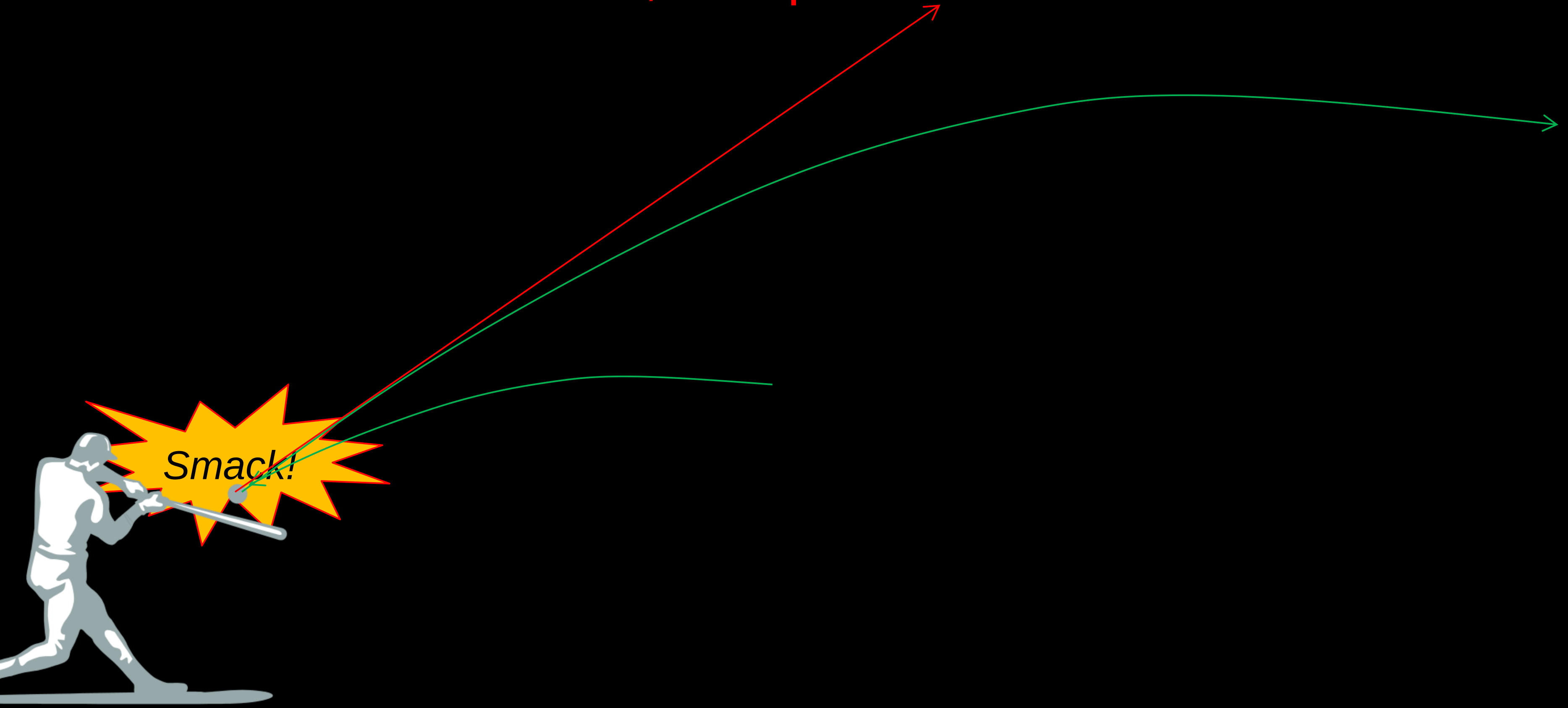
Mon.,	10.9-.10 Collision Complications	RE 10.c
Lab	L10 Collisions 1	EP9
Wed.,	10.5, .11 Different Reference Frames	RE 10.c
Fri.,	1.1 Translational Angular Momentum Quiz 10	RE 11.a; HW10: Pr's 13*, 21, 30, 35, "39"

* For *each part* of these problems, be very careful about what you choose as the system and what you are using as initial and final states.

Collisions

Short, Sharp Shocks

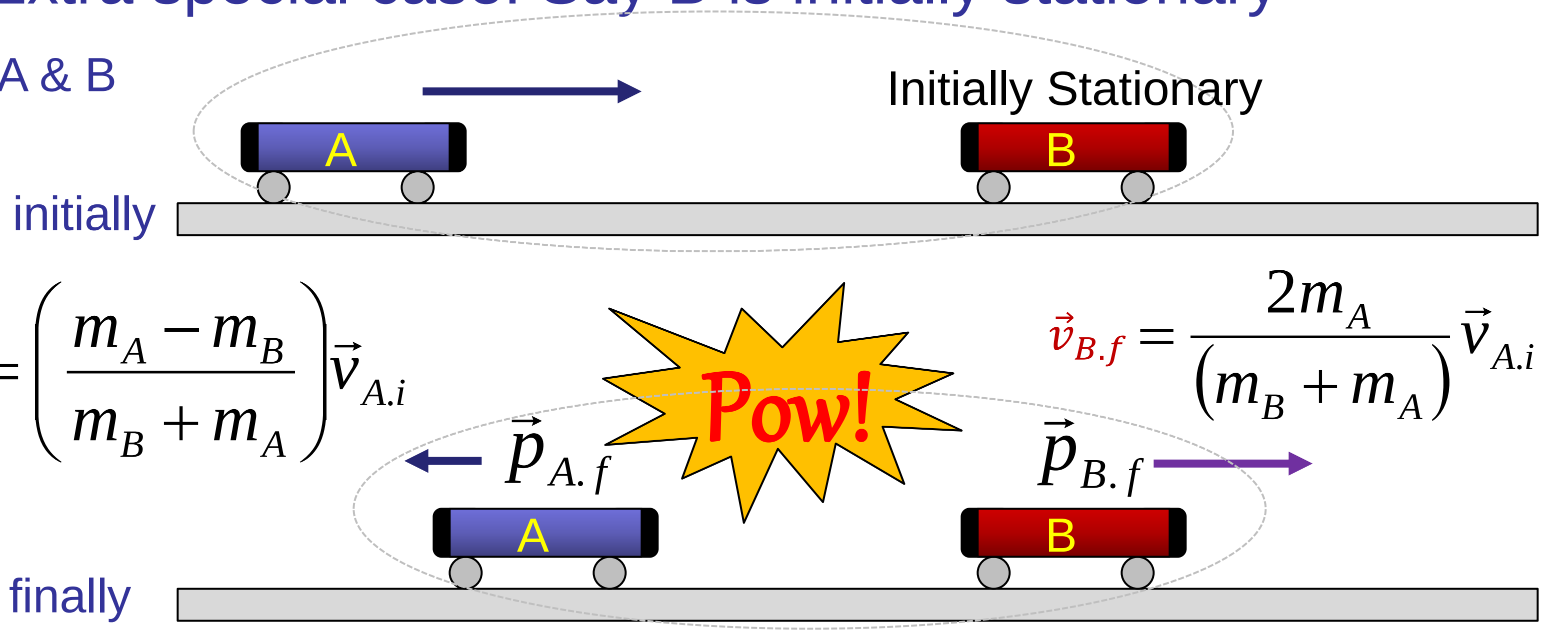


1-D Collision

Special Case: Perfectly Elastic (all internal changes ‘bounce back’)

Extra special case: Say B is initially stationary

System = carts A & B



$$\vec{v}_{A.f} = \left(\frac{m_A - m_B}{m_B + m_A}\right)\vec{v}_{A.i}$$

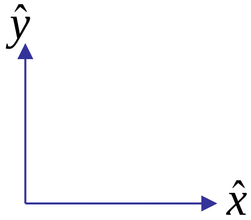
$$\vec{p}_{A.f} \qquad \vec{p}_{B.f}$$

$$\vec{v}_{B.f} = \frac{2m_A}{(m_B + m_A)}\vec{v}_{A.i}$$

Say cart A is *much* more massive than cart B. In that case, cart B comes away with…

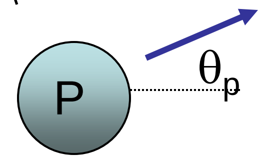
1) a lot higher speed than does cart A
2) about twice the speed of cart A
3) about the same speed as cart A
4) about half the speed of cart A
5) a lot less speed than cart A

2-D Collision: Scattering

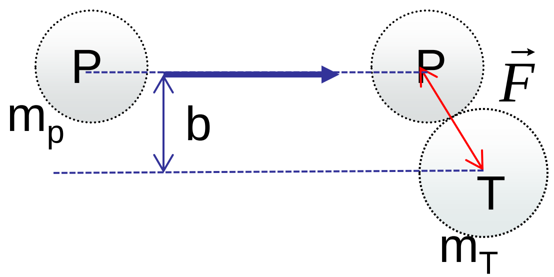


Slow (v<<C), Elastic Collision

$$\vec{p}_{p.f} = \langle p_{p.f}\cos\theta_p, p_{p.f}\sin\theta_p, 0\rangle$$



$$\vec{p}_{p.i} = \langle p_{p.i}, 0, 0\rangle$$



$$\vec{F} = \frac{d\vec{p}}{dt}$$

$$\vec{p}_{T.i} = 0$$

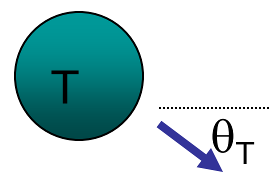
***Three* Equations / *Four* Unknowns**

Need another equation

Collision geometry and Impact Parameter

Component of momentum along line of contact changes, other component remains constant

b = Impact Parameter



$$\vec{p}_{T.f} = \langle p_{T.f}\cos\theta_T, -p_{T.f}|\sin\theta_T|, 0\rangle$$

Momentum Principle

$$\vec{p}_{p.f} + \vec{p}_{T.f} - \vec{p}_{p.i} = 0$$

$$\hat{x}: p_{p.f}\cos\theta_p + p_{T.f}\cos\theta_T - p_{p.i} = 0$$

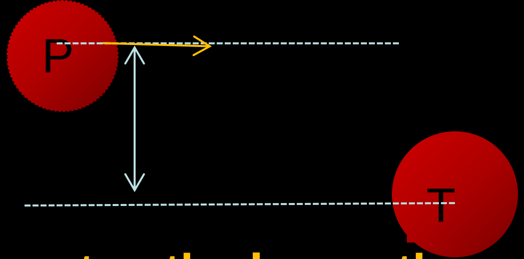
$$\hat{y}: \underline{p_{p.f}}\sin\underline{\theta_p} - \underline{p_{T.f}}|\sin\underline{\theta_T}| - 0 = 0$$

Energy Principle – *if Elastic*

$$(E_{p.f} + E_{T.f}) - (E_{p.i} + E_{T.i}) = 0$$

$$K_{p.f} + K_{T.f} - K_{p.i} = 0$$

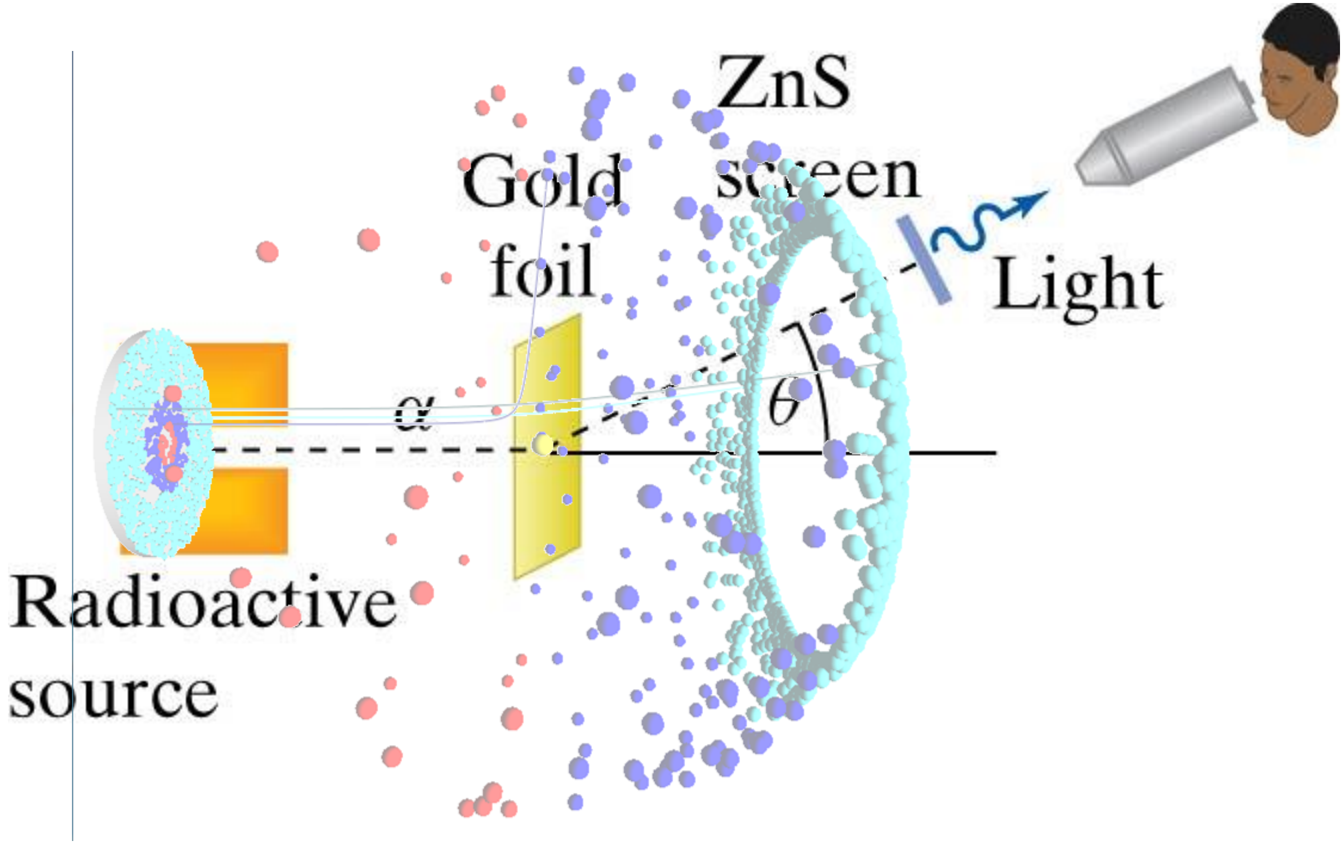
$$\frac{p_{p.f}^2}{2m_p} + \frac{p_{T.f}^2}{2m_T} - \frac{p_{p.i}^2}{2m_p} = 0$$



Which of these is true?

1) The larger the impact parameter, the larger the scattering angle (deflection).
2) The larger the impact parameter, the smaller the scattering angle (deflection).

2-D Collision: Scattering – Discovering Nucleus

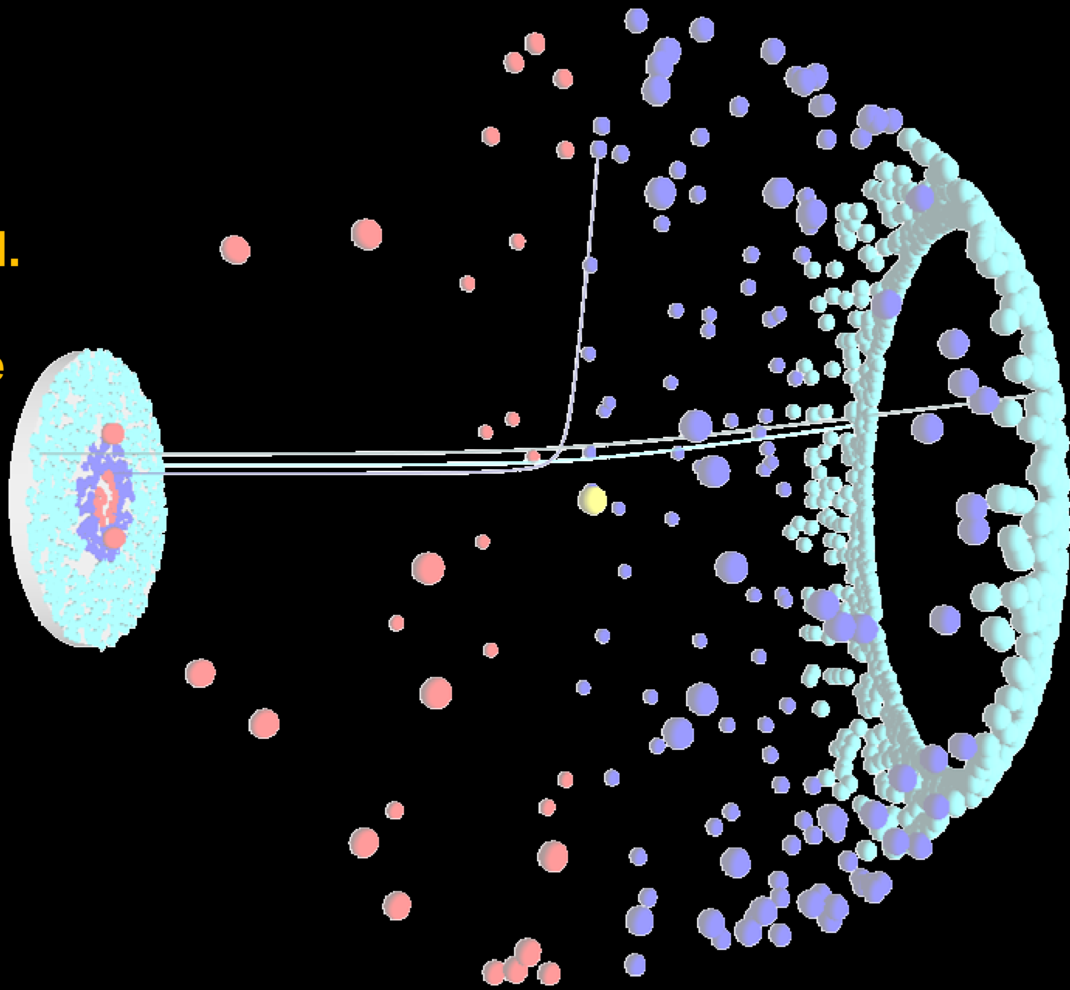


Prior to the Rutherford experiment (shooting alpha particles at a thin gold foil), the atom's positive charge was thought to be distributed through out the atom rather than concentrated in a small nucleus. So, what aspect of Rutherford's results was surprising to the experimenters?

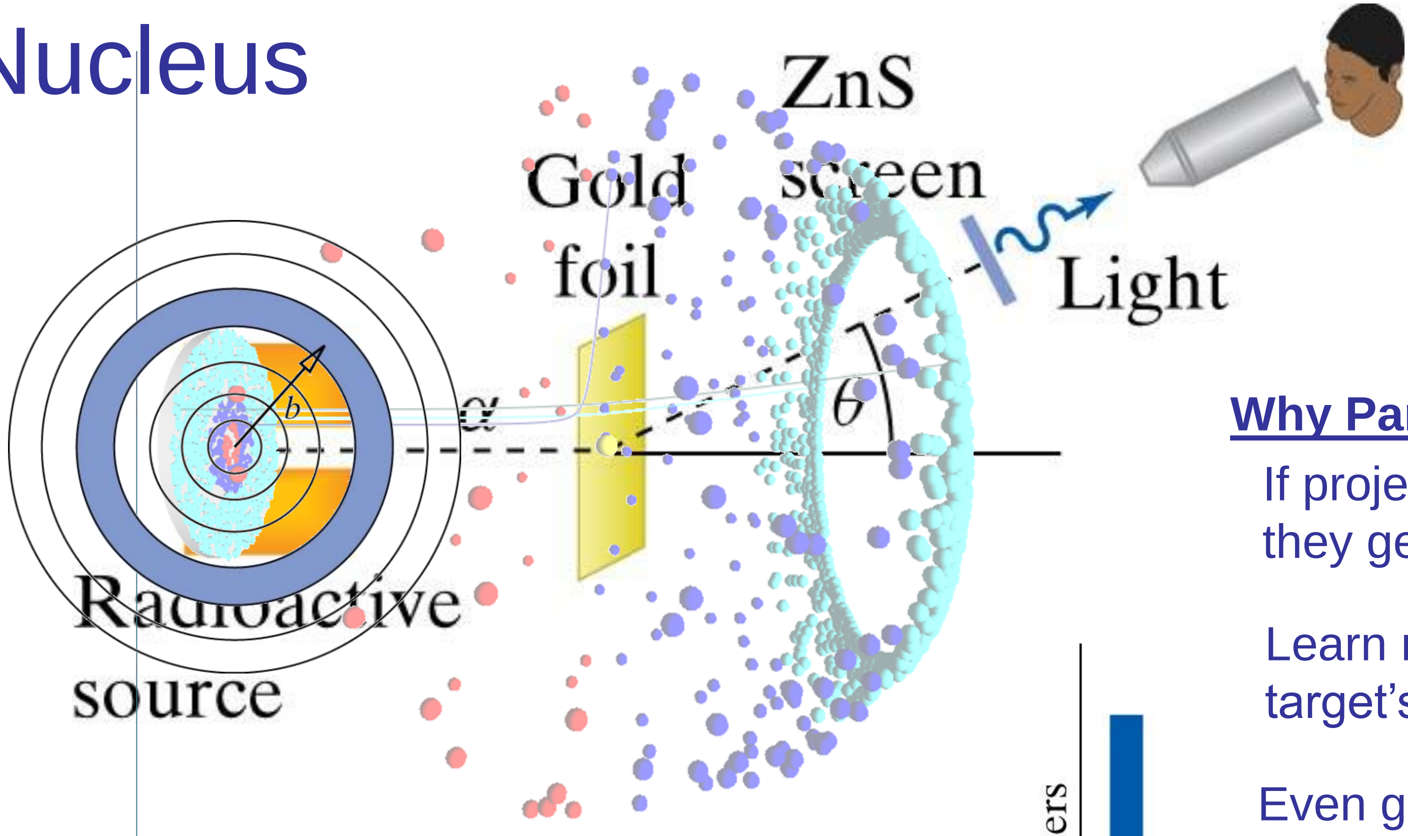
(1) Sometimes the alpha "rays" passed right through the gold foil.

(1) Sometimes the alpha "rays" were deflected slightly when they passed through the gold foil.

(1) Sometimes the alpha "rays" bounced back from the gold foil.



2-D Collision: Scattering – Discovering Nucleus

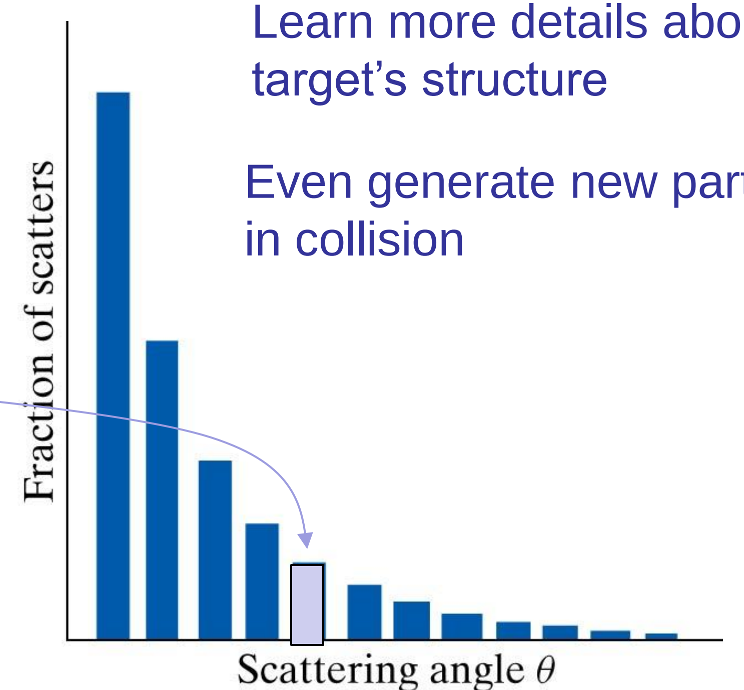


Why Particle Accelerators

If projectiles start **faster**, they get closer

Learn more details about target's structure

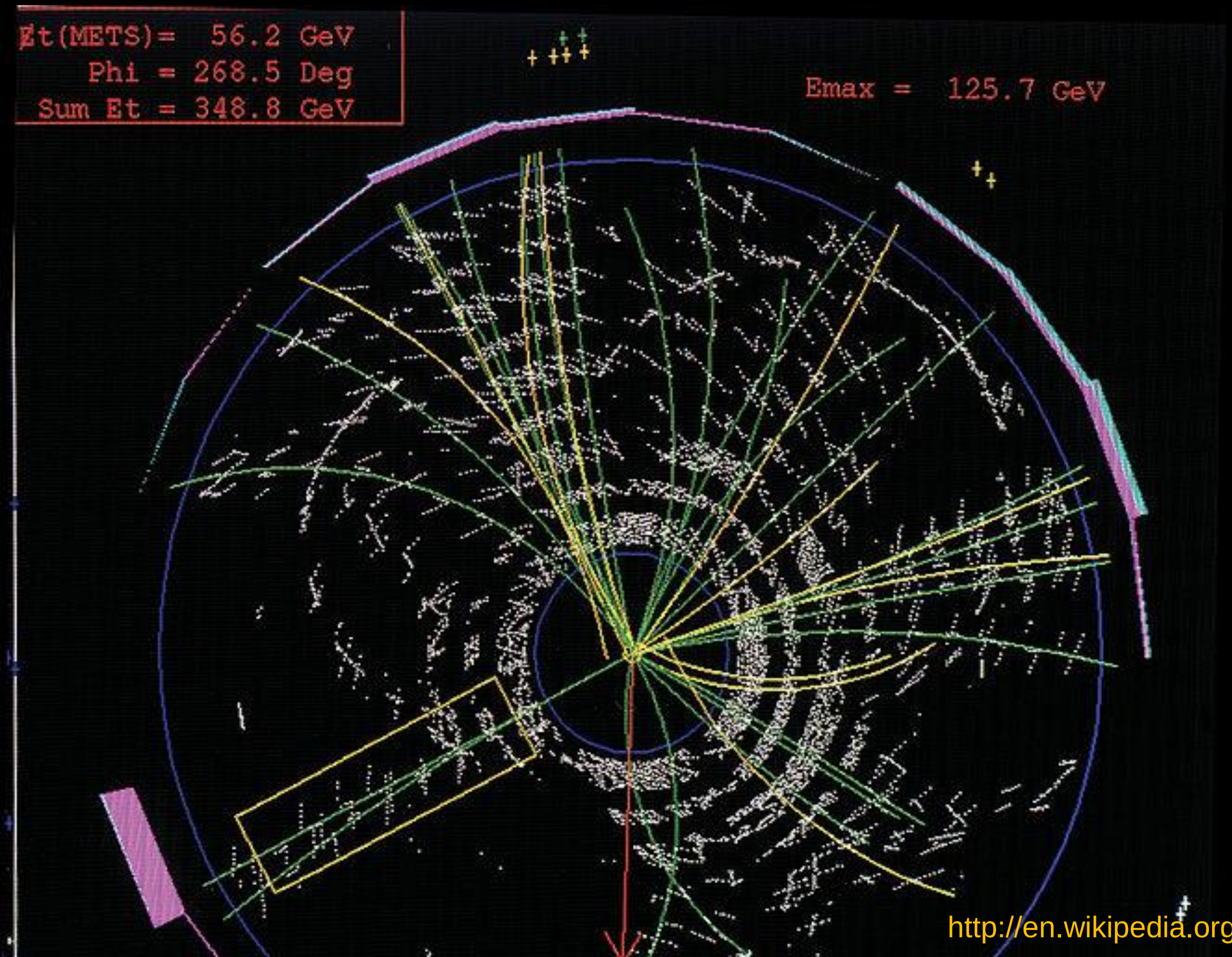
Even generate new particles in collision



Will simulate in Lab and explore probability / scattering-angle relation

Deeper Collisions

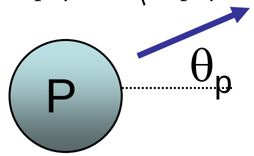
- relativistic speeds
- identifying 'mystery' particles
- the mathematical trick of analyzing in the center-of-mass reference frame



http://en.wikipedia.org/wiki/File:CDF_Top_Event.jpg

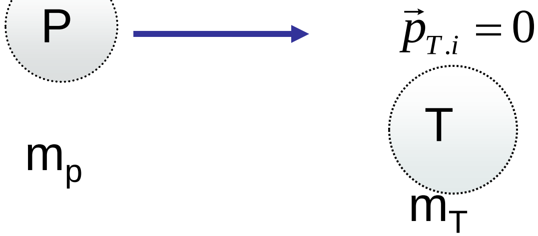
2-D Collision: Scattering Fast

$$\vec{p}_{p.f} = \langle p_{p.f}\cos\theta_p, p_{p.f}\sin\theta_p, 0\rangle$$



$$\vec{p}_{p.i} = \langle p_{p.i}, 0, 0\rangle$$

$$\vec{p}_{T.i} = 0$$



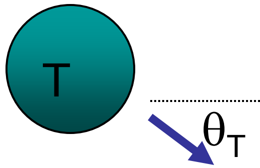
Conservation of Momentum

$$\vec{p}_{p.f} + \vec{p}_{T.f} - \vec{p}_{p.i} = 0$$

$$\hat{x}: p_{p.f}\cos\theta_p + p_{T.f}\cos\theta_T - p_{p.i} = 0$$

$$\hat{y}: p_{p.f}\sin\theta_p - p_{T.f}\left|\sin\theta_T\right| - 0 = 0$$

where $\vec{p} = \dfrac{m\vec{v}}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$



$$\vec{p}_{T.f} = \langle p_{T.f}\cos\theta_T, -p_{T.f}\left|\sin\theta_T\right|, 0\rangle$$

show $E = \sqrt{(pc)^2 + \left(mc^2\right)^2}$

By plugging that into it and recovering that.

Conservation of Energy

$$\left(E_{p.f} + E_{T.f}\right) - \left(E_{p.i} + E_{T.i}\right) = 0$$

where $E = \dfrac{mc^2}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$

$$\sqrt{\left(p_{p.f}c\right)^2 + \left(m_{p.f}c^2\right)^2} + \sqrt{\left(p_{T.f}c\right)^2 + \left(m_{T.f}c^2\right)^2} - \sqrt{\left(p_{p.i}c\right)^2 + \left(m_{p.i}c^2\right)^2} - m_{T.i}c^2 = 0$$

Note: initial and final masses differ for inelastic collisions

2-D Collision: Scattering Fast

Mass, Elastic & Inelastic

Recall particle energy: $E = K + E_{\text{int}}$

$$E_{\text{int}} = mc^2$$

Elastic: $\Delta E_{\text{int}} = \Delta mc^2 = 0$

$$\sqrt{(p_{p.f}c)^2 + (m_p c^2)^2} + \sqrt{(p_{T.f}c)^2 + (m_T c^2)^2} - \sqrt{(p_{p.i}c)^2 + (m_p c^2)^2} - m_T c^2 = 0$$

In-Elastic: $\Delta E_{\text{int}} = \Delta mc^2 \neq 0$

$$\sqrt{(p_{p.f}c)^2 + (m_{p.f} c^2)^2} + \sqrt{(p_{T.f}c)^2 + (m_{T.f} c^2)^2} - \sqrt{(p_{p.i}c)^2 + (m_{p.i} c^2)^2} - m_{T.i} c^2 = 0$$

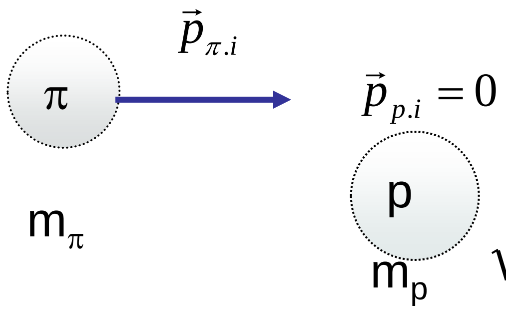
Or even have more or fewer particles finally than initially

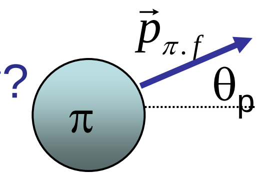
2-D Collision: Fast

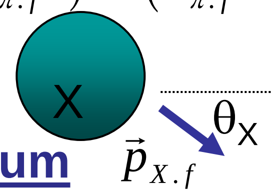
Example: A beam of high energy π^- (negative pions) is shot at a flask of liquid hydrogen, and sometimes a pion interacts through the strong interaction with a proton in the hydrogen, the reaction is $\pi^- + p^+ \rightarrow \pi^- + X^+$ where X^+ is a positively charged particle of unknown mass (a proton containing reoriented quarks.)

A proton's rest mass is 938MeV, and a pion's rest mass is 140 MeV. The incoming pion has momentum 3GeV/c. It scatters through 40°, and its momentum drops to 1.510 GeV/c.

What is the rest mass of the X^+?







Conservation of Energy

$$\left(E_{p.f} + E_{T.f}\right) - \left(E_{p.i} + E_{T.i}\right) = 0$$

$$\sqrt{\left(p_{\pi.f}c\right)^2 + \left(m_{\pi.f}c^2\right)^2} + \sqrt{\left(p_X c\right)^2 + \left(m_X c^2\right)^2} - \sqrt{\left(p_{\pi.i}c\right)^2 + \left(m_\pi c^2\right)^2} - m_{p.i}c^2 = 0$$

Conservation of Momentum

$$\hat{x}: p_{\pi.f}\cos\theta_\pi + p_{X.f}\cos\theta_X - p_{\pi.i} = 0 \qquad p_{X.f}\cos\theta_X = p_{\pi.i} - p_{\pi.f}\cos\theta_\pi$$

$$\hat{y}: p_{\pi.f}\sin\theta_\pi - p_{X.f}\left|\sin\theta_X\right| = 0 \qquad p_{X.f}\left|\sin\theta_X\right| = p_{\pi.f}\sin\theta_\pi$$

Cancel angle dependence using $(\cos\theta_X)^2 + (\sin\theta_X)^2 = 1$

$$\left(p_{X.f}\cos\theta_X\right)^2 + \left(p_{X.f}\left|\sin\theta_X\right|\right)^2 = \left(p_{\pi.i} - p_{\pi.f}\cos\theta_\pi\right)^2 + \left(p_{\pi.f}\sin\theta_\pi\right)^2$$

$$p_{X.f} = \sqrt{\left(p_{\pi.i} - p_{\pi.f}\cos\theta_\pi\right)^2 + \left(p_{\pi.f}\sin\theta_\pi\right)^2} = \sqrt{\left(p_{\pi.i}\right)^2 + \left(p_{\pi.f}\right)^2 - 2p_{\pi.i}p_{\pi.f}\cos\theta_\pi}$$

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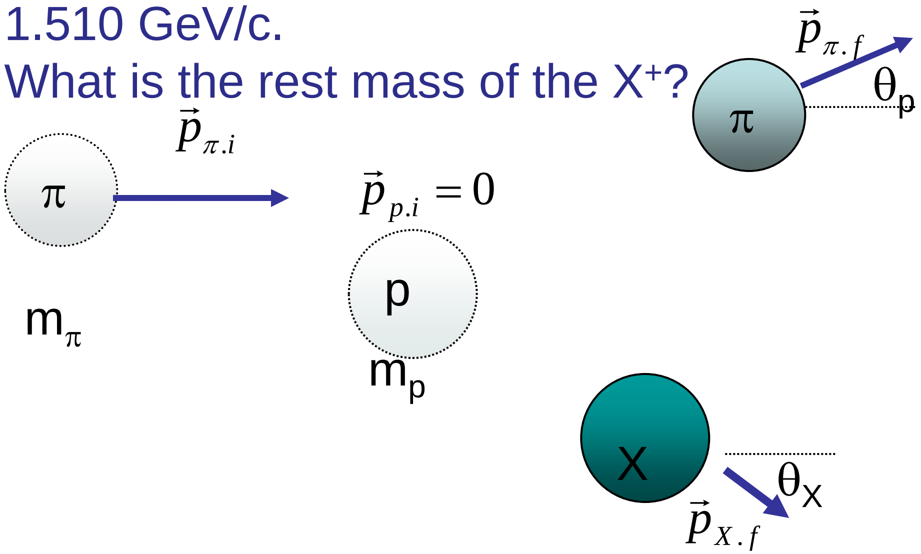


Conservation of Momentum

$$p_{X.f} = \sqrt{\left(p_{\pi.i}\right)^2 + \left(p_{\pi.f}\right)^2 - 2p_{\pi.i}p_{\pi.f}\cos\theta_\pi}$$

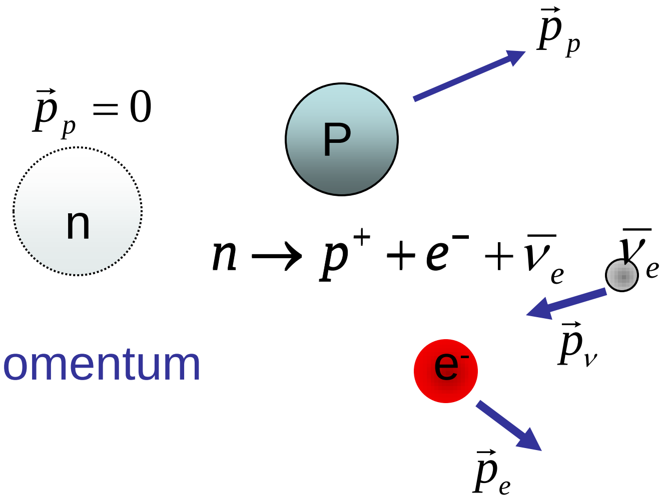
Conservation of Energy

$$\sqrt{\left(p_{\pi.f}c\right)^2 + \left(m_{\pi.f}c^2\right)^2} + \sqrt{\left(p_Xc\right)^2 + \left(m_Xc^2\right)^2} - \sqrt{\left(p_{\pi.i}c\right)^2 + \left(m_\pi c^2\right)^2} - m_{p.i}c^2 = 0$$

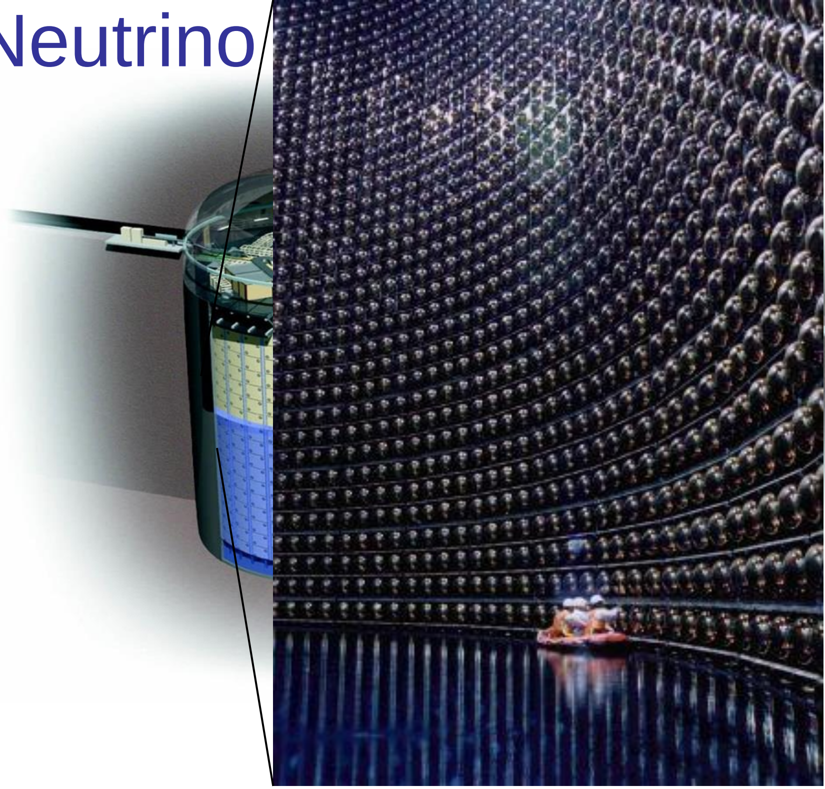
$$\sqrt{\left(p_Xc\right)^2 + \left(m_Xc^2\right)^2} = \sqrt{\left(p_{\pi.i}c\right)^2 + \left(m_\pi c^2\right)^2} + m_{p.i}c^2 - \sqrt{\left(p_{\pi.f}c\right)^2 + \left(m_{\pi.f}c^2\right)^2}$$

Could do algebraically further, or just plugin numbers and simplify

Deducing the invisible particle: Neutrino



$$n \rightarrow p^+ + e^- + \bar{\nu}_e$$



Conservation of Momentum

$$\vec{p}_p + \vec{p}_e \neq 0$$

$$\vec{p}_p + \vec{p}_e + \vec{p}_\nu = 0$$

Must be another particle with missing energy & momentum

Charge is conserved and detectors can't see it – must be neutral

Sometimes energy and momentum *almost* conserved – must be nearly massless

Conservation of Energy

$$m_n c^2 \neq \sqrt{(p_e c)^2 + (m_e c^2)^2} + \sqrt{(p_p c)^2 + (m_p c^2)^2}$$

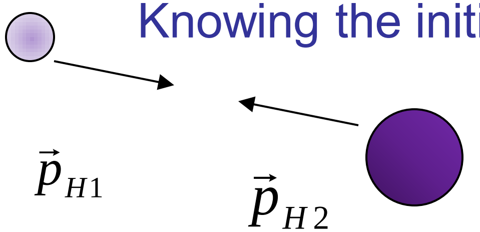
$$m_n c^2 = \sqrt{(p_e c)^2 + (m_e c^2)^2} + \sqrt{(p_p c)^2 + (m_p c^2)^2} + \sqrt{(p_\nu c)^2 + (m_\nu c^2)^2}$$

Center of Mass & Collisions
High Speeds

$$^1H + ^2H \rightarrow ^3He^* \rightarrow ^3He + \gamma$$

"gamma ray" = High energy photon

Knowing the initial momenta and masses, what's the mass of the excited He?



Stage 1

Conservation of Momentum:

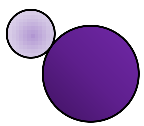
$$\vec{p}_{H1} + \vec{p}_{H2} = 0 \text{ in center of mass frame}$$

$$\vec{p}_{H1} = -\vec{p}_{H2}$$

Conservation of Energy:

$$E = \sqrt{(p_{H1}c)^2 + (m_{H1}c^2)^2} + \sqrt{(p_{H2}c)^2 + (m_{H2}c^2)^2}$$

$$E = \sqrt{(p_{H1}c)^2 + (m_{H1}c^2)^2} + \sqrt{(p_{H1}c)^2 + (m_{H2}c^2)^2}$$

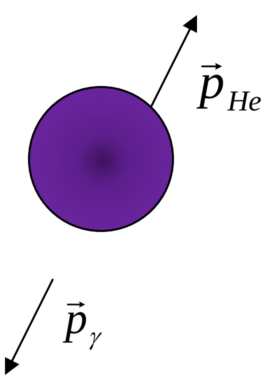


Stage 2

$$E = m_{He^*}c^2 \quad \text{or} \quad \boxed{m_{He^*} = E / c^2}$$

What's the photon's momentum?

$$\vec{p}_{He} + \vec{p}_{\gamma} = 0$$



Stage 3

$$E = \sqrt{(p_{He}c)^2 + (m_{He}c^2)^2} + \sqrt{(p_{\gamma}c)^2 + (m_{\gamma}c^2)^2}$$

photon is massless

$$E = \sqrt{(p_{\gamma}c)^2 + (m_{He}c^2)^2} + p_{\gamma}c$$

$$(E - p_{\gamma}c)^2 = (p_{\gamma}c)^2 + (m_{He}c^2)^2$$

$$E^2 - 2Ep_{\gamma}c + (p_{\gamma}c)^2 = (p_{\gamma}c)^2 + (m_{He}c^2)^2$$

$$\boxed{\frac{E^2 - (m_{He}c^2)^2}{2Ec} = p_{\gamma}}$$

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