ed. ab i.	10.1-.4 Introducing Collisions Quiz 9 L9 Multi-particle Systems 10.6-.8 Scattering	RE 10.a RE 10.b
on., ab ed., i.,	10.9-.10 Collision Complications L10 Collisions 1 10.5, .11 Different Reference Frames 1.1 Translational Angular Momentum Quiz 10	RE 10.c EP9 RE 11.a; HW10: Pr’s 13*, 21, 30, “39

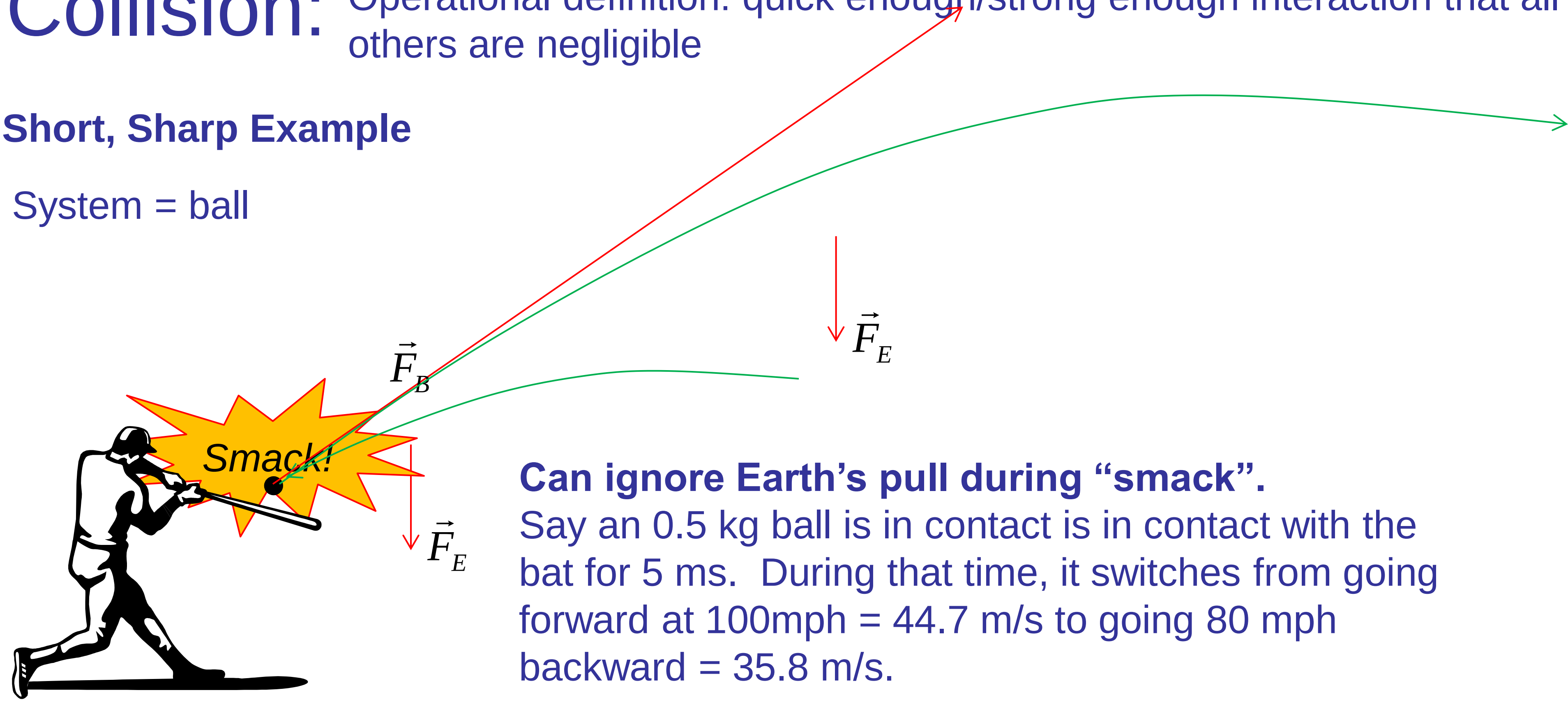
Collisions!

Short, Sharp Shocks

Collision: Operational definition: quick enough/strong enough interaction that all others are negligible

Short, Sharp Example

System = ball



Can ignore Earth's pull during "smack".

Say an 0.5 kg ball is in contact is in contact with the bat for 5 ms. During that time, it switches from going forward at 100mph = 44.7 m/s to going 80 mph backward = 35.8 m/s.

$$\left|F_{net}\right| = \left|\frac{\Delta\vec{p}}{\Delta t}\right| = \left|\frac{0.5kg\cdot(-35.8m/s) - 0.5kg\cdot(44.7m/s)}{0.005s}\right| = 8{,}050N$$

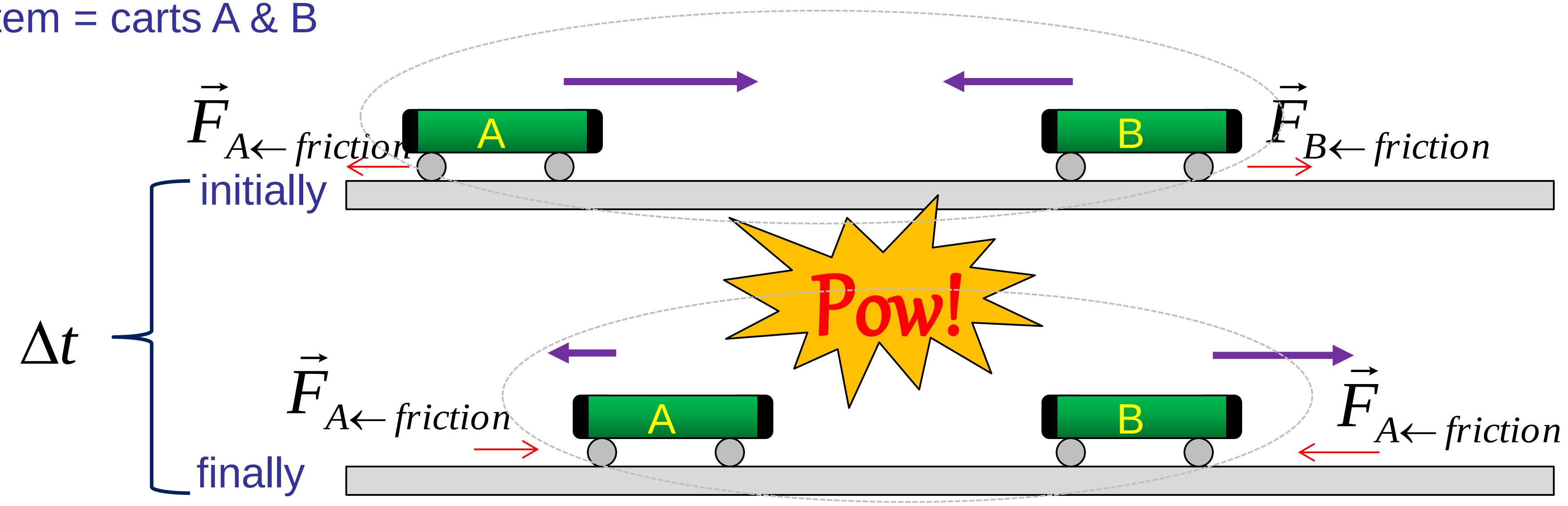
Of that, the Earth's pull accounts for only

$$\left|F_{b\leftarrow E}\right| = mg = (0.5kg)(9.8m/s^2) = 4.9N$$

1-D Collision

Example

System = carts A & B



Often know initial motion, want to predict final motion

Two unknowns: $\vec{v}_{A.f}$ and $\vec{v}_{B.f}$

Generally need *two* equations to solve for them

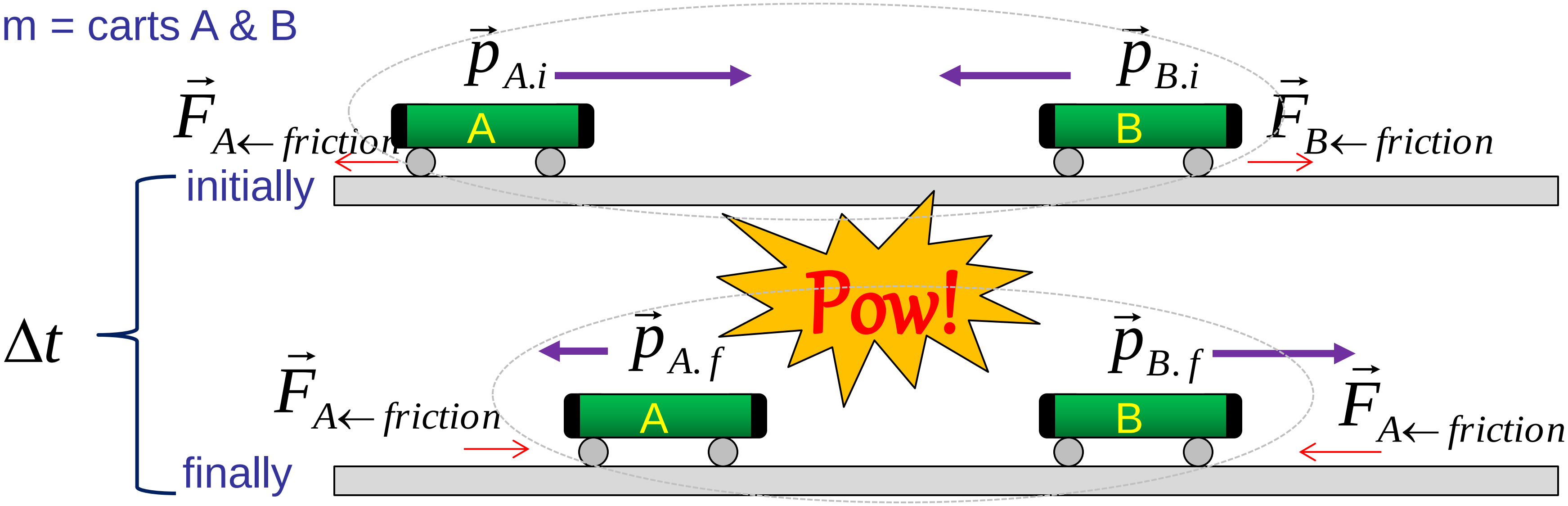
1-D Collision

One of the Equations: Momentum Principle

$$\frac{d\vec{p}_{system}}{dt} = \vec{F}_{net.ext}$$

Example

System = carts A & B

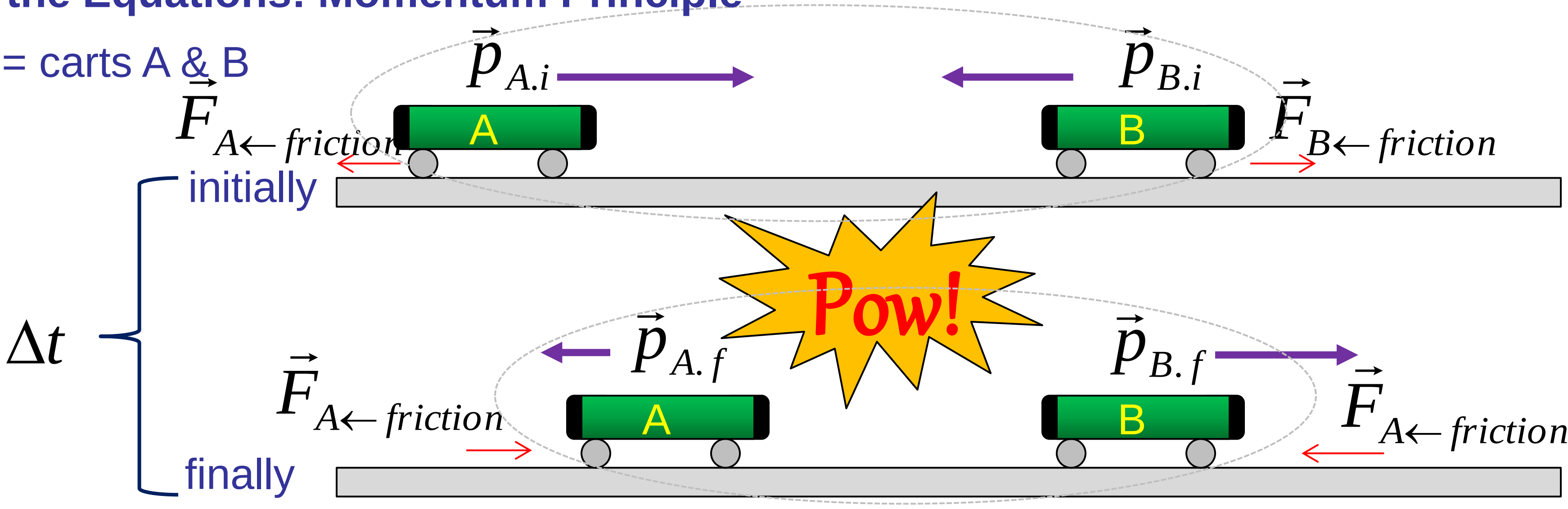


$$\Delta\vec{p}_{system} = \vec{F}_{system\leftarrow net.ext}\Delta t$$

1-D Collision

One of the Equations: Momentum Principle

System = carts A & B



$$\Delta\vec{p}_A + \Delta\vec{p}_B \approx 0 \qquad (v's << c)$$

$$\left(m_A\vec{v}_{A.f} - m_A\vec{v}_{A.i}\right) + \left(m_B\vec{v}_{B.f} - m_B\vec{v}_{B.i}\right) \approx 0$$

$$\left(m_A\vec{v}_{A.f} - m_A\vec{v}_{A.i}\right) + \left(m_B\vec{v}_{B.f} - m_B\vec{v}_{B.i}\right) \approx 0$$

A space satellite of mass 500 kg has velocity < 12, 0, –8 > m/s just before being struck by a rock of mass 3 kg with velocity < –3000, 0, 900 > m/s.

After the collision the rock's velocity is < 700, 0, –300 > m/s. Now what is the velocity of the space satellite?

a. < –5100, 0, –400 > m/s

b. < –10.2, 0, –0.8 > m/s

c. < 10.2, 0, 0.8 > m/s

d. < –3688, 0, 1191 > m/s

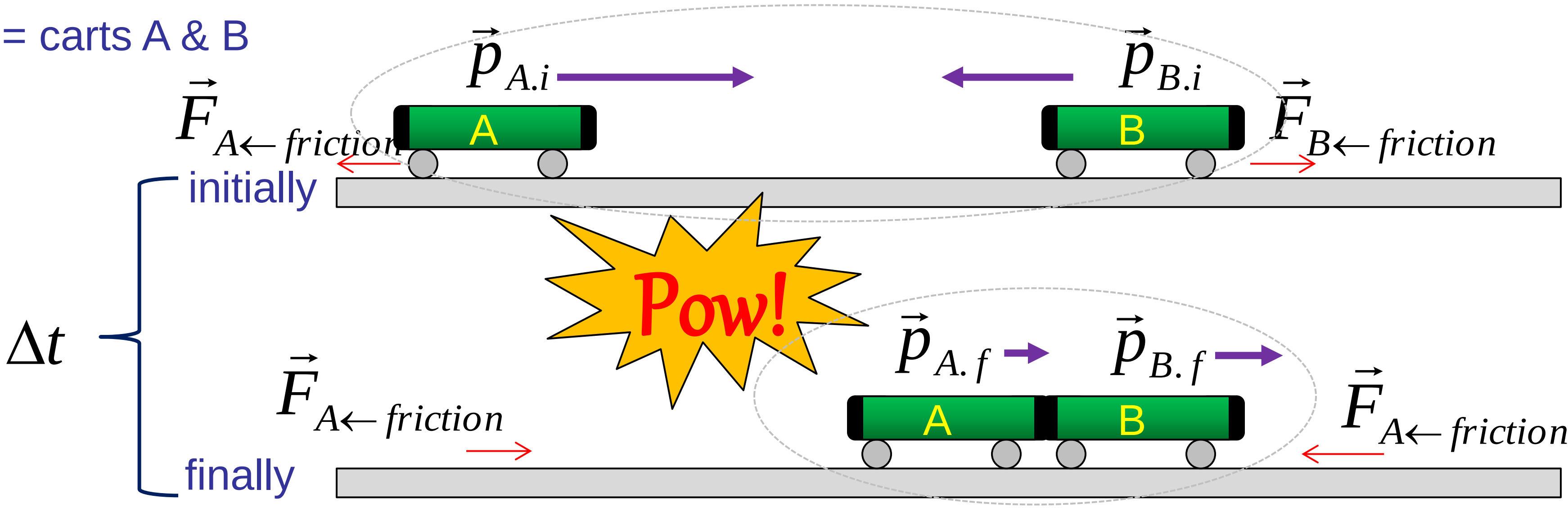
e. < 3688, 0, –1192 > m/s

1-D Collision

One of the Equations: Momentum Principle

Special Case: “Maximally Inelastic” – hit & stick

System = carts A & B



$$\Delta\vec{p}_A + \Delta\vec{p}_B \approx 0 \qquad (v's << c)$$

$$\underbrace{\left(m_A\vec{v}_{A.f} - m_A\vec{v}_{A.i}\right)}_{\Delta\vec{p}_A} + \underbrace{\left(m_B\vec{v}_{B.f} - m_B\vec{v}_{B.i}\right)}_{\Delta\vec{p}_B} \approx 0$$

Other Equations: for “Maximally Inelastic”

$$\vec{v}_{Af} = \vec{v}_{Bf} \equiv \vec{v}_f$$

A squishy clay ball collides in midair with a baseball, and sticks to the baseball, which keeps going.

Initial momenta:

$\vec{p}_{1\_CLAY}$ and $\vec{p}_{1\_BALL}$

Final momentum of clay+ball :

$\vec{p}_2$

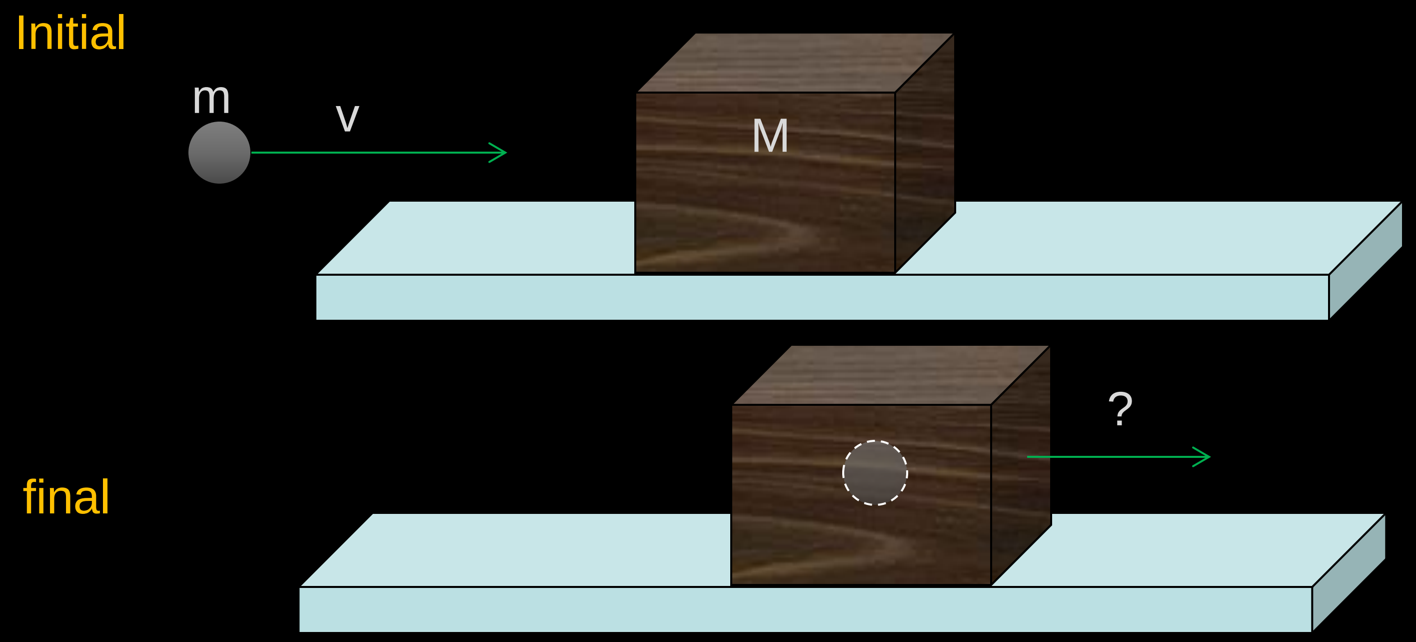
Which equation correctly describes this collision?

1) $\vec{p}_2 = \vec{p}_{1\_CLAY} + \vec{p}_{1\_BALL}$

2) $\vec{p}_2 > \vec{p}_{1\_CLAY} + \vec{p}_{1\_BALL}$

3) $\vec{p}_2 < \vec{p}_{1\_CLAY} + \vec{p}_{1\_BALL}$

Maximally Inelastic



A bullet of mass *m* traveling horizontally at a very high speed *v* embeds itself in a block of mass *M* that is sitting at rest on a nearly frictionless surface. What is the speed of the block just after the bullet embeds itself in the block?

(1) v

(2) $\dfrac{m}{M}v$

(3) $\sqrt{\dfrac{m}{M+m}}v$

(4) $\dfrac{m}{M+m}v$

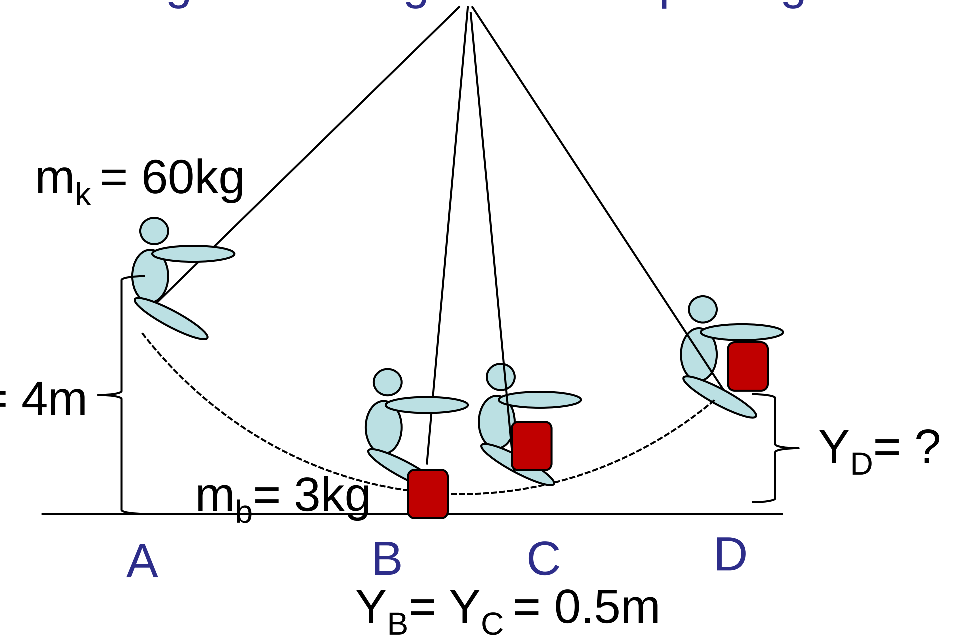
(5) $\dfrac{M+m}{m}v$

1-D Collision

Special Case: “Maximally Inelastic” – hit & stick

Multi-step Example: A 60kg kid’s on a swing, say she’s gotten herself going so that at her highest her center of mass is 4m above the ground and when she swoops down her center of mass is just 0.5 m above the ground. On her down-sweep, she reaches down and picks up her 3kg backpack that’s sitting on the ground (maybe her cell phone started ringing).

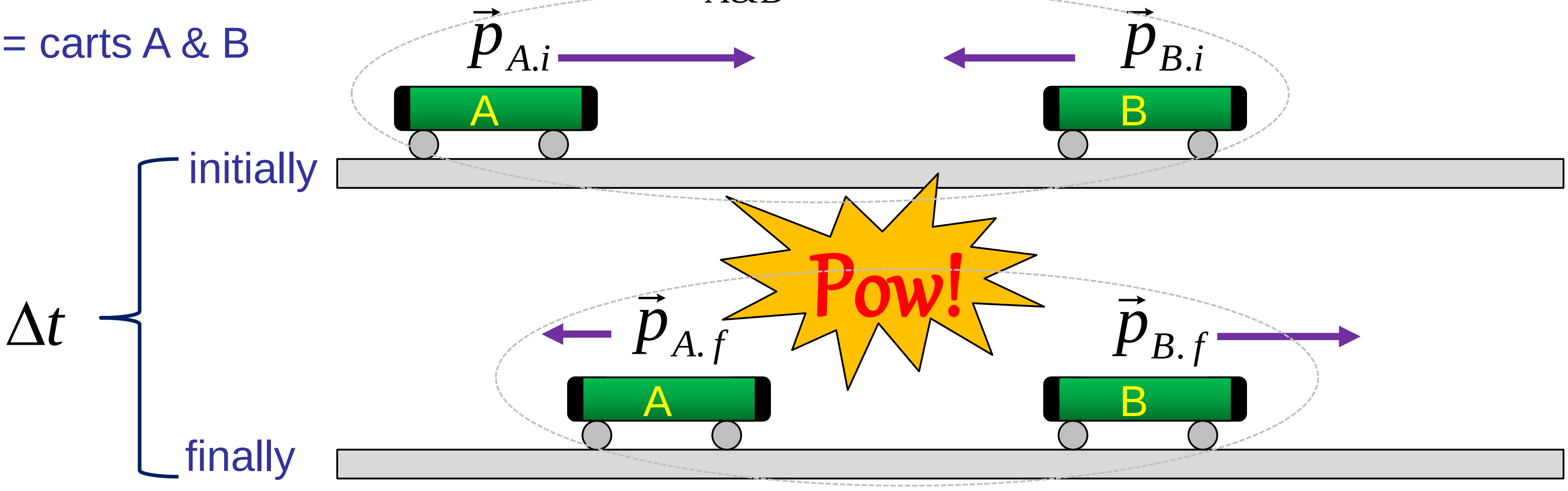
How high will she get on her upswing with the pack in her lap?



1-D Collision

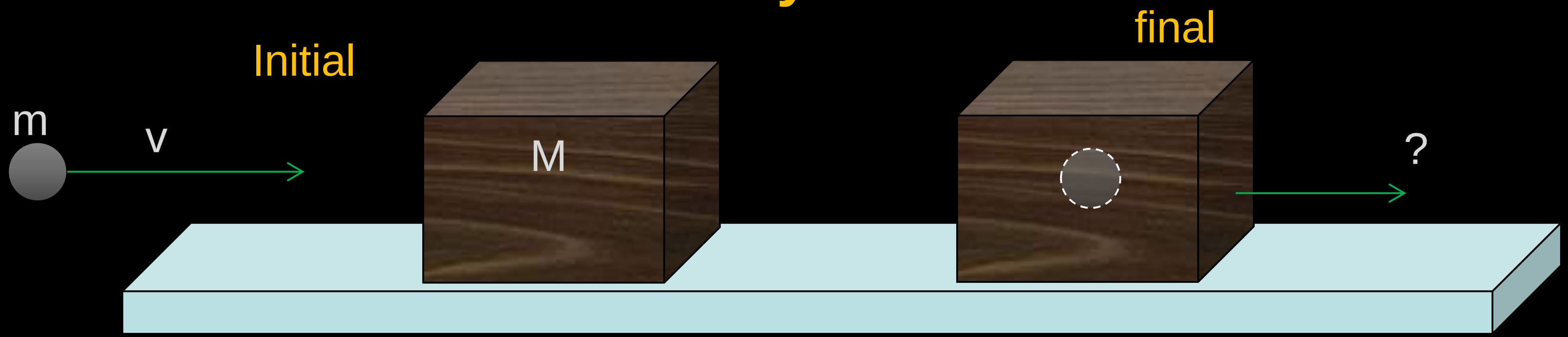
Second Equations: Energy Principle $\Delta E_{A\&B} \approx 0$

System = carts A & B



$$\Delta E_{A\&B} = \Delta K_A + \Delta E_{A.\text{int}} + \Delta K_B + \Delta E_{B.\text{int}} + \Delta U_{A\&B}$$

Maximally Inelastic



Which is an accurate energy equation for this collision for the system of bullet + block?

(1) $$\frac{1}{2}(M+m)v_f^2 = \frac{1}{2}mv_i^{\,2}$$

(2) $$\frac{1}{2}Mv_f^2 = \frac{1}{2}mv_i^{\,2}$$

(3) $$\frac{1}{2}(M+m)v_f^2 + \Delta E_{\text{internal}} = \frac{1}{2}mv_i^{\,2}$$

(4) $$\Delta E_{\text{internal}} = \frac{1}{2}mv_i^{\,2}$$

(5) $$\frac{1}{2}(M+m)v_f^2 = \frac{1}{2}mv_i^{\,2} + \Delta E_{\text{internal}}$$

A squishy clay ball collides in midair with a baseball, and sticks to the baseball, which keeps going.

Initial kinetic energies:

K_{1clay}, $K_{1baseball}$

Final kinetic energy of clay+ball :

Which equation correctly describes this collision?

final initial

1) $K_{clay+ball} = K_{1clay} + K_{1baseball}$

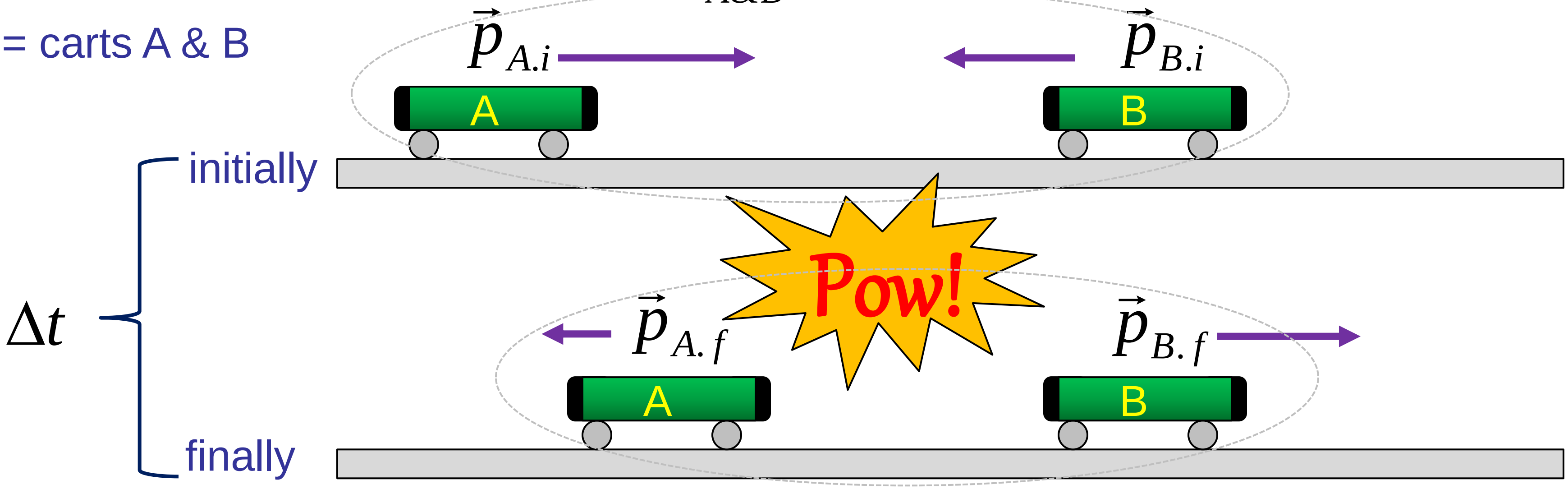
2) $K_{clay+ball} > K_{1clay} + K_{1baseball}$

3) $K_{clay+ball} < K_{1clay} + K_{1baseball}$

1-D Collision

Second Equations: Energy Principle $\Delta E_{A\&B} \approx 0$

System = carts A & B



$$\Delta E_{A\&B} = \Delta K_A + \Delta E_{A.\text{int}} + \Delta K_B + \Delta E_{B.\text{int}} + \Delta U_{A\&B}$$

Special Case: Perfectly Elastic (all internal changes 'bounce back')

$$\Delta E_{A\&B} = \Delta K_A + \cancel{\Delta E_{A.\text{int}}} + \Delta K_B + \cancel{\Delta E_{B.\text{int}}} + \cancel{\Delta U_{A\&B}}$$

$(v's << c)$

$$\left(\tfrac{1}{2} m_A v_{A.f}^2 - \tfrac{1}{2} m_A v_{A.i}^2\right) + \left(\tfrac{1}{2} m_B v_{B.f}^2 - \tfrac{1}{2} m_B v_{B.i}^2\right) \approx 0$$

$$\left(\frac{p_{A.f}^2}{2m_A} - \frac{p_{A.i}^2}{2m_A}\right) + \left(\frac{p_{B.f}^2}{2m_B} - \frac{p_{B.i}^2}{2m_B}\right) \approx 0$$

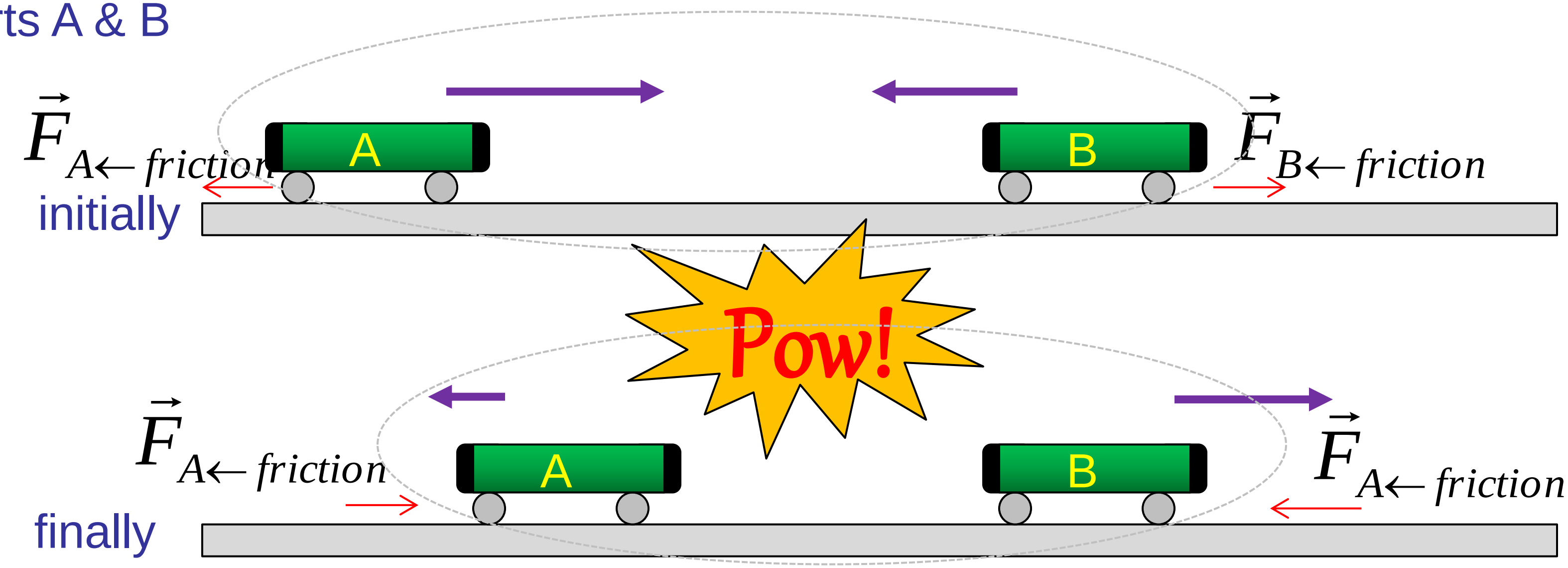
Which of the following is a property of all “elastic” collisions?

(1) The colliding objects interact through springs.
(2) The kinetic energy of one of the objects doesn’t change.
(3) The total kinetic energy is constant at all times -- before, during, and after the collision.
(4) The total kinetic energy after the collision is equal to the total kinetic energy before the collision.
(5) The elastic spring energy after the collision is greater than the elastic spring energy before the collision.

1-D Collision

Example

System = carts A & B



Often know initial motion, want to predict final motion

Two unknowns: $\vec{v}_{A.f}$ and $\vec{v}_{B.f}$

Generally need *two* equations to solve for them

True for *all* collisions

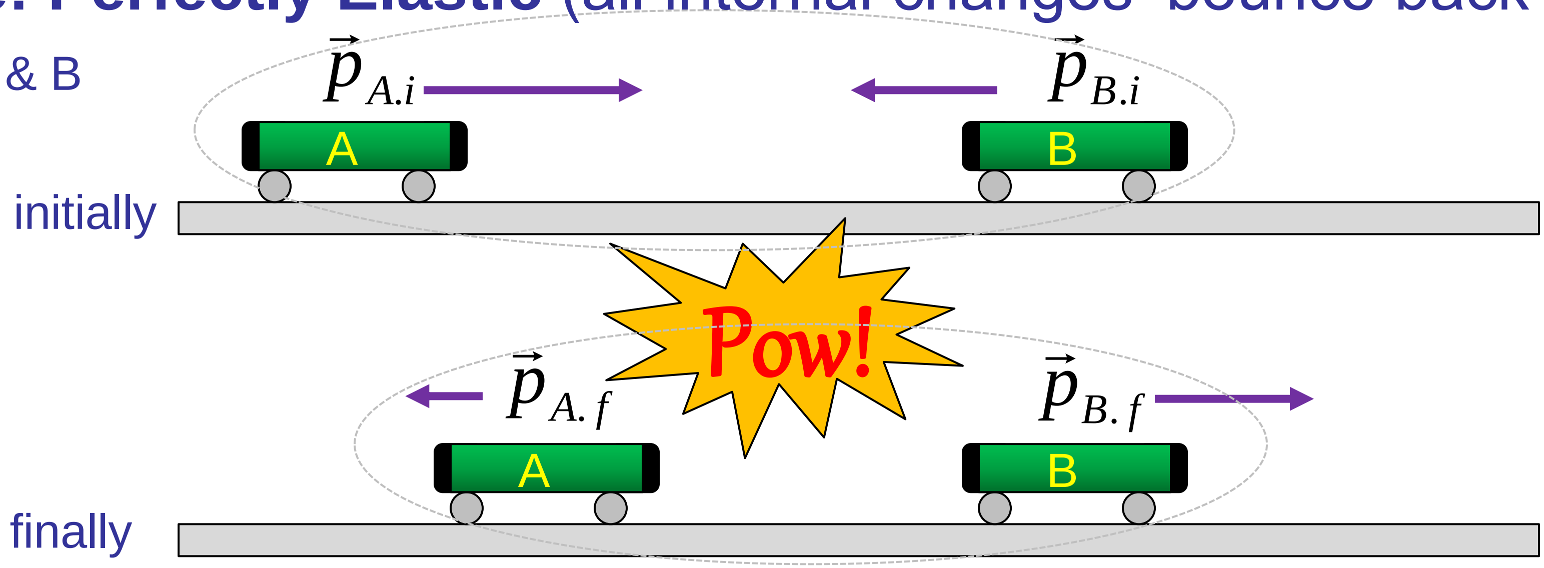
$$\Delta\vec{p}_{A\&B} = \Delta\vec{p}_A + \Delta\vec{p}_B \approx 0$$

$$\Delta E_{A\&B} = \underbrace{\Delta E_A}_{\Delta K_A + \Delta E_{\mathrm{int}.A}} + \underbrace{\Delta E_B}_{\Delta K_B + \Delta E_{\mathrm{int}.B}} + \Delta U_{A\&B} \approx 0$$

1-D Collision

Special Case: Perfectly Elastic (all internal changes ‘bounce back’)

System = carts A & B



$$\Delta E_{A\&B} = \Delta K_A + \cancel{\Delta E_{A.\text{int}}} + \Delta K_B + \cancel{\Delta E_{B.\text{int}}} + \cancel{\Delta U_{A\&B}}$$

$$\left(\tfrac{1}{2} m_A v_{A.f}^2 - \tfrac{1}{2} m_A v_{A.i}^2\right) + \left(\tfrac{1}{2} m_B v_{B.f}^2 - \tfrac{1}{2} m_B v_{B.i}^2\right) \approx 0 \quad \left(v's << c\right)$$

Equation 1 $$\left(\frac{p_{A.f}^2}{2m_A} - \frac{p_{A.i}^2}{2m_A}\right) + \left(\frac{p_{B.f}^2}{2m_B} - \frac{p_{B.i}^2}{2m_B}\right) \approx 0$$

Equation 2 $$\vec{p}_{A.f} + \vec{p}_{B.f} = \vec{p}_{A.i} + \vec{p}_{B.i}$$

1-D Collision

Special Case: Perfectly Elastic (all internal changes ‘bounce back’)

Deriving relation for Final Speed

Equation 1 $$\left(\frac{p_{A.f}^2}{2m_A}-\frac{p_{A.i}^2}{2m_A}\right)+\left(\frac{p_{B.f}^2}{2m_B}-\frac{p_{B.i}^2}{2m_B}\right)\approx 0$$

Equation 2 $$\vec{p}_{A.f}=\vec{p}_{A.i}+\vec{p}_{B.i}-\vec{p}_{B.f}$$

$$p_{A.f}^2=\left(\vec{p}_{A.i}+\vec{p}_{B.i}-\vec{p}_{B.f}\right)^2$$

$$p_{A.f}^2=p_{A.i}^2+p_{B.i}^2+p_{B.f}^2+2\vec{p}_{A.i}\cdot\vec{p}_{B.i}-2\vec{p}_{B.i}\cdot\vec{p}_{B.f}-2\vec{p}_{A.i}\cdot\vec{p}_{B.f}$$

$$p_{B.f}^2\underbrace{\frac{1}{2}\left(\frac{1}{m_A}+\frac{1}{m_B}\right)}_{a}-\vec{p}_{B.f}\cdot\underbrace{\frac{\left(\vec{p}_{B.i}+\vec{p}_{A.i}\right)}{m_A}}_{b}+\underbrace{\frac{p_{B.i}^2}{2}\left(\frac{1}{m_A}-\frac{1}{m_B}\right)+\frac{\vec{p}_{A.i}\cdot\vec{p}_{B.i}}{m_A}}_{c}\approx 0$$

Form of $$p_{B.f}^2a+p_{B.f}b+c\approx 0$$

Solved by $$p_{B.f}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Ack!

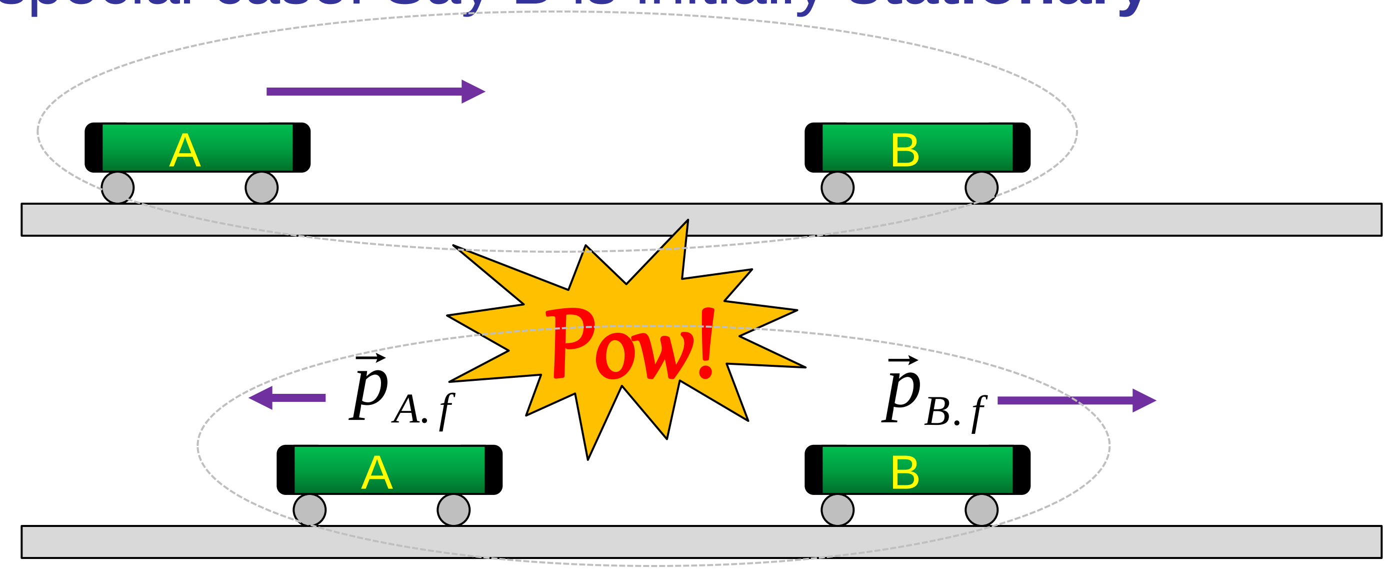
1-D Collision

Special Case: Perfectly Elastic (all internal changes 'bounce back')

Extra special case: Say B is initially **stationary**

System = carts A & B

initially



finally

$$p_{B.f}^2 \frac{1}{2}\left(\frac{1}{m_A}+\frac{1}{m_B}\right) - \vec{p}_{B.f}\cdot\frac{\left(\cancel{\vec{p}_{B.i}}+\vec{p}_{A.i}\right)}{m_A} + \frac{\cancel{p_{B.i}^2}}{2}\left(\frac{1}{m_A}-\frac{1}{m_B}\right) + \frac{\vec{p}_{A.i}\cdot\cancel{\vec{p}_{B.i}}}{m_A} \approx 0$$

$$p_{B.f}^{\cancel{2}} \frac{1}{2}\left(\frac{1}{m_A}+\frac{1}{m_B}\right) - \underset{1}{\cancel{\vec{p}_{B.f}}}\cdot\frac{\vec{p}_{A.i}}{m_A} \approx 0$$

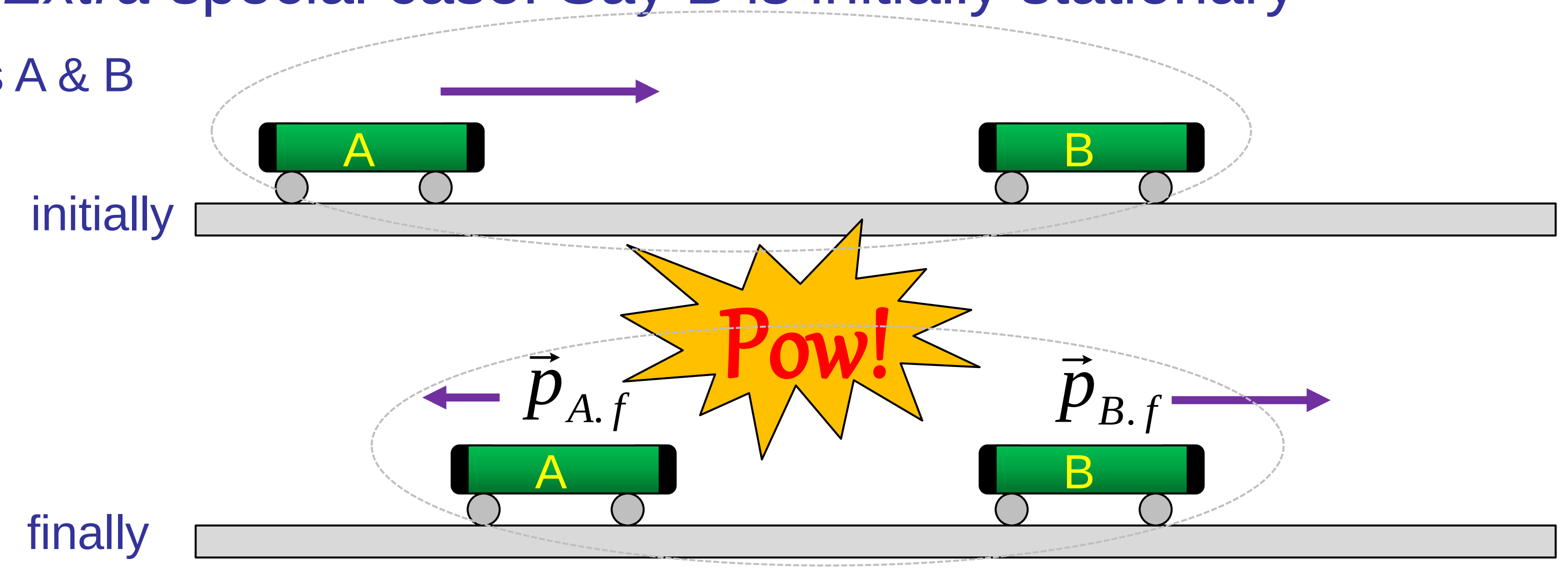
$$\vec{p}_{B.f} = 2\frac{\vec{p}_{A.i}}{m_A\left(\frac{1}{m_A}+\frac{1}{m_B}\right)} = 2\frac{m_B\vec{p}_{A.i}}{\left(m_B+m_A\right)}$$

1-D Collision

Special Case: Perfectly Elastic (all internal changes 'bounce back')

Extra special case: Say B is initially stationary

System = carts A & B



$$\vec{p}_{B.f} = 2\frac{m_B \vec{p}_{A.i}}{(m_B + m_A)}$$

$$\vec{p}_{A.f} = \vec{p}_{A.i} - \vec{p}_{B.f}$$

$$\vec{v}_{B.f} = \frac{2m_A}{(m_B + m_A)}\vec{v}_{A.i}$$

$$\vec{p}_{A.f} = \vec{p}_{A.i} - 2\frac{m_B \vec{p}_{A.i}}{(m_B + m_A)}$$

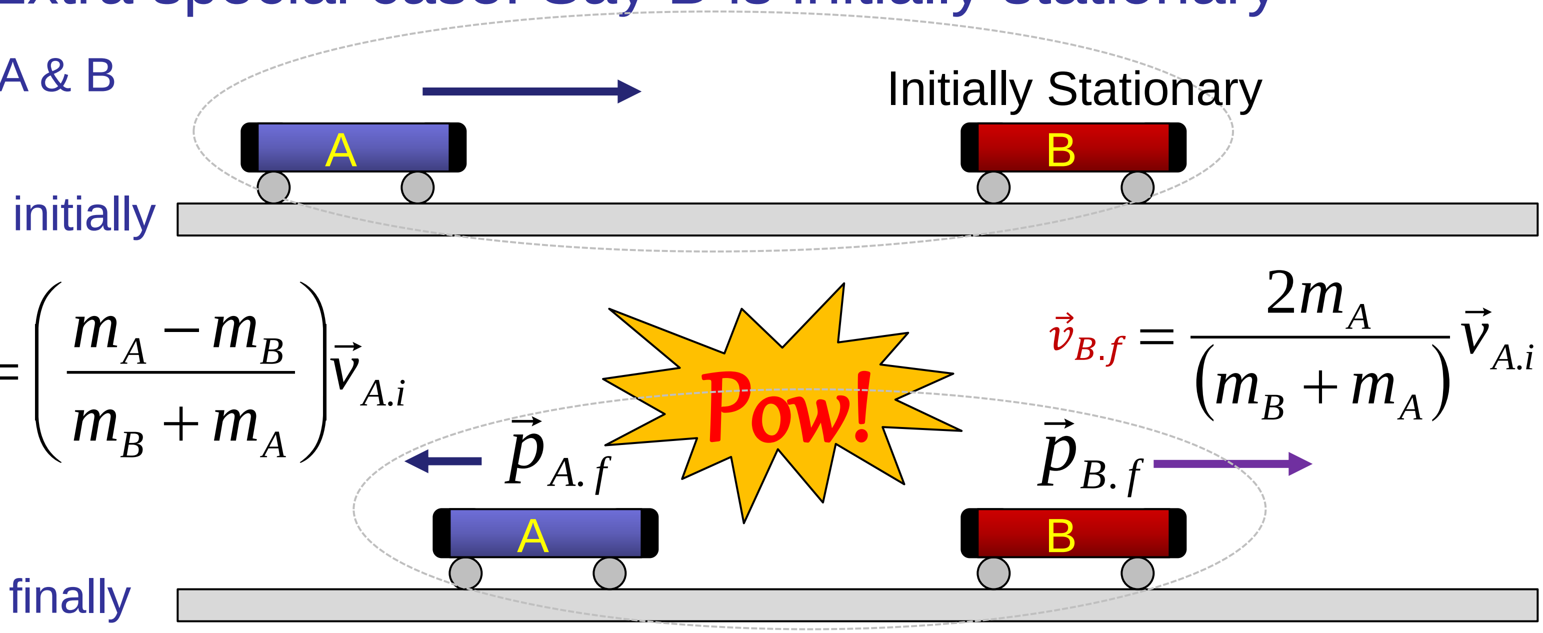
$$\vec{v}_{A.f} = \left(\frac{m_A - m_B}{m_B + m_A}\right)\vec{v}_{A.i} \longleftarrow \vec{p}_{A.f} = \left(\frac{m_A - m_B}{m_B + m_A}\right)\vec{p}_{A.i}$$

1-D Collision

Special Case: Perfectly Elastic (all internal changes 'bounce back')

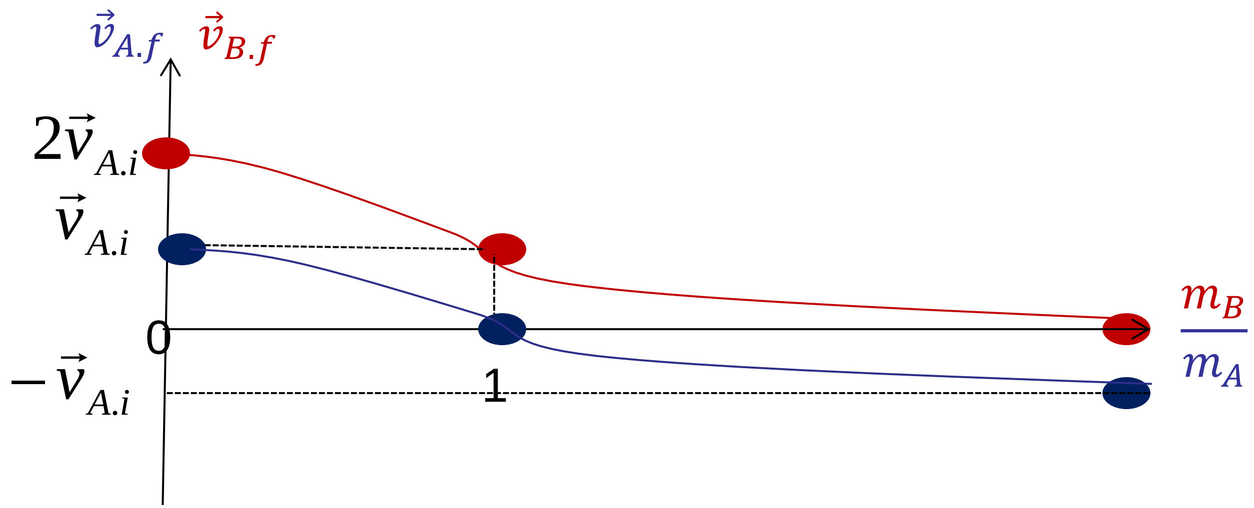
Extra special case: Say B is initially stationary

System = carts A & B



$$\vec{v}_{A.f} = \left(\frac{m_A - m_B}{m_B + m_A}\right)\vec{v}_{A.i}$$

$$\vec{v}_{B.f} = \frac{2m_A}{(m_B + m_A)}\vec{v}_{A.i}$$

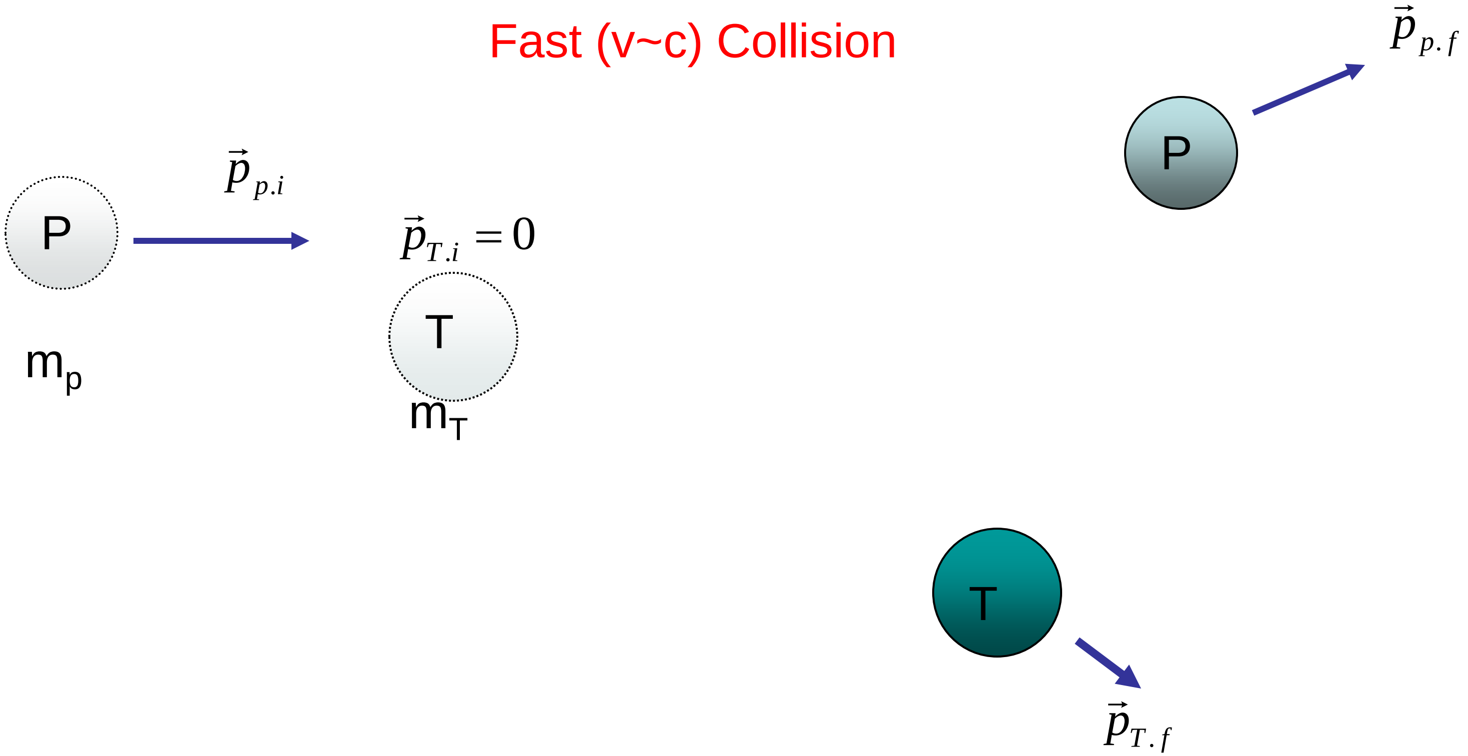


ed. ab i.	10.1-.4 Introducing Collisions Quiz 9 L9 Multi-particle Systems 10.6-.8 Scattering	RE 10.a RE 10.b
on., ab ed., i.,	10.9-.10 Collision Complications L10 Collisions 1 10.5, .11 Different Reference Frames 1.1 Translational Angular Momentum Quiz 10	RE 10.c EP9 RE 11.a; HW10: Pr’s 13*, 21, 30, “39

Collisions!

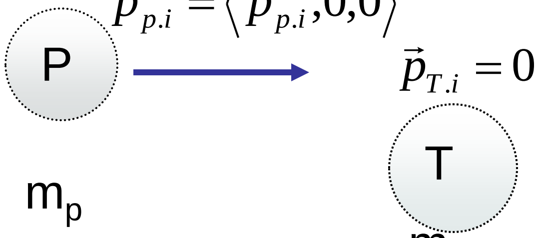
Short, Sharp Shocks

Fast (v~c) Collision



Fast (v~c) Collision

$$\vec{p}_{p.i} = \left\langle p_{p.i}, 0, 0 \right\rangle$$

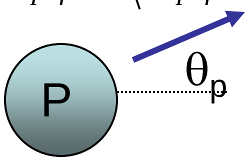


$$\vec{p}_{T.i} = 0$$

m_p

m_T

$$\vec{p}_{p.f} = \left\langle p_{p.f}\cos\theta_p, p_{p.f}\sin\theta_p, 0 \right\rangle$$

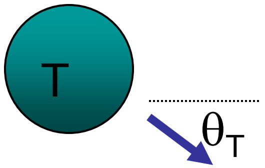


Conservation of Momentum

$$\vec{p}_{p.f} + \vec{p}_{T.f} - \vec{p}_{p.i} = 0$$

$$\hat{x} : p_{p.f}\cos\theta_p + p_{T.f}\cos\theta_T - p_{p.i} = 0$$

$$\hat{y} : p_{p.f}\sin\theta_p - p_{T.f}\left|\sin\theta_T\right| - 0 = 0$$

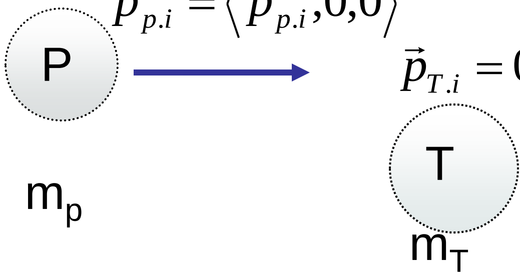


$$\vec{p}_{T.f} = \left\langle p_{T.f}\cos\theta_T, -p_{T.f}\left|\sin\theta_T\right|, 0 \right\rangle$$

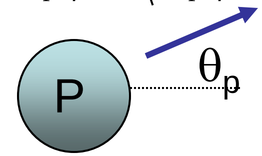
Fast (v~c) Collision

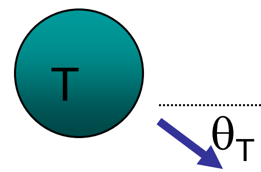
$$\vec{p}_{p.i} = \langle p_{p.i}, 0, 0 \rangle$$

$$\vec{p}_{T.i} = 0$$



$$\vec{p}_{p.f} = \langle p_{p.f}\cos\theta_p, p_{p.f}\sin\theta_p, 0 \rangle$$





$$\vec{p}_{T.f} = \langle p_{T.f}\cos\theta_T, -p_{T.f}\left|\sin\theta_T\right|, 0 \rangle$$

Conservation of Momentum

$$\vec{p}_{p.f} + \vec{p}_{T.f} - \vec{p}_{p.i} = 0$$

$$\hat{x}: p_{p.f}\cos\theta_p + p_{T.f}\cos\theta_T - p_{p.i} = 0$$

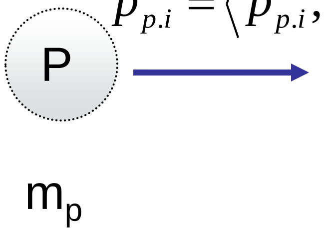
$$\hat{y}: p_{p.f}\sin\theta_p - p_{T.f}\left|\sin\theta_T\right| - 0 = 0$$

where

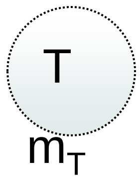
$$\vec{p} = \frac{m\vec{v}}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$

Fast (v~c) Collision

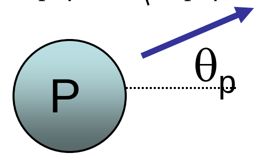
$$\vec{p}_{p.i} = \langle p_{p.i}, 0, 0 \rangle$$

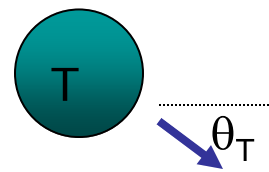


$$\vec{p}_{T.i} = 0$$



$$\vec{p}_{p.f} = \langle p_{p.f}\cos\theta_p, p_{p.f}\sin\theta_p, 0 \rangle$$





$$\vec{p}_{T.f} = \langle p_{T.f}\cos\theta_T, -p_{T.f}|\sin\theta_T|, 0 \rangle$$

Conservation of Momentum

$$\vec{p}_{p.f} + \vec{p}_{T.f} - \vec{p}_{p.i} = 0$$

$$\hat{x} : p_{p.f}\cos\theta_p + p_{T.f}\cos\theta_T - p_{p.i} = 0$$

$$\hat{y} : p_{p.f}\sin\theta_p - p_{T.f}|\sin\theta_T| - 0 = 0$$

where

$$\vec{p} = \frac{m\vec{v}}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$

Conservation of Energy

$$\left(E_{p.f} + E_{T.f}\right) - \left(E_{p.i} + E_{T.i}\right) = 0$$

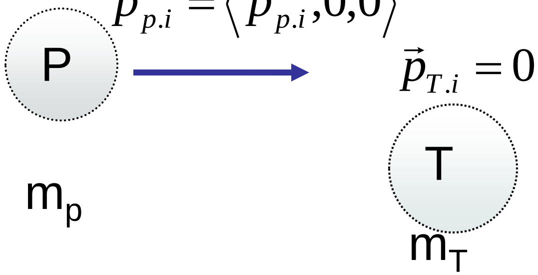
where

$$E = \frac{mc^2}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$

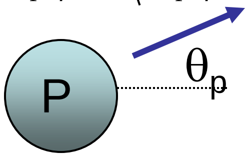
Fast (v~c) Collision

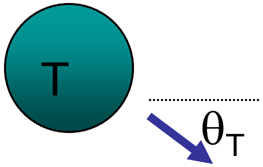
$$\vec{p}_{p.i} = \langle p_{p.i}, 0, 0 \rangle$$

$$\vec{p}_{T.i} = 0$$



$$\vec{p}_{p.f} = \langle p_{p.f}\cos\theta_p, p_{p.f}\sin\theta_p, 0 \rangle$$





$$\vec{p}_{T.f} = \langle p_{T.f}\cos\theta_T, -p_{T.f}\left|\sin\theta_T\right|, 0 \rangle$$

Conservation of Momentum

$$\vec{p}_{p.f} + \vec{p}_{T.f} - \vec{p}_{p.i} = 0$$

$$\hat{x} : p_{p.f}\cos\theta_p + p_{T.f}\cos\theta_T - p_{p.i} = 0$$

$$\hat{y} : p_{p.f}\sin\theta_p - p_{T.f}\left|\sin\theta_T\right| - 0 = 0$$

where

$$\vec{p} = \frac{m\vec{v}}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$

Conservation of Energy

$$\left(E_{p.f} + E_{T.f}\right) - \left(E_{p.i} + E_{T.i}\right) = 0$$

where

$$E = \frac{mc^2}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$$

CW: show $E = \sqrt{(pc)^2 + \left(mc^2\right)^2}$

By plugging that into it and recovering that.