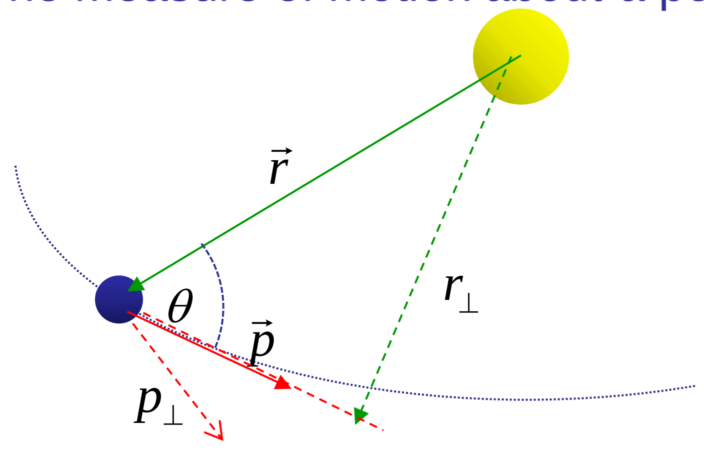
Mon. Tues.	11.2-.3, (.12) Rotational + Translational	RE 11.b EP10
Mon. Tues. Wed. Lab Fri.	11.4-.6, (.13) Angular Momentum & Torque 11.7 - .9, (.11) Torque L11 Rotation Course Evals 11.10 Quantization, Quiz 11	RE 11.c EP11 RE 11.d RE 11.e
Mon.	Review for Final (1-11)	HW11: Ch 11 Pr’s 39, 57, 64, 74, 78 & Practice Exam

Using Angular Momentum

The measure of motion *about* a point



Magnitude and Direction

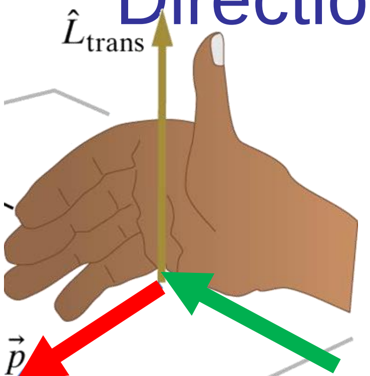
$$\vec{L} = \vec{r} \times \vec{p}$$

$$\vec{L} = \langle (p_z r_y - p_y r_z), (p_x r_z - p_z r_x), (p_y r_x - p_x r_y) \rangle$$

Magnitude

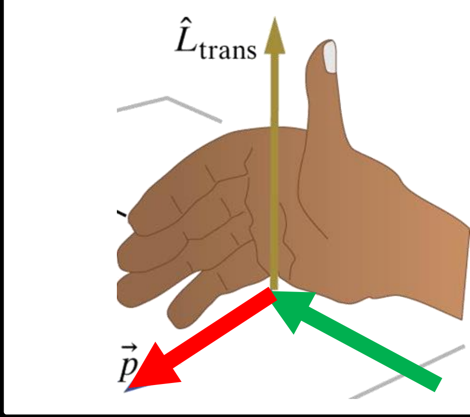
$$|L| = |p_\perp||r| = |p||r_\perp| = |p||r|\sin(\theta)$$

Direction



Orient Right hand so fingers curl from the axis and with motion, then thump points in direction of angular momentum.

$$\vec{L} = \langle (p_z r_y - p_y r_z), (p_x r_z - p_z r_x), (p_y r_x - p_x r_y) \rangle$$



What is the direction of

$\vec{p}$ $\vec{r}$

< 0, 4, 0> x < 0, 0, 3>?

What is the direction of

$\vec{p}$ $\vec{r}$

< 0, 0, 4> x < 0, 0, 3>?

1) +x
2) –x
3) +y
4) –y
5) +z
6) –z
7) zero magnitude

$$\vec{L} = \vec{r} \times \vec{p} = \langle (p_z r_y - p_y r_z), (p_x r_z - p_z r_x), (p_y r_x - p_x r_y) \rangle$$

$$|L| = |p_\perp||r| = |p||r_\perp| = |p||r|\sin(\theta)$$

If an object is traveling at a constant speed in a vertical circle, how does the object's angular momentum change as the object goes from the top of the circle to the bottom of the circle?

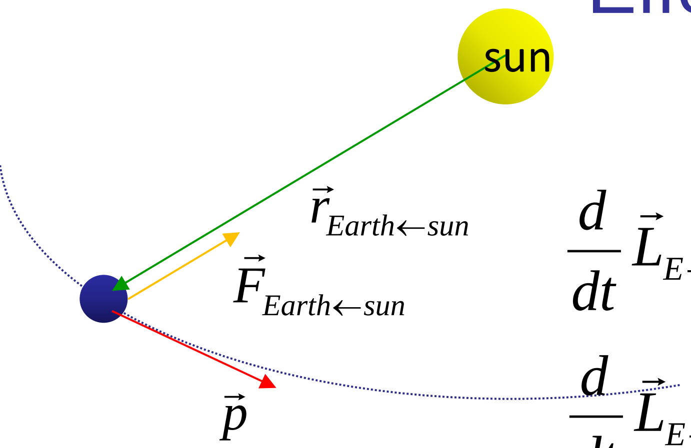
1. $|\vec{L}|$ increases
2. $|\vec{L}|$ decreases
3. $|\vec{L}|$ stays the same but the direction of $\vec{L}$ changes
4. The direction and magnitude of $\vec{L}$ remain the same

Using Angular Momentum

The measure of motion *about* a point

Effect of a radial force

(like gravity or electric)



$$\frac{d}{dt}\vec{L}_{E-S} = \frac{d}{dt}\left(\vec{r}_{E-S} \times \vec{p}_E\right)$$

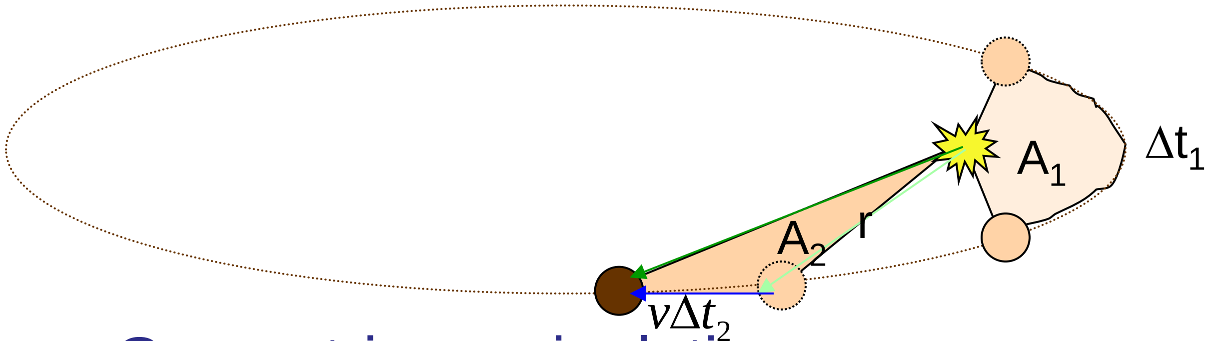
$$\frac{d}{dt}\vec{L}_{E-S} = \underbrace{\vec{v}_E \times \vec{p}_E}_{\text{Parallel}} + \underbrace{\vec{r}_{E-S} \times \vec{F}_{Earth\leftarrow sun}}_{\text{Parallel}} = 0$$

$$\vec{L}_{E-S} = \text{constant}$$

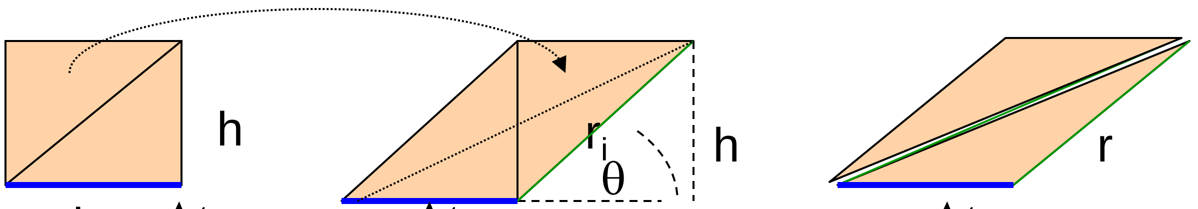
Orbit noncircular.py

Kepler and Planetary Orbits:

Sweeping our equal area in equal time: If $\Delta t_1 = \Delta t_2$, then $A_1 = A_2$



Some Geometric manipulation



b= $v\Delta t_2$

A=b*h= $v\Delta t_2 h$

$v\Delta t_2$

A= $v\Delta t_2 r\sin\theta$

$v\Delta t_2$

A_2= $\frac{1}{2} v\Delta t_2 r\sin\theta$

Some mathematical manipulation…

$$A_2 = \frac{\frac{1}{2}\Delta t_2}{m} mvr\sin\theta = \frac{\frac{1}{2}\Delta t_2}{m} pr\sin\theta = \frac{\frac{1}{2}\Delta t_2}{m}\left|\vec{L}\right|$$

Since L is constant (and m is constant), A is the same for the same time interval

Relating Energy, Radius and Angular Momentum in Circular Orbit

Angular Momentum: $L_{orbit} = rp = rmv$ (r & p perpendicular)

Kinetic and Gravitational Potential Energy: $E = K + U$

Kinetic energy: $K = \frac{1}{2}mv^2 = \frac{p^2}{2m} = \frac{L^2}{2mr^2}$

Potential energy: $U = -G\frac{Mm}{r}$

Gravitational Force and Circular motion: $\left|F_{net}\right| = \frac{mv^2}{r}$

$$-\frac{U}{r} = G\frac{Mm}{r^2} = \frac{2K}{r}$$

$$-U = 2K \quad \Rightarrow K + U = K - 2K = -K$$

$$G\frac{Mm}{r} = 2\frac{L^2}{2mr^2}$$

$$K + U = -\frac{L^2}{2mr^2}$$

$$\boxed{r = \frac{L^2}{m^2GM}}$$

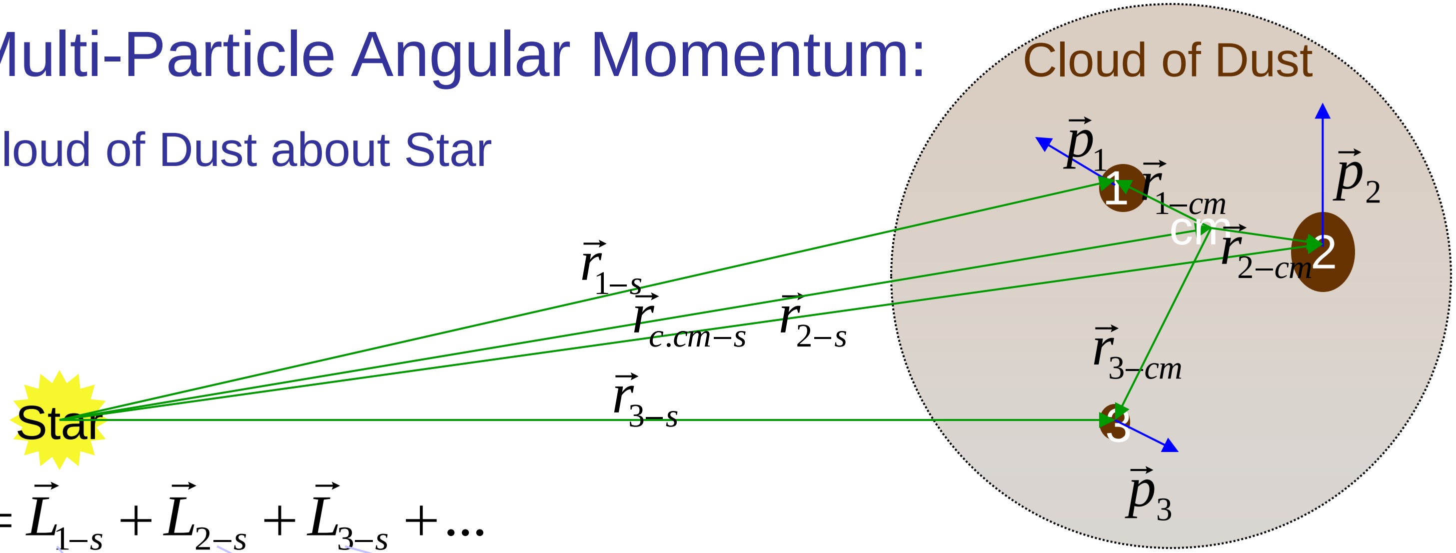
$$K + U = -\frac{L^2}{2m\left(\frac{L^2}{mGMm}\right)^2}$$

$$\boxed{K + U = -\frac{1}{2}\left(\frac{GmM}{L}\right)^2}$$

Later in chapter will apply same reasoning to electric interaction

Multi-Particle Angular Momentum:

Cloud of Dust about Star



$$\vec{L}_{c-s} = \vec{L}_{1-s} + \vec{L}_{2-s} + \vec{L}_{3-s} + \ldots$$

$$\vec{L}_{c-s} = \left(\vec{r}_{1-s} \times \vec{p}_1\right) + \left(\vec{r}_{2-s} \times \vec{p}_2\right) + \left(\vec{r}_{3-s} \times \vec{p}_2\right) + \ldots$$

$$\vec{L}_{c-s} = \left(\left(\vec{r}_{1-cm} + \vec{r}_{cm-s}\right) \times \vec{p}_1\right) + \left(\left(\vec{r}_{2-cm} + \vec{r}_{cm-s}\right) \times \vec{p}_2\right) + \left(\left(\vec{r}_{3-cm} + \vec{r}_{cm-s}\right) \times \vec{p}_3\right) + \ldots$$

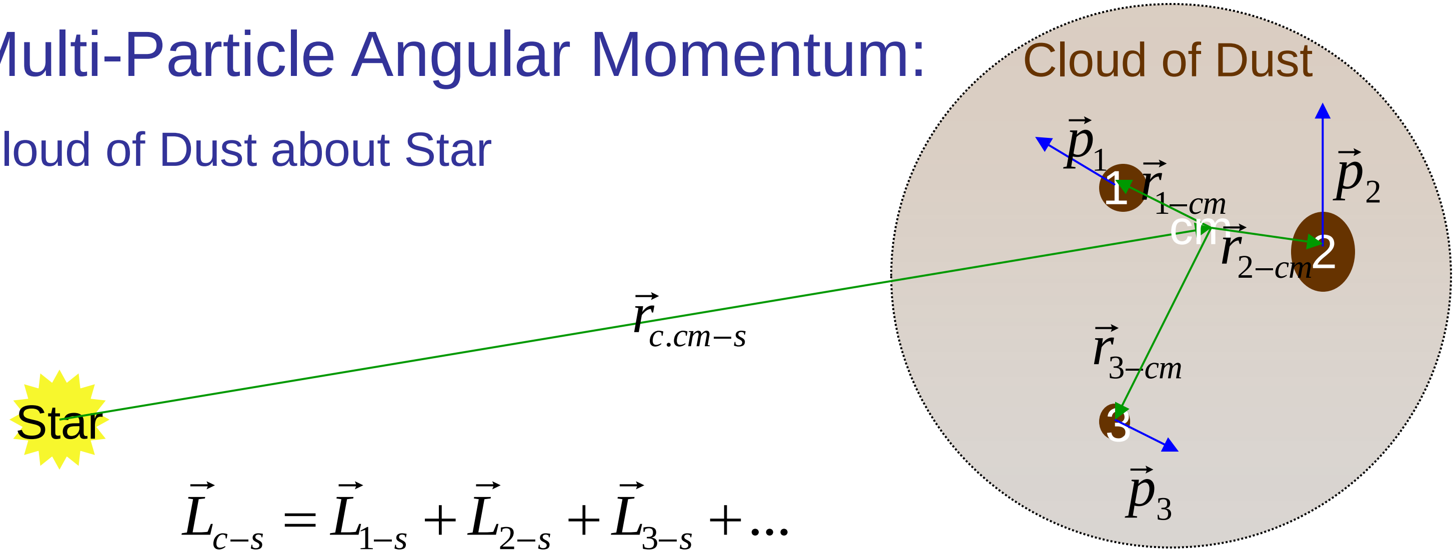
$$\vec{L}_{c-s} = \left(\vec{r}_{cm-s} \times \left(\vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \ldots\right)\right) + \left(\left(\vec{r}_{1-cm} \times \vec{p}_1\right) + \left(\vec{r}_{2-cm} \times \vec{p}_2\right) + \left(\vec{r}_{3-cm} \times \vec{p}_3\right) + \ldots\right)$$

$$\vec{L}_{c-s} = \left(\vec{r}_{cm-s} \times \vec{p}_{tot}\right) + \left(\vec{L}_{1.cm} + \vec{L}_{2.cm} + \vec{L}_{3.cm} + \ldots\right)$$

$$\vec{L}_{c-s} = \vec{L}_{cm-s} + \vec{L}_{c-cm} = \vec{L}_{translational.c-s} + \vec{L}_{rotational.c}$$

Multi-Particle Angular Momentum:

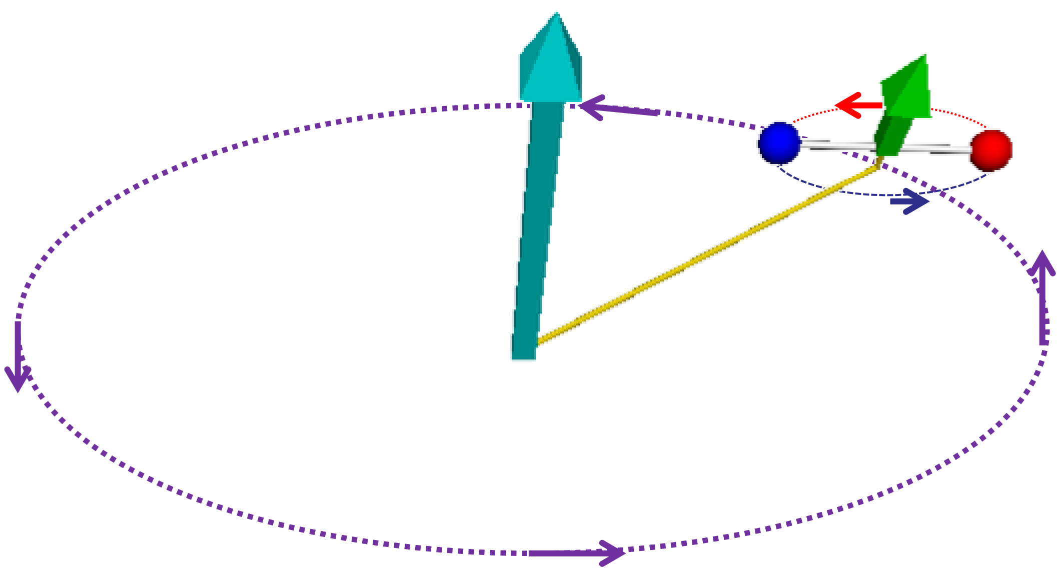
Cloud of Dust about Star



$$\vec{L}_{c-s} = \vec{L}_{1-s} + \vec{L}_{2-s} + \vec{L}_{3-s} + \ldots$$

$$\vec{L}_{c-s} = \left(\vec{r}_{cm-s} \times \vec{p}_{cm}\right) + \left(\left(\vec{r}_{1-cm} \times \vec{p}_1\right) + \left(\vec{r}_{2-cm} \times \vec{p}_2\right) + \left(\vec{r}_{3-cm} \times \vec{p}_3\right) + \ldots\right)$$

$$\vec{L}_{c-s} = \vec{L}_{translational.c-s} + \vec{L}_{rotational.c}$$



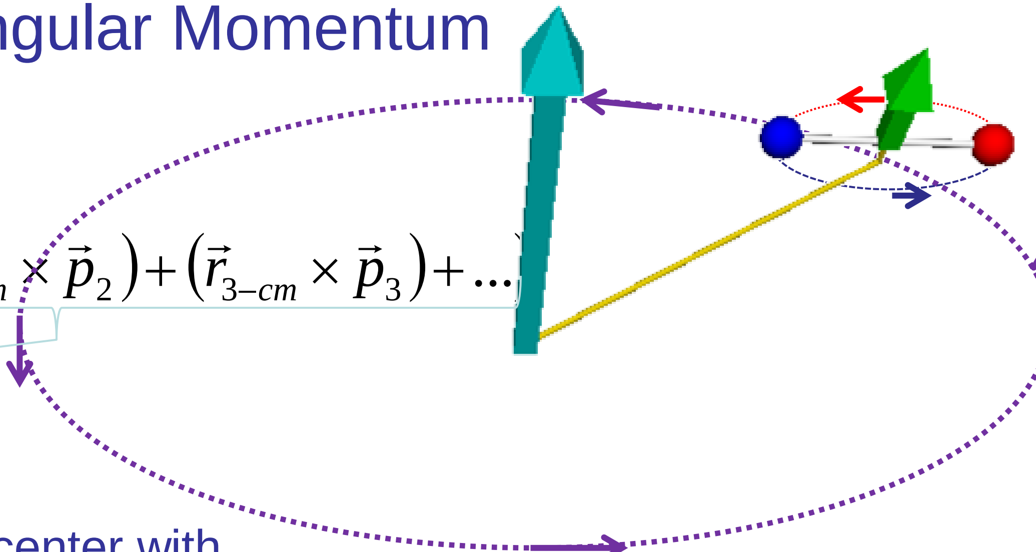
09_barbell_ang_mom.py

Multi-Particle Angular Momentum

$$\vec{L}_{c-s} = \vec{L}_{1-s} + \vec{L}_{2-s} + \vec{L}_{3-s} + \ldots$$

$$\vec{L}_{c-s} = \underbrace{\left(\vec{r}_{cm-s} \times \vec{p}_{cm}\right)} + \underbrace{\left(\left(\vec{r}_{1-cm} \times \vec{p}_1\right) + \left(\vec{r}_{2-cm} \times \vec{p}_2\right) + \left(\vec{r}_{3-cm} \times \vec{p}_3\right) + \ldots\right)}$$

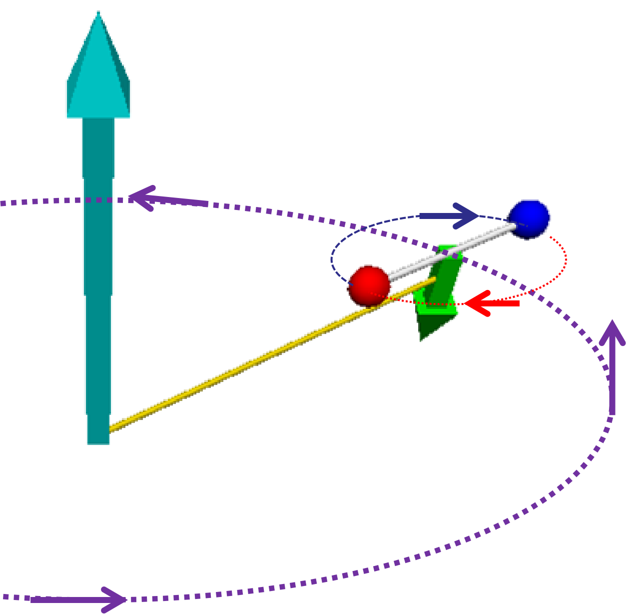
$$\vec{L}_{c-s} = \vec{L}_{translational.c-s} + \vec{L}_{rotational.c}$$



Example: the baton spins about its own center with $\vec{L}_{rot} = \langle 0,0,2\rangle kg \cdot m^2/s$ and about the person who's holding it with $\vec{L}_{trans} = \langle 0,0,5\rangle kg \cdot m^2/s$

Its total angular momentum about the person is $\vec{L}_{tot} = \vec{L}_{trans} + \vec{L}_{rot} = \langle 0,0,2+5\rangle kg \cdot m^2/s$

$$= \langle 0,0,7\rangle kg \cdot m^2/s$$



Example: the baton spins about its own center in the *opposite* direction, with $\vec{L}_{rot} = \langle 0,0,-2\rangle kg \cdot m^2/s$ and about the person who's holding it with $\vec{L}_{trans} = \langle 0,0,5\rangle kg \cdot m^2/s$

Its total angular momentum about the person is

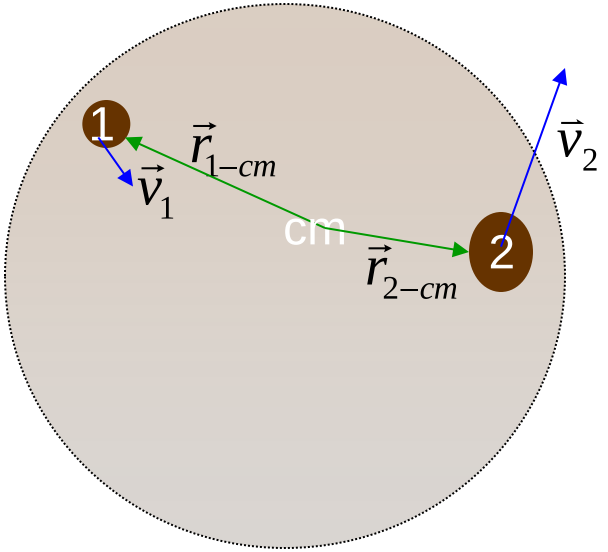
$$\vec{L}_{tot} = \vec{L}_{trans} + \vec{L}_{rot} = \langle 0,0,-2+5\rangle kg \cdot m^2/s$$

$$= \langle 0,0,3\rangle kg \cdot m^2/s$$

09_barbell_ang_mom.py

Rotational Angular Momentum

$$\vec{L}_{rot.cm} = \left(\vec{r}_{1-cm} \times \vec{p}_1\right) + \left(\vec{r}_{2-cm} \times \vec{p}_2\right) + \left(\vec{r}_{3-cm} \times \vec{p}_3\right) + \ldots$$



Example: say we have two particles, what's $\vec{L}_{rot}$?

m_1 = 0.2kg, $\vec{r}_{1\leftarrow cm} = \langle 0,2,1 \rangle m$ and $\vec{v}_1 = \langle 3,0,0 \rangle m/s$

$$\vec{L} = \langle (p_z r_y - p_y r_z), (p_x r_z - p_z r_x), (p_y r_x - p_x r_y) \rangle$$

$$\vec{L}_{1.cm} = \langle (0 \cdot r_y - 0 \cdot r_z), ((0.2kg \cdot 3m/s)1m - 0 \cdot 0), (0 \cdot 0 - (0.2kg \cdot 3m/s)2m) \rangle$$

$$\vec{L}_{1.cm} = \langle 0,\ 0.6,\ -1.2 \rangle kg \cdot \frac{m^2}{s}$$

m_2 = 0.1kg, $\vec{r}_{2\leftarrow cm} = \langle 0, -4, -2 \rangle m$ and $\vec{v}_2 = \langle 0,4,0 \rangle m/s$

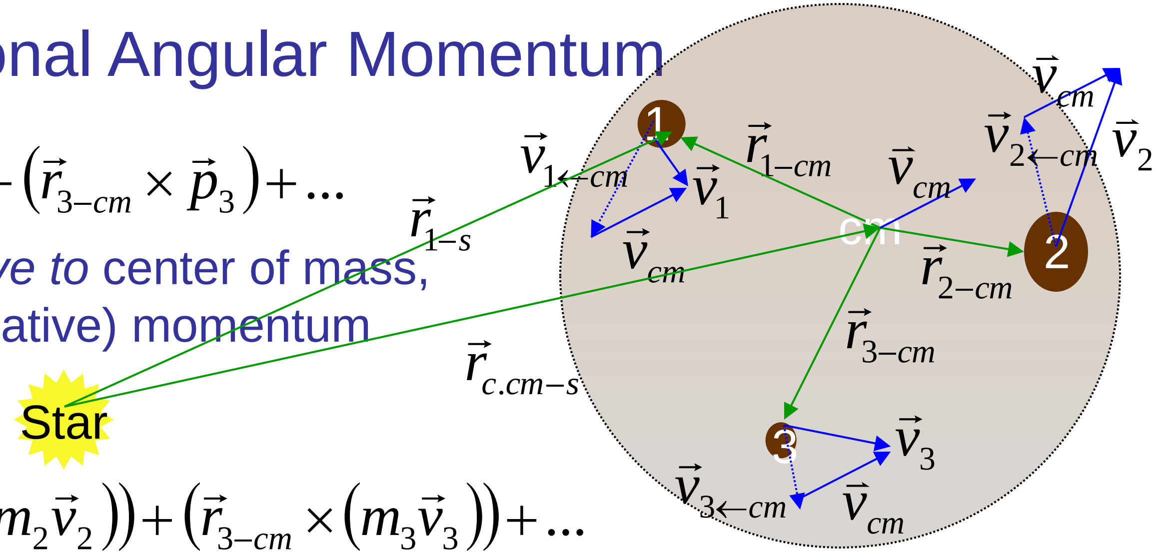
$$\vec{L} = \langle (p_z r_y - p_y r_z), (p_x r_z - p_z r_x), (p_y r_x - p_x r_y) \rangle$$

$$\vec{L}_{2.cm} = \langle (0 \cdot r_y - (0.1kg \cdot 4m/s) \cdot (-2m)), (0 \cdot r_z - 0 \cdot 0), (p_y \cdot 0 - 0 \cdot r_y) \rangle$$

$$\vec{L}_{2.cm} = \langle 0.8,\ 0,\ 0 \rangle kg \cdot \frac{m^2}{s}$$

$$\vec{L}_{rot.cm} = \vec{L}_{1.cm} + \vec{L}_{2.cm} = \langle 0.8,\ 0,\ 0 \rangle kg \cdot \frac{m^2}{s} + \langle 0,\ 0.6,\ -1.2 \rangle kg \cdot \frac{m^2}{s} = \langle 0.8,\ 0.6,\ -1.2 \rangle kg \cdot \frac{m^2}{s}$$

Focus on Rotational Angular Momentum



$$\vec{L}_{rot.cm} = \left(\vec{r}_{1-cm} \times \vec{p}_1\right) + \left(\vec{r}_{2-cm} \times \vec{p}_2\right) + \left(\vec{r}_{3-cm} \times \vec{p}_3\right) + \ldots$$

While it depends on position *relative to* center of mass, appears to depend on total (not relative) momentum

For v<<c, show that's *not* the case

$$\vec{L}_{rot.cm} = \left(\vec{r}_{1-cm} \times \left(m_1\vec{v}_1\right)\right) + \left(\vec{r}_{2-cm} \times \left(m_2\vec{v}_2\right)\right) + \left(\vec{r}_{3-cm} \times \left(m_3\vec{v}_3\right)\right) + \ldots$$

Focusing on just one particle

$$\vec{L}_{1.cm} = \left(m_1\vec{r}_{1-cm} \times \left(\vec{v}_{cm} + \vec{v}_{1\leftarrow cm}\right)\right) = \left(m_1\vec{r}_{1-cm} \times \vec{v}_{cm}\right) + \left(m_1\vec{r}_{1-cm} \times \vec{v}_{1\leftarrow cm}\right)$$

$$m_1\left(\vec{r}_1 - \vec{r}_{cm}\right) \times \vec{v}_{cm} + \left(m_1\vec{r}_{1-cm} \times \vec{v}_{1\leftarrow cm}\right)$$

$$\left(m_1\vec{r}_1 - m_1\vec{r}_{cm}\right) \times \vec{v}_{cm} + \left(m_1\vec{r}_{1-cm} \times \vec{v}_{1\leftarrow cm}\right)$$

Similarly for all other particles, so

$$\vec{L}_{rot.cm} = \left(\left(m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \ldots\right) - \left(m_1 + m_2 + m_3 + \ldots\right)\vec{r}_{cm}\right) \times \vec{v}_{cm} + \left(m_1\vec{r}_{1-cm} \times \vec{v}_{1\leftarrow cm}\right) + \left(m_2\right.$$

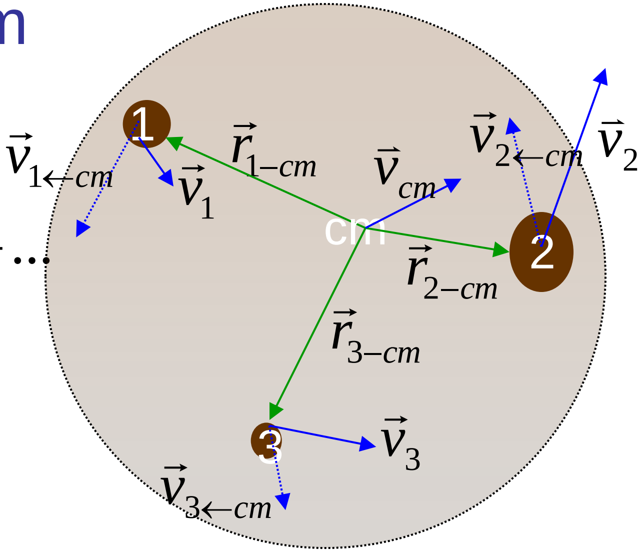
but $\vec{r}_{cm} \equiv \dfrac{\left(m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \ldots\right)}{\left(m_1 + m_2 + m_3 + \ldots\right)}$ so 0

$$\vec{L}_{rot.cm} = \left(m_1\vec{r}_{1-cm} \times \vec{v}_{1\leftarrow cm}\right) + \left(m_2\vec{r}_{2-cm} \times \vec{v}_{2\leftarrow cm}\right) + \left(m_3\vec{r}_{3-cm} \times \vec{v}_{3\leftarrow cm}\right) + \ldots$$

Really only depends on position and momentum *relative of center of mass.*

$$\vec{L}_{rot.cm} = \left(\vec{r}_{1-cm} \times \vec{p}_{1\leftarrow cm}\right) + \left(\vec{r}_{2-cm} \times \vec{p}_{2\leftarrow cm}\right) + \left(\vec{r}_{3-cm} \times \vec{p}_{3\leftarrow cm}\right) + \ldots$$

Rotational Angular Momentum



$$\vec{L}_{rot.cm} = (\vec{r}_{1-cm} \times \vec{p}_1) + (\vec{r}_{2-cm} \times \vec{p}_2) + (\vec{r}_{3-cm} \times \vec{p}_3) + \ldots$$

For v<<c

$$\vec{L}_{rot.cm} = (\vec{r}_{1-cm} \times \vec{p}_{1\leftarrow cm}) + (\vec{r}_{2-cm} \times \vec{p}_{2\leftarrow cm}) + (\vec{r}_{3-cm} \times \vec{p}_{3\leftarrow cm}) + \ldots$$

$$\vec{L}_{rot.cm} = \vec{L}_{1\leftarrow cm} + \vec{L}_{2\leftarrow cm} + \vec{L}_{3\leftarrow cm} + \ldots$$

Example: say we have two particles

$m_1 = 0.2kg$, $\vec{r}_{1\leftarrow cm} = \langle 0,2,1\rangle m$ and $\vec{v}_1 = \langle 3,0,0\rangle m/s$

$$\vec{v}_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2} = \frac{(0.2kg)\langle 3,0,0\rangle m/s + (0.1kg)\langle 0,4,0\rangle m/s}{(0.2kg) + (0.1kg)} = \langle 2, \tfrac{4}{3}, 0\rangle m/s$$

$$\vec{v}_{1\leftarrow cm} = \vec{v}_1 - \vec{v}_{cm} = \langle 1, -\tfrac{4}{3}, 0\rangle m/s$$

$$\vec{L} = \langle (p_z r_y - p_y r_z), (p_x r_z - p_z r_x), (p_y r_x - p_x r_y) \rangle$$

$$\vec{L}_{1\leftarrow cm} = \left\langle (0 \cdot r_y - \left(0.2kg \cdot \left(-\tfrac{4}{3}\tfrac{m}{s}\right)\right) \cdot 1m), ((0.2kg \cdot 1\tfrac{m}{s})1m - 0 \cdot 0), (p_y \cdot 0 - (0.2kg \cdot 1\tfrac{m}{s})2m) \right\rangle$$

$$\vec{L}_{1\leftarrow cm} = \langle \tfrac{0.8}{3},\ 0.2,\ -0.4\rangle kg \cdot \frac{m^2}{s}$$

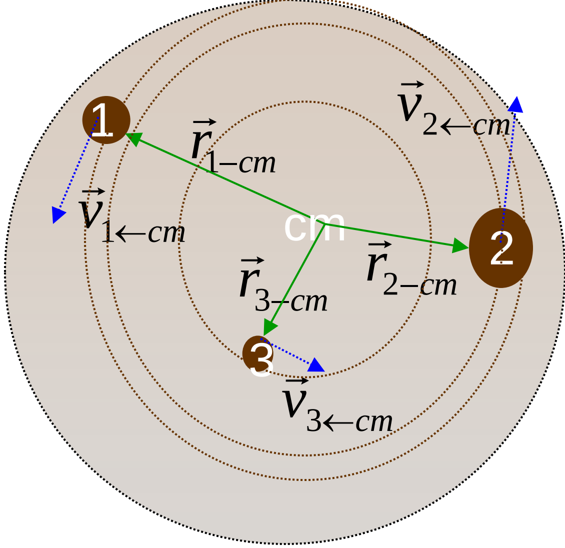
$m_2 = 0.1kg$, $\vec{r}_{2\leftarrow cm} = \langle 0,-4,-2\rangle m$ and $\vec{v}_2 = \langle 0,4,0\rangle m/s$ $\quad \vec{v}_{2\leftarrow cm} = \vec{v}_2 - \vec{v}_{cm} = \langle -2, \tfrac{8}{3}, 0\rangle m/s$

$$\vec{L}_{1\leftarrow cm} = \left\langle (0 \cdot r_y - \left(0.1kg \cdot \left(\tfrac{8}{3}\tfrac{m}{s}\right)\right) \cdot (-2m)), (0 \cdot r_z - 0 \cdot 0), (p_y \cdot 0 - \left(0.1kg \cdot \left(-2\tfrac{m}{s}\right)\right) \cdot (-4m)) \right\rangle$$

$$\vec{L}_{1\leftarrow cm} = \left\langle \frac{1.6}{3}, 0,\ -0.8 \right\rangle kg \cdot \frac{m^2}{s}$$

$$\vec{L}_{rot.cm} = \vec{L}_{1\leftarrow cm} + \vec{L}_{2\leftarrow cm} = \left(\langle \tfrac{0.8}{3},\ 0.2,\ -0.4\rangle + \left\langle \frac{1.6}{3}, 0,\ -0.8\right\rangle\right) kg \cdot \frac{m^2}{s} = \langle 0.8,\ 0.6,\ -1.2\rangle kg \cdot \frac{m^2}{s}$$

Rotational Angular Momentum



$$\vec{L}_{rot.cm} = (\vec{r}_{1-cm} \times \vec{p}_1) + (\vec{r}_{2-cm} \times \vec{p}_2) + (\vec{r}_{3-cm} \times \vec{p}_3) + \ldots$$

For v<<c

$$\vec{L}_{rot.cm} = (\vec{r}_{1-cm} \times \vec{p}_{1\leftarrow cm}) + (\vec{r}_{2-cm} \times \vec{p}_{2\leftarrow cm}) + (\vec{r}_{3-cm} \times \vec{p}_{3\leftarrow cm}) + \ldots$$

$$\vec{L}_{rot.cm} = \vec{L}_{1\leftarrow cm} + \vec{L}_{2\leftarrow cm} + \vec{L}_{3\leftarrow cm} + \ldots$$

Special case: rigid body

Particle velocities *must* be perpendicular to their position vectors (otherwise they'd be moving in and out)

$$\left|\vec{L}_{1\leftarrow cm}\right| = m_1\left|(\vec{r}_{1-cm} \times \vec{v}_{1-cm})\right| = m_1\left|\vec{r}_{1-cm}\right|\left|\vec{v}_{1-cm}\right|$$

all particles travel their circles in the same period, *T*

$$\left|\vec{v}_{1-cm}\right| = \frac{2\pi r_{1-cm}}{T} = \omega r_{1-cm}$$

$$\left|\vec{L}_{1\leftarrow cm}\right| = m_1 r_{1-cm}(r_{1-cm}\omega) = m_1 r_{1-cm}^2\ \omega$$

$$\left|\vec{L}_{rot.cm}\right| = m_1 r_{1-cm}^2\ \omega + m_2 r_{2-cm}^2\ \omega + m_3 r_{3-cm}^2\ \omega + \ldots = \left(\sum_i^{all.particles} m_i r_{i-cm}^2\right)\omega$$

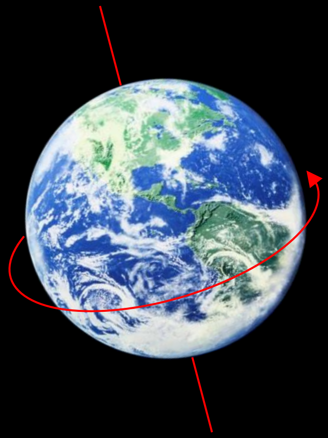
Moment of Inertia (again) $I_{cm} \equiv \sum_i^{all.particles} m_i r_{i-cm}^2$

$$\boxed{\vec{L}_{rot.cm} = I_{cm}\vec{\omega}}$$

Angular Speed Refresher

$$\omega = \frac{2\pi}{T}$$

The Earth rotates on its axis once every 24 hours. What is its angular speed?
Radius: 6.4e6 m   Mass: 6e24 kg



1) ω = 2 π / (24*60*60)

2) ω = 2 π * 6.4e6 / (24*60*60)

3) ω = (6e24) * 2 π * 6.4e6 / (24*60*60)

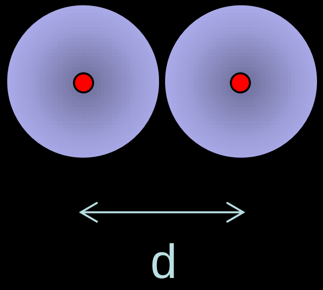
4) ω = (6e25) * (6.4e6)2 * 2 π / (24*60*60)

Moment of Inertia Refresher

$$I_{cm} \equiv \sum_{i}^{all.particles} m_i r_{i-cm}^2$$

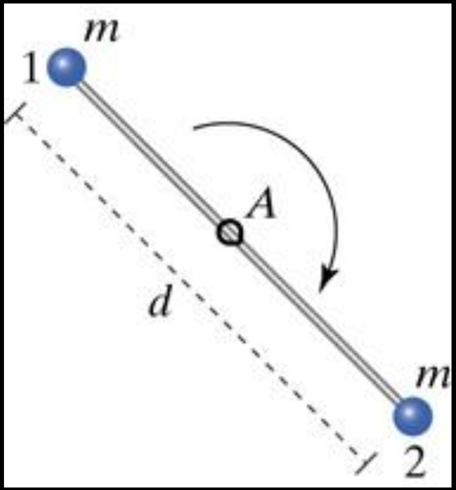
A diatomic molecule such as molecular nitrogen (N_2) consists of two atoms each of mass *M*, whose nuclei are a distance *d* apart. What is the moment of inertia of the molecule about its center of mass?

a) Md^2
b) $2Md^2$
c) ½ Md^2
d) ¼ Md^2
e) 4 Md^2



Rotational Angular Momentum
Special case: rigid body

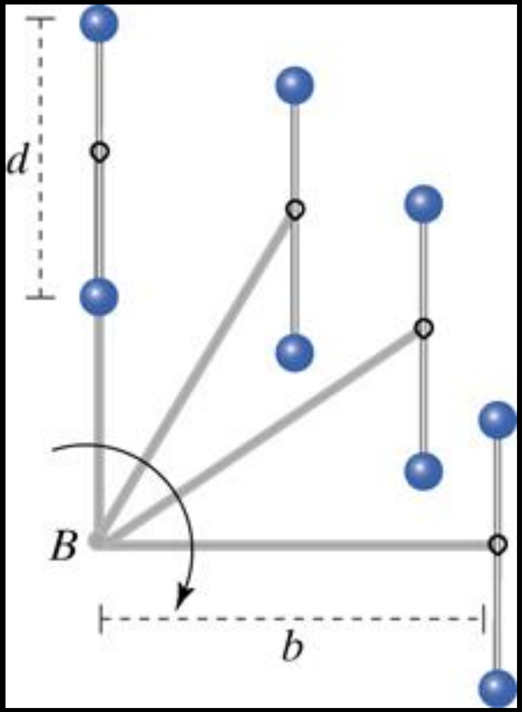
Example: A barbell spins around a pivot at its center at *A*. The barbell consists of two small balls, each with mass 500 grams (0.5 kg), at the ends of a very low mass rod of length 50 cm (0.5 m). The barbell spins clockwise with angular speed ω = 120 radians/s.



a) What is the moment of inertia about A?
b) What is the direction of the angular momentum?
c) What is the translational angular momentum?
d) What is the rotational angular momentum?
e) What is the total angular momentum?

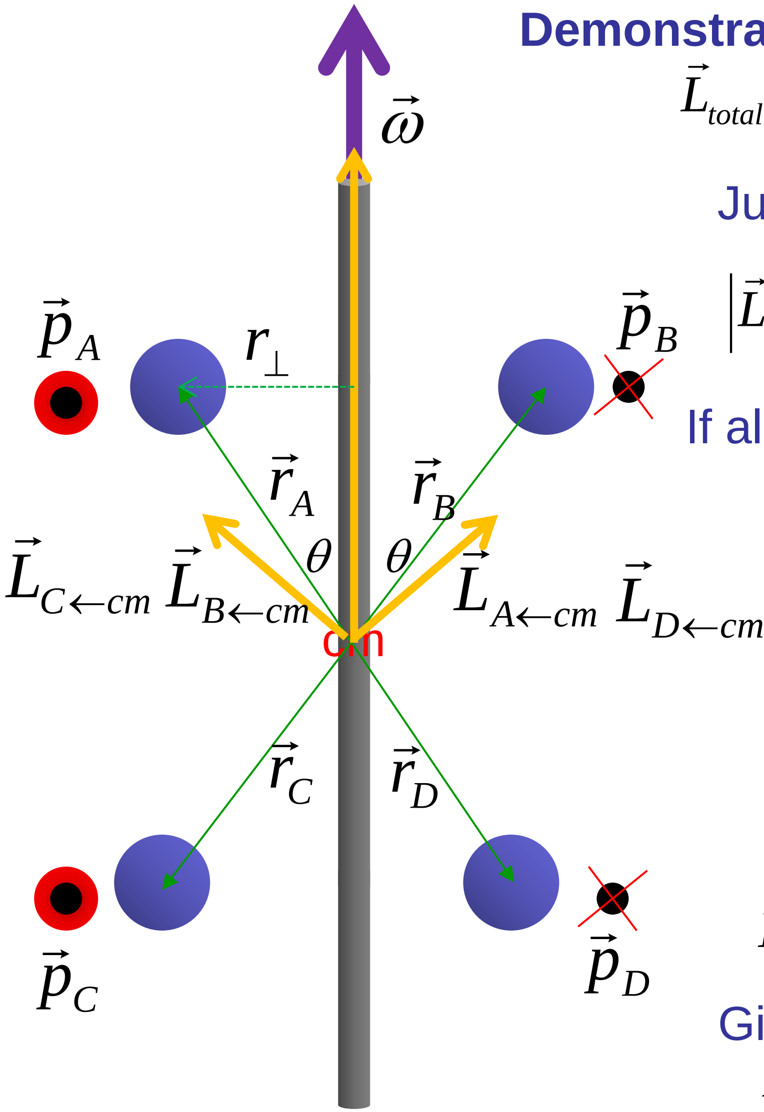
Rotational Angular Momentum
Special case: rigid body

Example: Next the center of the barbell (of length 50 cm, two 0.5kg masses) is mounted on the end of a low mass rigid rod of length *b* = 1 m. The apparatus is started in such a way that although the rod rotates clockwise with angular speed ω = 90 radians/s, the barbell maintains its vertical orientation.



a) What is the moment of inertia about B?
b) What is the direction of the angular momentum?
c) What is the translational angular momentum?
d) What is the rotational angular momentum?
e) What is the total angular momentum?

If the Masses Don't Lie in a Plane



Demonstrating I_{axis}

$$\vec{L}_{total\leftarrow cm} = \vec{r}_A \times \vec{p}_A + \vec{r}_B \times \vec{p}_B + \vec{r}_C \times \vec{p}_C + \vec{r}_D \times \vec{p}_D$$

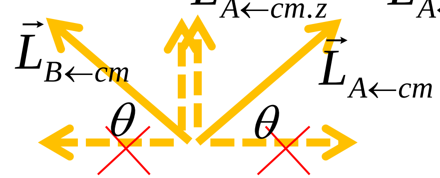
Just looking at magnitude for ball A

$$\left|\vec{L}_{A\leftarrow cm}\right| = \left|\vec{r}_A \times \vec{p}_A\right| = r_A p_A = r_A m_A\left(\frac{2\pi r_{A\perp}}{T}\right) = r_A m_A r_{A\perp}\omega$$

If all masses, distances, and speeds are the same,

$$\left|\vec{L}_{A\leftarrow cm}\right| = \left|\vec{L}_{b\leftarrow cm}\right| = \left|\vec{L}_{C\leftarrow cm}\right| = \left|\vec{L}_{D\leftarrow cm}\right| = rmr_\perp\omega$$

$$L_{A\leftarrow cm.z} = L_{A\leftarrow cm}\sin\theta = rmr_\perp\omega\sin\theta$$



$$\vec{L}_{total\leftarrow cm} = \left(L_{A\leftarrow cm.z} + L_{B\leftarrow cm.z} + L_{C\leftarrow cm.z} + L_{D\leftarrow cm.z}\right)\hat{z}$$

Given the symmetry,

$$\vec{L}_{total\leftarrow cm} = 4rmr_\perp\omega\sin\theta\hat{z}$$

but $r_\perp = r\sin\theta$

$$\vec{L}_{total\leftarrow cm} = 4mr_\perp^2\omega\hat{z} = I_{axis}\vec{\omega}$$

Generally, it's the moment of inertia about the *rotational axis* through cm

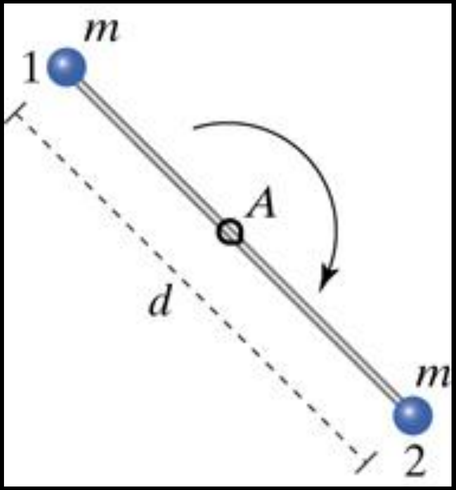
Rotational Angular Momentum and Rotational Energy

		Analogous to
Recall	$K_{rot} = \frac{1}{2} I\omega^2$	$K = \frac{1}{2} mv^2$
now	$\vec{L}_{rot} = I\vec{\omega}$	$\vec{p} = m\vec{v}$
so	$K_{rot} = \dfrac{L^2}{2I}$	$K = \dfrac{p^2}{2m}$

Mon. Tues.	11.2-.3, (.12) Rotational + Translational	RE 11.b EP10
Mon. Tues. Wed. Lab Fri.	11.4-.6, (.13) Angular Momentum & Torque 11.7 - .9, (.11) Torque L11 Rotation Course Evals 11.10 Quantization, Quiz 11	RE 11.c EP11 RE 11.d RE 11.e
Mon.	Review for Final (1-11)	HW11: Ch 11 Pr's 39, 57, 64, 74, 78 & Practice Exam

Rotational Angular Momentum and Kinetic Energy
Special case: rigid body

Example: A barbell spins around a pivot at its center at *A*. The barbell consists of two small balls, each with mass 500 grams (0.5 kg), at the ends of a very low mass rod of length 50 cm (0.5 m). The barbell spins clockwise with angular speed ω = 120 radians/s.



a) What is the moment of inertia about A?
b) What is the direction of the angular velocity?
c) What is the translational angular momentum?
d) What is the rotational angular momentum?
e) What is the total angular momentum?
f) What is the rotational kinetic energy?