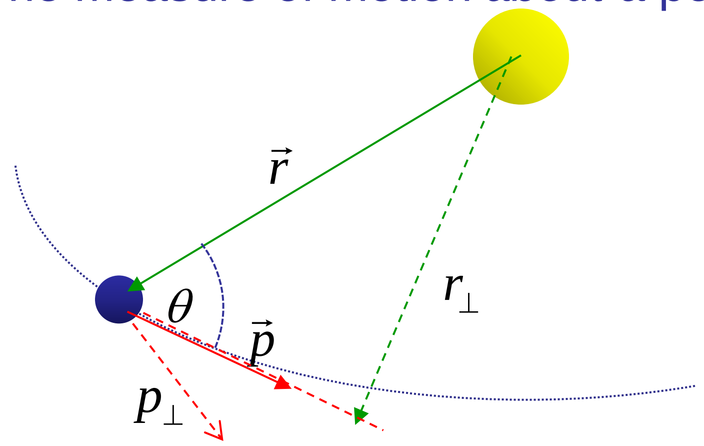
Wed.	11.7 - .9, (.11) Motion With & Without Torque	RE 11.d
Lab	L11 Rotation Lab Evals	
Fri.	11.10 Quantization, Quiz 11, Lect Evals	RE 11.e
Mon.	Review for Final (1-11)	HW11: Pr's 39, 57, 64, 74, 78
Sat.	9 a.m.	Final Exam (Ch. 1-11)

Using Angular Momentum

The measure of motion *about* a point



Magnitude and Direction

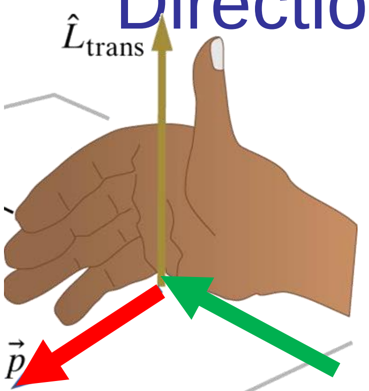
$$\vec{L} = \vec{r} \times \vec{p}$$

$$\vec{L} = \langle (p_z r_y - p_y r_z), (p_x r_z - p_z r_x), (p_y r_x - p_x r_y) \rangle$$

Magnitude

$$|L| = |p_\perp||r| = |p||r_\perp| = |p||r|\sin(\theta)$$

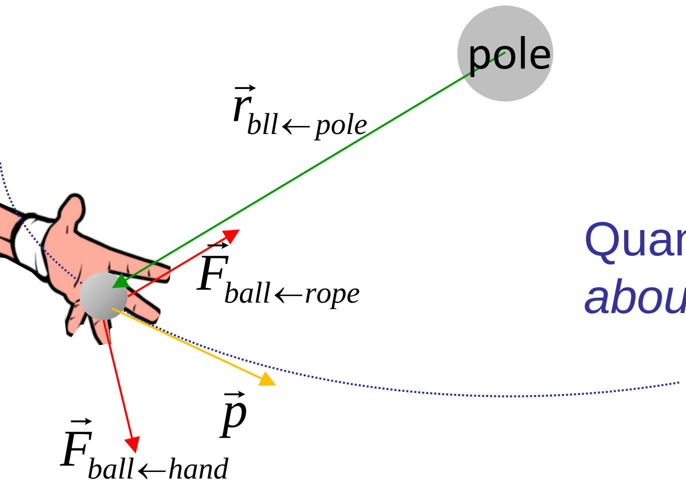
Direction



Orient Right hand so fingers curl from the axis and with motion, then thump points in direction of angular momentum.

Angular Momentum Principle

$$\frac{d}{dt}\vec{L}_{(about)A} = \sum_{net} \vec{\tau}_{(about)A}$$



Quantifies motion *about* a point

Interaction that changes motion *about* a point

Torque

where, $\vec{\tau}_{(about)A} \equiv \vec{r}_{(from)A} \times \vec{F}$

Magnitude

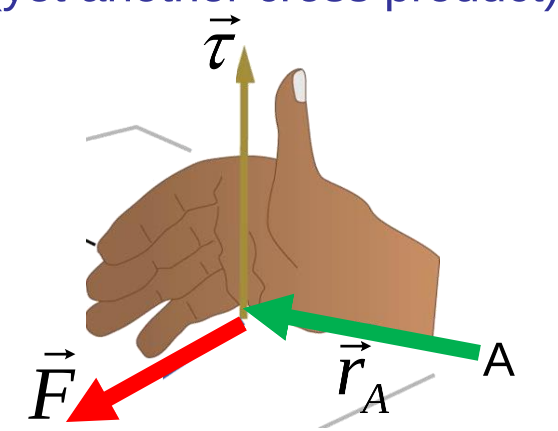
(yet another cross product)

$$|\tau_A| = |r_A||F_\perp| \quad = |r_{A\perp}||F|$$

$$|\tau_A| = |r_A|\left(|F|\sin\theta\right) = \left(|r_A|\sin\theta\right)|F| \quad = |r_A||F|\sin\theta$$

Direction

(yet another cross product)

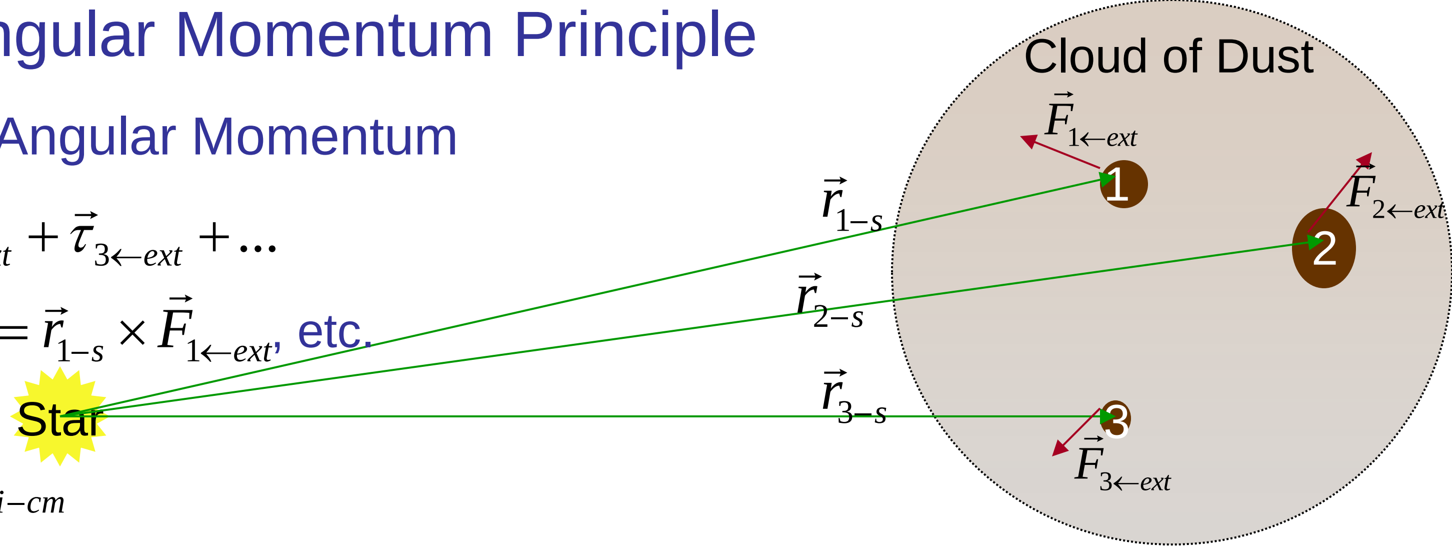


Multi-Particle Angular Momentum Principle

With Multi-Particle Angular Momentum

$$\frac{d\vec{L}_{c-s}}{dt} = \vec{\tau}_{1\leftarrow ext} + \vec{\tau}_{2\leftarrow ext} + \vec{\tau}_{3\leftarrow ext} + \ldots$$

where $\vec{\tau}_{1\leftarrow ext} = \vec{r}_{1-s} \times \vec{F}_{1\leftarrow ext}$, etc.



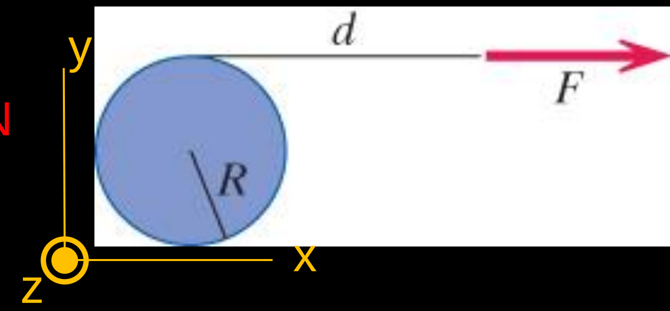
$$\vec{L}_{c-s} = \vec{L}_{cm-s} + \sum_i \vec{L}_{i-cm}$$

$$\vec{L}_{c-s} = \vec{L}_{trans-s} + \vec{L}_{rot-cm}$$

for rigid objects

$$\vec{L}_{rot-cm} = I_{cm}\vec{\omega}_{cm}$$

Example: A uniform solid disk with radius 9 cm has mass 0.9 kg (moment of inertia $I = ½MR^2$). A constant force 12 N is applied as shown. At the instant shown, the angular velocity of the disk is 45 radians/s in the –z direction. The length of the string d is 18 cm.



(z-axis points out of page)

At this instant, what are the magnitude and direction of the angular momentum about the center of the disk?

What are the magnitude and direction of the torque on the disk, about the center of mass of the disk?

The string is pulled for 0.2 s. What are the magnitude and direction of the angular impulse applied to the disk during this time?

After the torque has been applied for 0.2 s, what are the magnitude and direction of the angular momentum about the center of the disk?

At this later time, what are the magnitude and direction of the angular velocity of the disk?

Zero-Torque Systems

Demo: spinning dumb bells

Spinning Skater

Initial

Final

$$\vec{L}_{rot.f} - \vec{L}_{rot.i} = \vec{\tau}_{ave}\Delta t$$

$$\vec{L}_{rot.f} - \vec{L}_{rot.i} \approx 0$$

$$\vec{L}_{rot.i} \approx \vec{L}_{rot.f}$$

$$I_i\vec{\omega}_i \approx I_f\vec{\omega}_f$$

mass farther from axis: I_i larger
ω_i smaller

mass closer to axis:
I_f smaller
ω_i larger

What about the energy?

$$\Delta E \approx 0$$

$$\Delta K_{rot} + \Delta E_{\text{int}.other} \approx 0$$

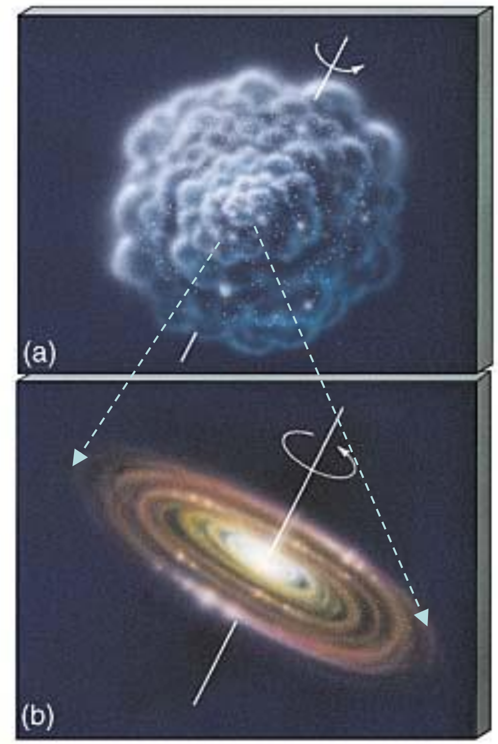
$$\Delta K_{rot} \approx -\Delta E_{\text{int}.other}$$

$$\frac{L^2}{2I_f} - \frac{L^2}{2I_i} \approx -\Delta E_{\text{int}.other}$$

$$\frac{L^2}{2}\left(\frac{1}{I_f} - \frac{1}{I_i}\right) \approx -\Delta E_{\text{int}.other}$$

Increased kinetic energy fueled by change in internal energy (Wheaties)

Solar system formation

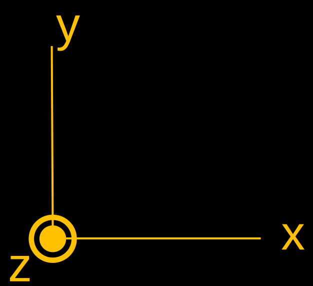


http://lifeng.lamost.org/courses/astrotoday/CHAISSON/AT315/HTML/AT31502.HTM

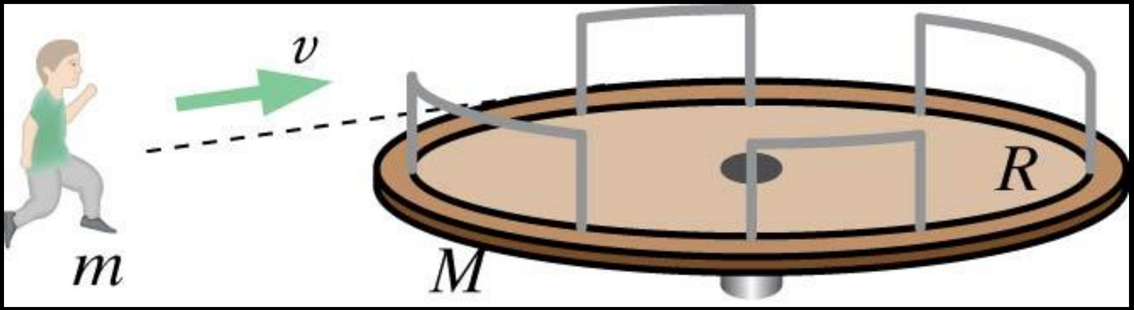
Increased rotational kinetic energy fueled by change in internal energy (gravitational potential)

Also consider: diver, Sit-spin & flip spinning wheel

Completely Inelastic Collision & Angular Motion



(z-axis points out of page)



Child runs and jumps on playground merry-go-round. For the system of the child + disk (excluding the axle and the Earth), which statement is true from just before to just after impact?

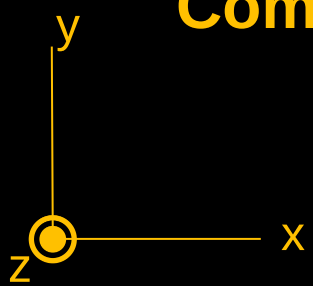
K is total (macroscopic) kinetic energy
$\vec{P}$ is total (linear) momentum
$\vec{L}$ is total angular momentum (about the axle)

a. K, $\vec{P}$, and $\vec{L}$ do not change
b. $\vec{P}$, and $\vec{L}$ do not change
c. $\vec{L}$ does not change
d. K and $\vec{P}$ do not change
e. K and $\vec{L}$ do not change

What is the initial angular momentum of the child + disk about the axle?

a) < 0, 0, 0 >
b) < 0, –Rmv, 0 >
c) < 0, Rmv, 0 >
d) < 0, 0, –Rmv >
e) < 0, 0, Rmv >

Completely Inelastic Collision & Angular Motion



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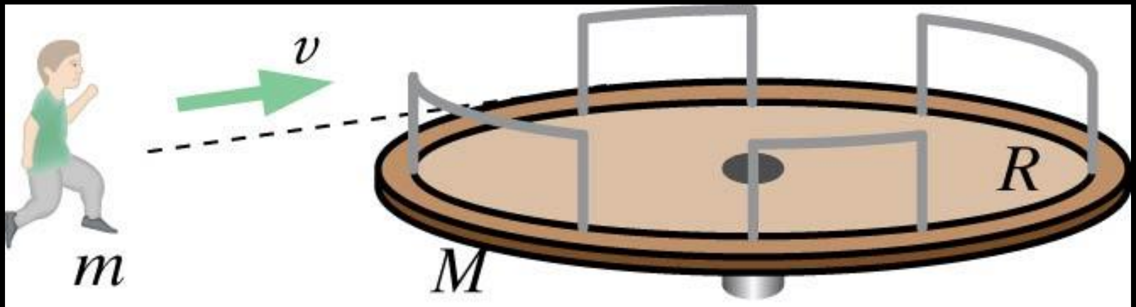
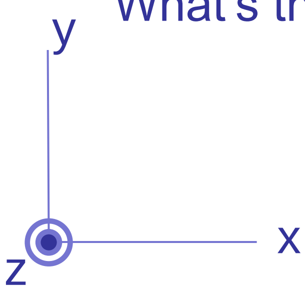


The disk has moment of inertia I, and after the collision it is rotating with angular speed ω. The rotational angular momentum of the disk alone (not counting the child) is	a. < 0, 0, 0 > b. < 0, –Iω, 0 > c. < 0, Iω, 0 > d. < 0, 0, –Iω > e. < 0, 0, Iω >
After the collision, what is the speed (in m/s) of the child?	a. ωR b. ω c. ωR^2 d. ω / R e. $\omega^2 R$

After the collision, what is translational angular momentum of the child about the axle?	a. < 0, 0, 0 > b. < 0, –Rmω, 0 > c. < 0, Rmω, 0 > d. < 0, –Rm(ωR), 0 > e. < 0, Rm(ωR), 0 >

Completely Inelastic Collision & Angular Motion

What's the final angular speed of the merry-go-round after the kid jumps on?



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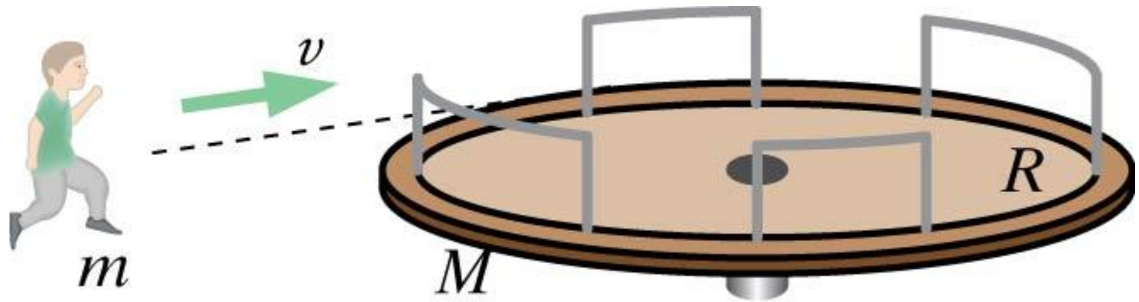


System: child + merry-go-round

Active environment: none that change angular momentum

Approximations: negligible frictional torque at axel

Axis: Axle

$$\vec{L}_{A.f} = \vec{L}_{A.i} + \cancel{\vec{\tau}_A}\Delta t$$

$$\vec{L}_{m.g.r-A.f} + \vec{L}_{child-A.f} = \vec{L}_{child-A.i}$$

$$\langle 0, -I_{disk.cm}\omega, 0\rangle + \langle 0, -(mR\omega)R, 0\rangle = \langle 0, -mvR, 0\rangle$$

$$I_{disk.cm} = \tfrac{1}{2}MR^2$$

$$\left\langle 0, -\tfrac{1}{2}MR^2\omega, 0\right\rangle + \left\langle 0, -mR^2\omega, 0\right\rangle = \langle 0, -mvR, 0\rangle$$

$$\left\langle 0, -\left(m + \tfrac{1}{2}M\right)R^2\omega, 0\right\rangle = \langle 0, -mvR, 0\rangle$$

$$-\left(m + \tfrac{1}{2}M\right)R^2\omega = -mvR$$

$$\omega = \frac{mv}{\left(m + \tfrac{1}{2}M\right)R} = \frac{1}{\left(1 + \tfrac{1}{2}\tfrac{M}{m}\right)}\frac{v}{R}$$

Reasonable?

Two-Step Example: A blob of clay (mass *m*) is dropped a distance h to land on and stick to a wheel (mass M, radius *R*) horizontally ½ R off axis. What's the wheel's angular speed just after the collision?

Step 1: Ball falls to wheel; use energy

System: Ball + Earth

Active environment: none

Approximations: negligible drag

$$\Delta E_{E\&B} = 0$$

$$\Delta K_B + \Delta E_{B.int} + \Delta E_E + \Delta U_{E,B} = 0$$

$$\left(\tfrac{1}{2} m v_{B.1}^2 - \tfrac{1}{2} m v_{B.0}^2\right) - mgh = 0 \quad \Rightarrow \quad v_{B.1} = \sqrt{2gh}$$

Step 2: Ball & wheel collide; use angular momentum

Axis: C   System: Ball + Wheel   Active environment: no torques

Approximations: time interval small enough axel friction and Earth's gravitation have negligible effect

$$\vec{L}_{W\&B-C.f} = \vec{L}_{B-C.i}$$

By right-hand-rule, all in the +z direction

$$I_{W\&B.C}\,\omega_2 = m v_B r_\perp$$

Before collision, $r_\perp = \tfrac{1}{2} R$

$$I_{W\&B.C} = I_{ring.cm} + I_{m.cm}$$

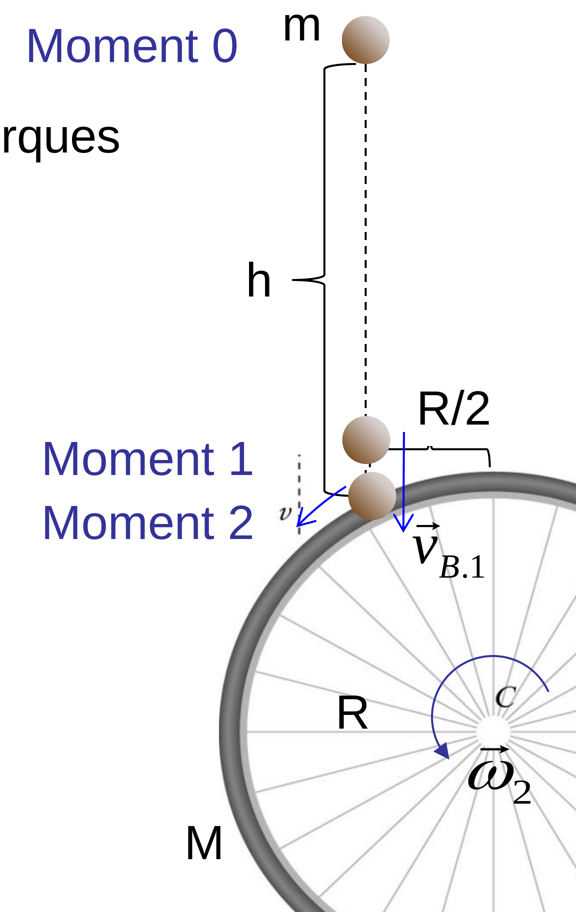
$$I_{W\&B.C} = MR^2 + mR^2$$

$$I_{W\&B.C}\,\omega_2 = m\left(\sqrt{2gh}\right)\left(\tfrac{1}{2} R\right)$$

$$(M+m)R^2\omega_2 = m\left(\sqrt{2gh}\right)\left(\tfrac{1}{2} R\right)$$

$$(M+m)R\omega_2 = m\tfrac{1}{2}\sqrt{2gh} \quad \Rightarrow \omega_2 = \frac{m\sqrt{2gh}}{2(M+m)R} = \frac{\sqrt{2gh}}{2\left(\frac{M}{m}+1\right)R}$$

Reasonable?



Three Fundamental Principles

Angular Momentum:

$$\frac{d}{dt}\vec{L}_{(about)A} = \sum_{net}\vec{\tau}_{(about)A}$$

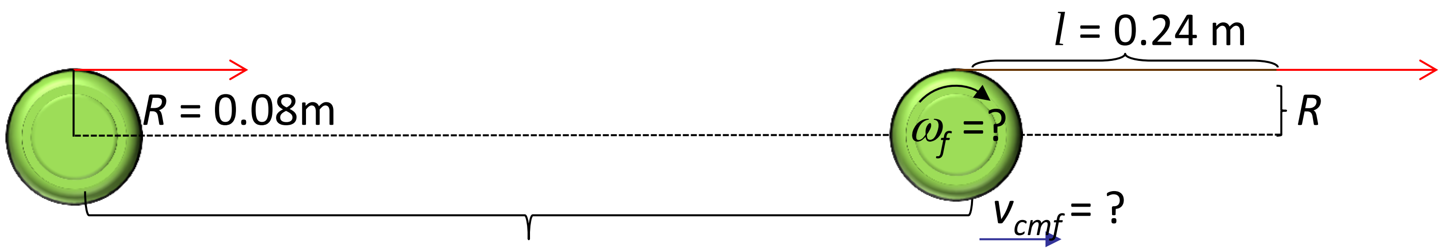
(linear) Momentum:

$$\frac{d}{dt}\vec{p} = \sum_{net}\vec{F}$$

Energy:

$$\Delta E = \sum_{net} W \qquad \Delta K_{cm} = \sum_{net} "W_{cm}"$$

Example all three together! Say we have a uniform 0.4 kg puck with an 8 cm radius. A 24 cm string is initially wrapped around its circumference. If it's on a frictionless surface and a 10 N force is applied to the end of the string until it's unwound…



$I_{puck} = \frac{1}{2}mR^2$ $\quad d = ? \quad \Delta t = ?$

a. What will be its rate of rotation when the string is fully unwound?

Energy Principle

$$\Delta E_{total} = W$$

$$\Delta K_{trans} + \Delta E_{\text{int}} = \vec{F}\cdot\Delta\vec{r}_F$$

$$\vec{F}\cdot\Delta\vec{r}_{cm} + \Delta K_{rot} = F(d+l)$$

$$Fd + \left(\tfrac{1}{2}I\omega_f^{\,2} - \tfrac{1}{2}I\omega_i^{\,2}\right) = Fd + Fl$$

$$\tfrac{1}{2}I\omega_f^{\,2} = Fl$$

$$\omega_f = \sqrt{\frac{2Fl}{I}}$$

$$\omega_f = \sqrt{\frac{2Fl}{\frac{1}{2}R^2 m}} = \frac{2}{R}\sqrt{\frac{Fl}{m}}$$

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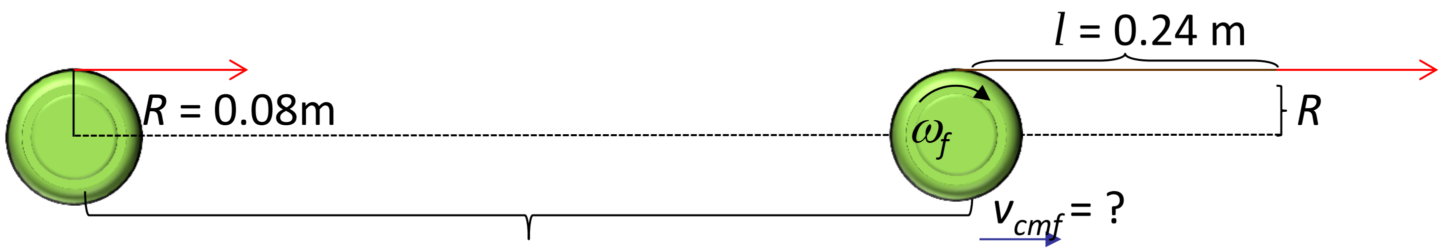

$$I_{puck} = \tfrac{1}{2} mR^2 \qquad d = ? \qquad \Delta t = ?$$

a. What will be its rate of rotation when the string is fully unwound? $\omega_f = \frac{2}{R}\sqrt{\frac{Fl}{m}}$

b. How long was the force applied? Angular Momentum Principle (axis through final location of cm)

$$\Delta\vec{L} = \int \vec{\tau}dt$$

$$\vec{L}_f - \vec{L}_i = \vec{\tau}\Delta t$$

$$(\vec{L}_{f.trans} + \vec{L}_{f.rot}) - (\vec{L}_{i.trans} + \vec{L}_{i.rot}) = \vec{\tau}\Delta t$$

$$I\vec{\omega}_f = \left(\vec{r}_{F-a} \times \vec{F}\right)\Delta t$$

Torque and final angular velocity in –z direction

$$I\omega_f = (RF)\Delta t \quad \Rightarrow \quad \Delta t = \frac{I\omega_f}{RF} = \frac{\frac{1}{2}R^2 m\omega_f}{RF} = \frac{Rm\omega_f}{2F} = \frac{Rm\frac{2}{R}\sqrt{\frac{Fl}{m}}}{2F} = \sqrt{\frac{ml}{F}}$$

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Angular Momentum:

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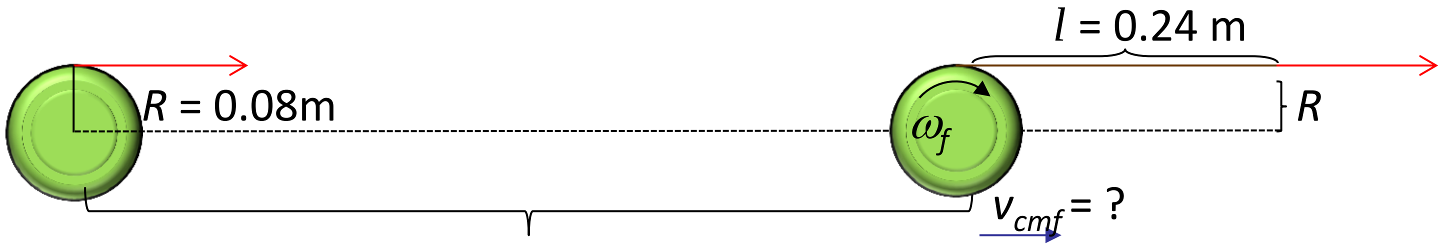

$$I_{puck} = \tfrac{1}{2} mR^2$$

a. What will be its rate of rotation when the string is fully unwound? $\omega_f = \frac{2}{R}\sqrt{\frac{Fl}{m}}$

b. How long was the force applied? $\Delta t = \sqrt{\frac{ml}{F}}$

You try:

c. How quickly is the puck finally sliding, $v_{cm\,f}$?

d. How far has the puck moved, d?

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