Mon.	6.14-.17 Electric and Rest Energy	RE 6.e
Tues.		EP6, HW6: Pr's 58, 59, 91, 99(a-c), 105(a-c)
Wed.	7.1-.4 Macroscopic Energy Quiz 6	RE 7.a bring laptop, smartphone, pad,…
Lab	L6 Work and Energy	laptop
Fri.	7.5-.9 Energy Transfer	RE 7.b

Example Gravitational System: Earth and Sun

$\vec{v}_{E.i}$ m_E $\vec{v}_{E.f}$

$r_{E\leftarrow S.i}$ $r_{E\leftarrow S.f}$

m_S

System= Earth + Sun

Active environment = none

$$\Delta E = W_{system\leftarrow ext} = 0$$

Not changing

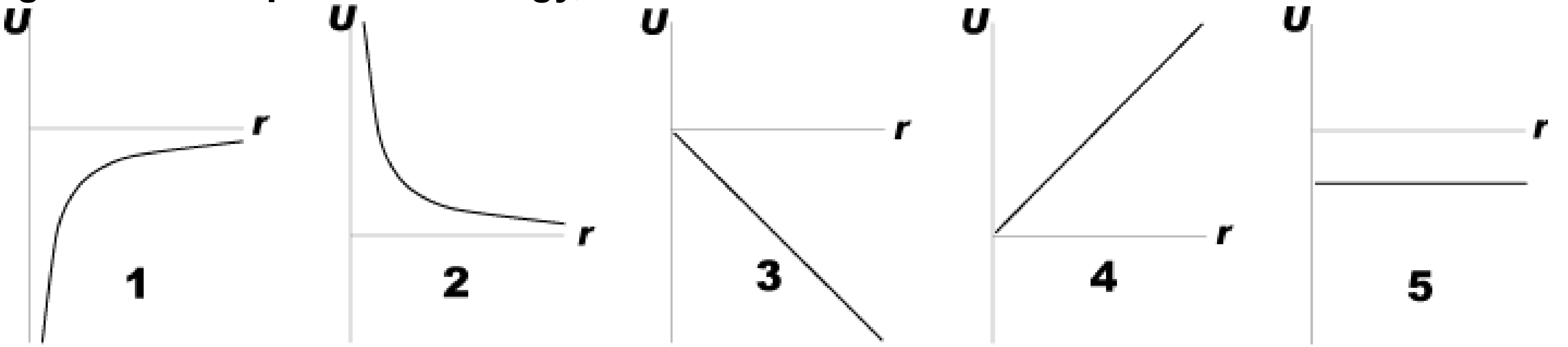
$$\Delta E_{E,S} = \Delta E_{rest.E} + \Delta E_{rest.S} + \Delta K_E + \Delta K_S + \Delta U_{E,S} = 0$$

$$\Delta E_{E,S} = \Delta K_E + \Delta U_{E,S} = 0$$

$$K_{E.f} + U_{E,S.f} = K_{E.i} + U_{E,S.i}$$

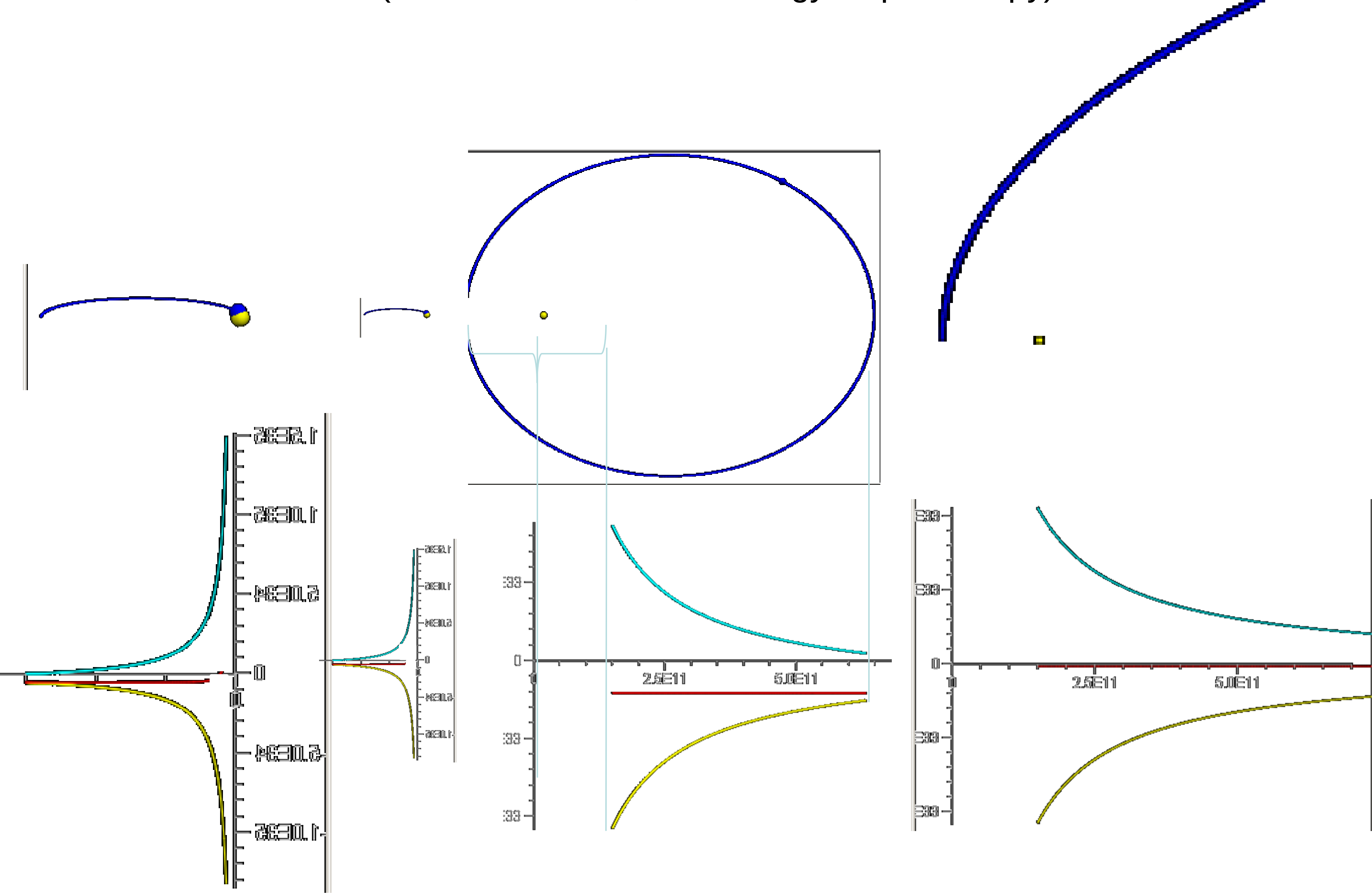
$$\tfrac{1}{2} m_E v_{E.f}^2 - G\frac{m_E m_s}{r_{ESf}} = \tfrac{1}{2} m_E v_{E.f}^2 - G\frac{m_E m_s}{r_{ESi}}$$

Which of the following graphs of *U* vs *r* represents the gravitational potential energy, $U = -GMm/r$?

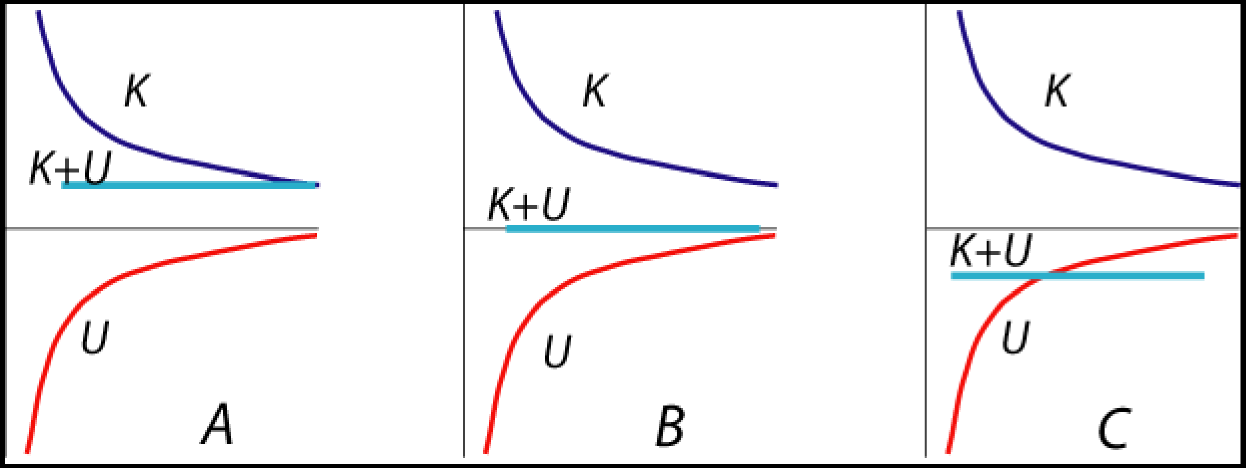


Different Initial Speeds / kinetic Energies, Different Paths

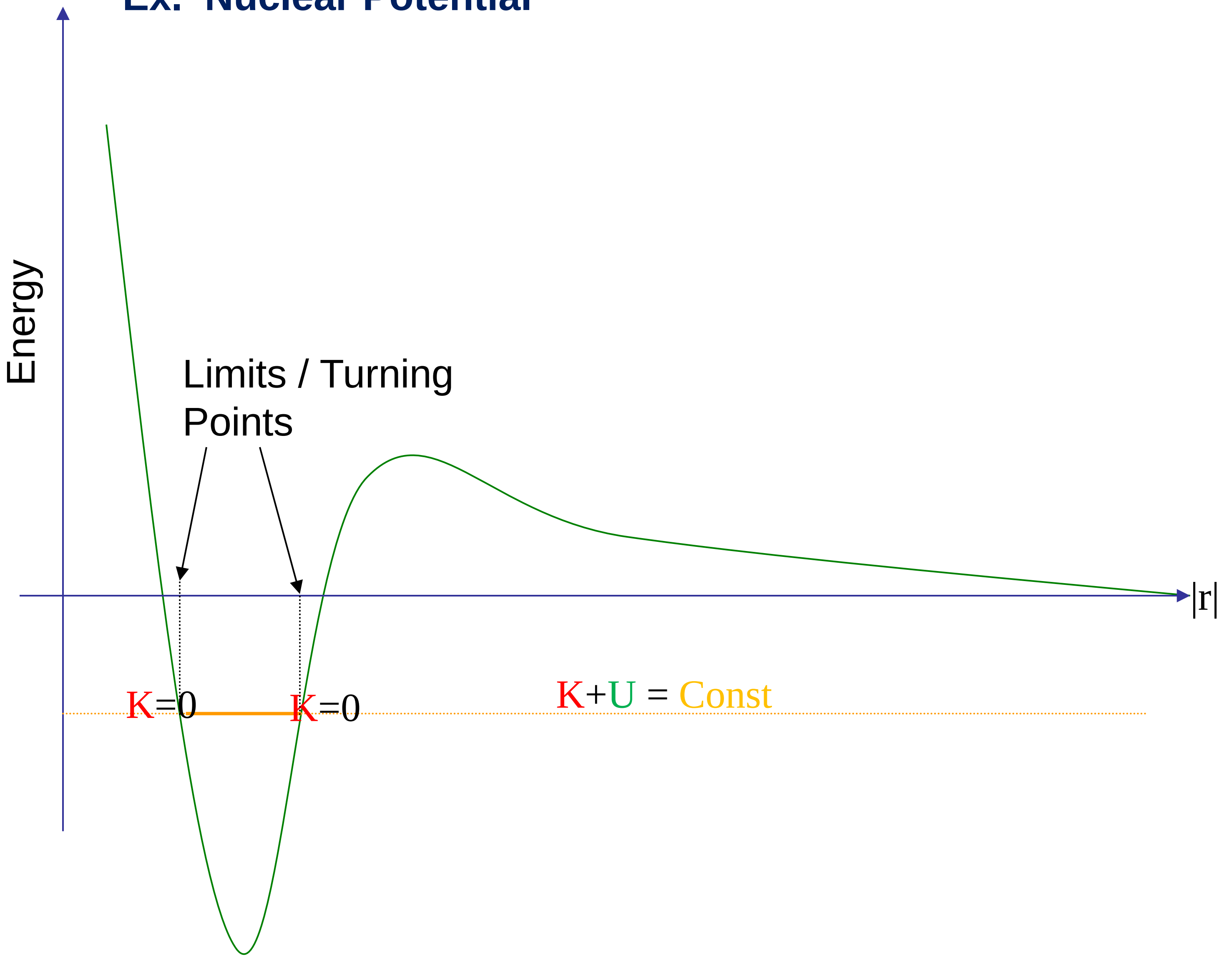
(orbit noncircular, with energy vs position.py)



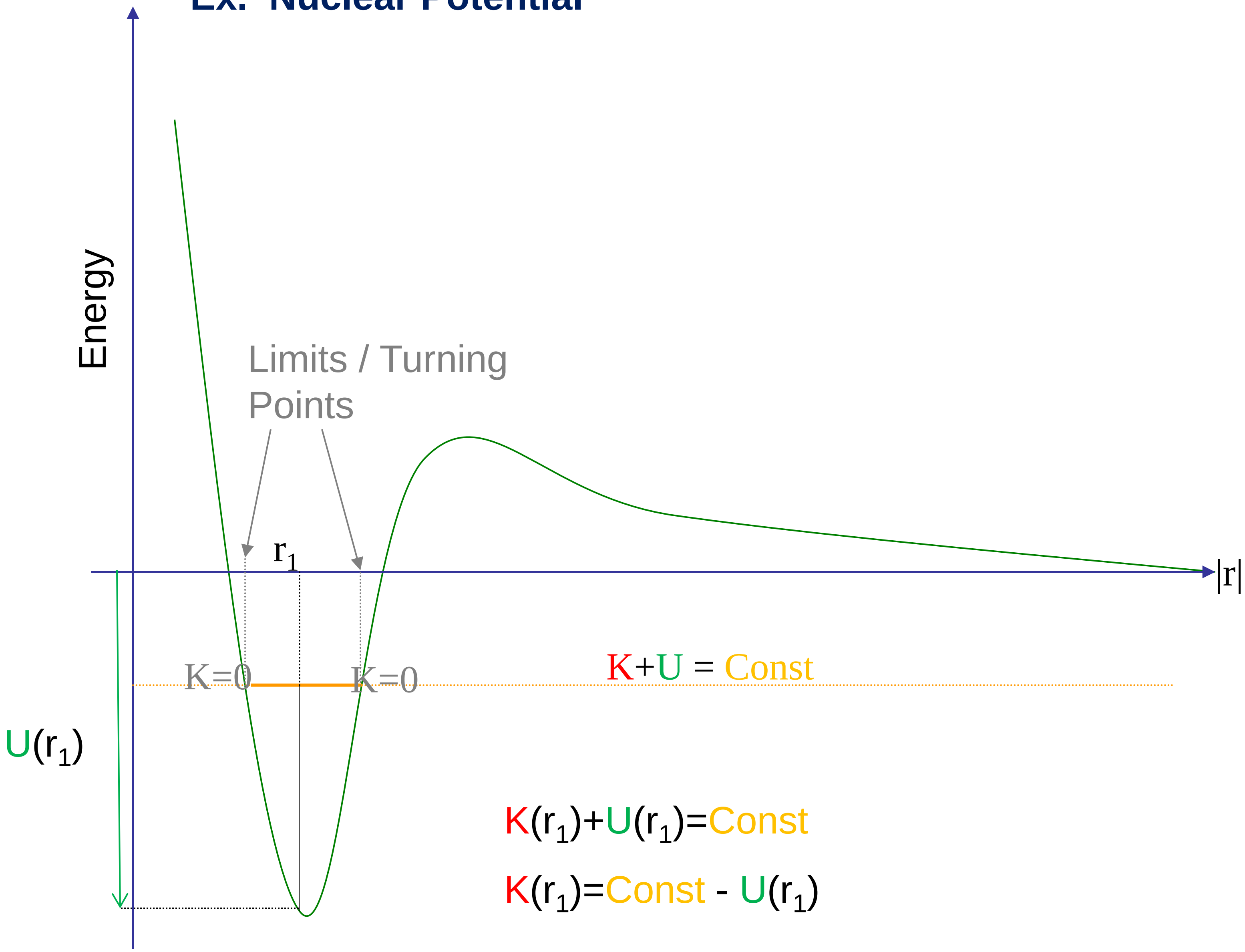
In which graph does the cyan line correctly represent the sum of kinetic energy plus potential energy?



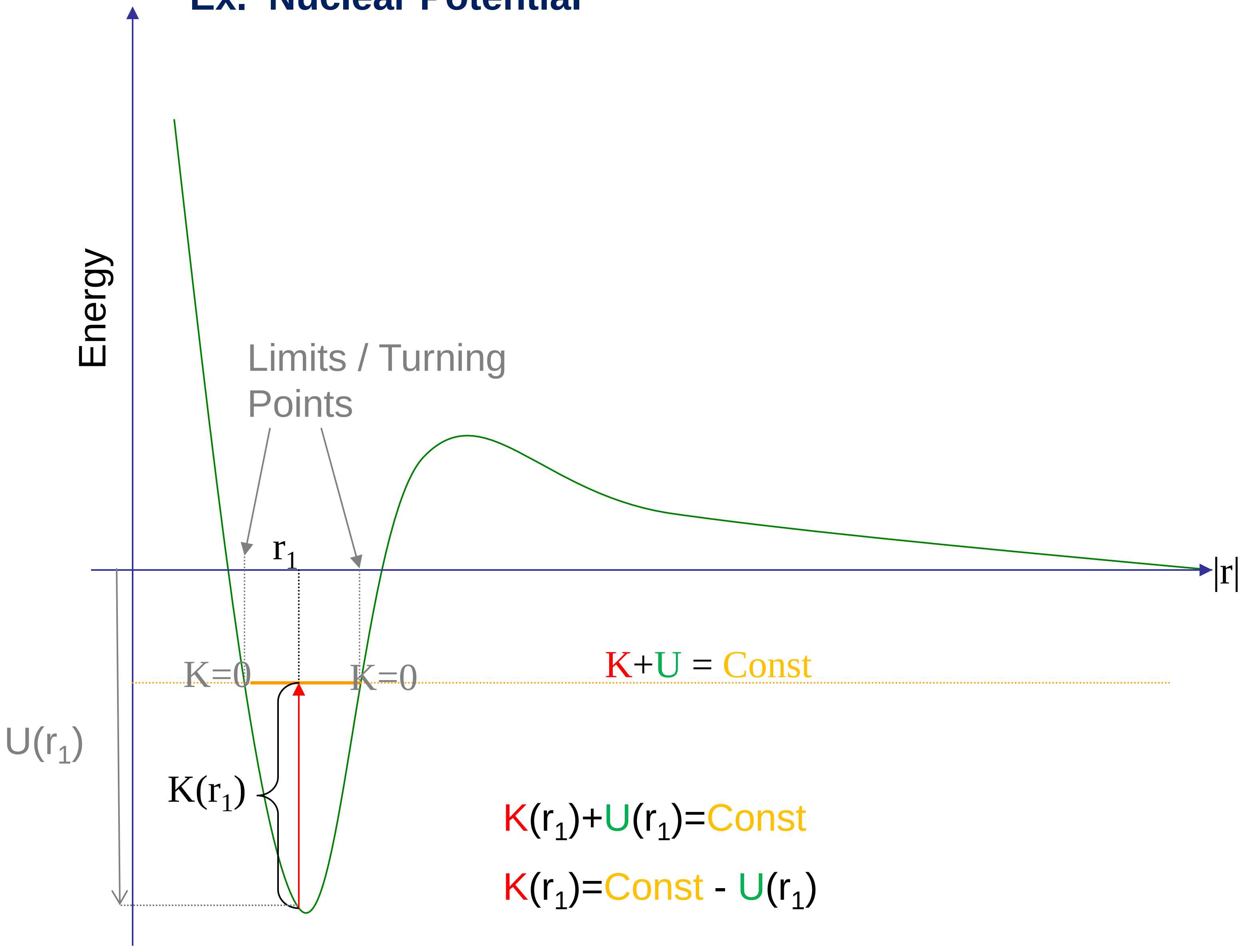
Conceptual Understanding from Energy Diagrams
Ex. Nuclear Potential



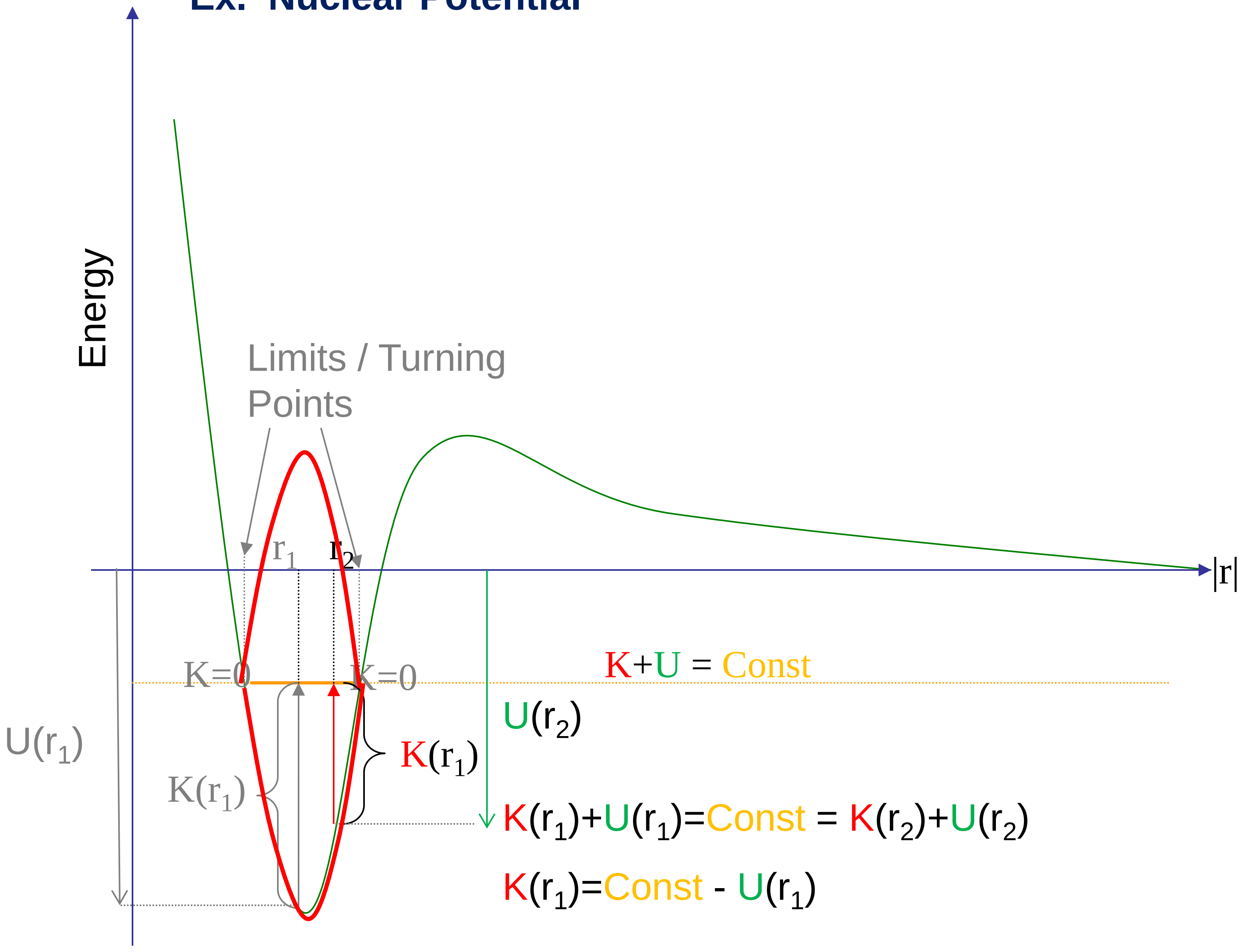
Conceptual Understanding from Energy Diagrams
Ex. Nuclear Potential



Conceptual Understanding from Energy Diagrams
Ex. Nuclear Potential

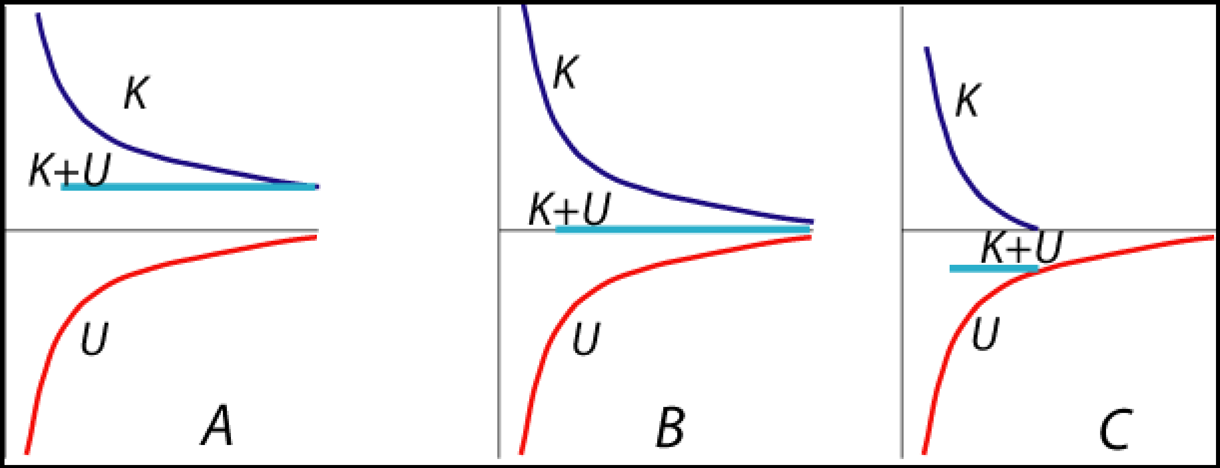


Conceptual Understanding from Energy Diagrams
Ex. Nuclear Potential

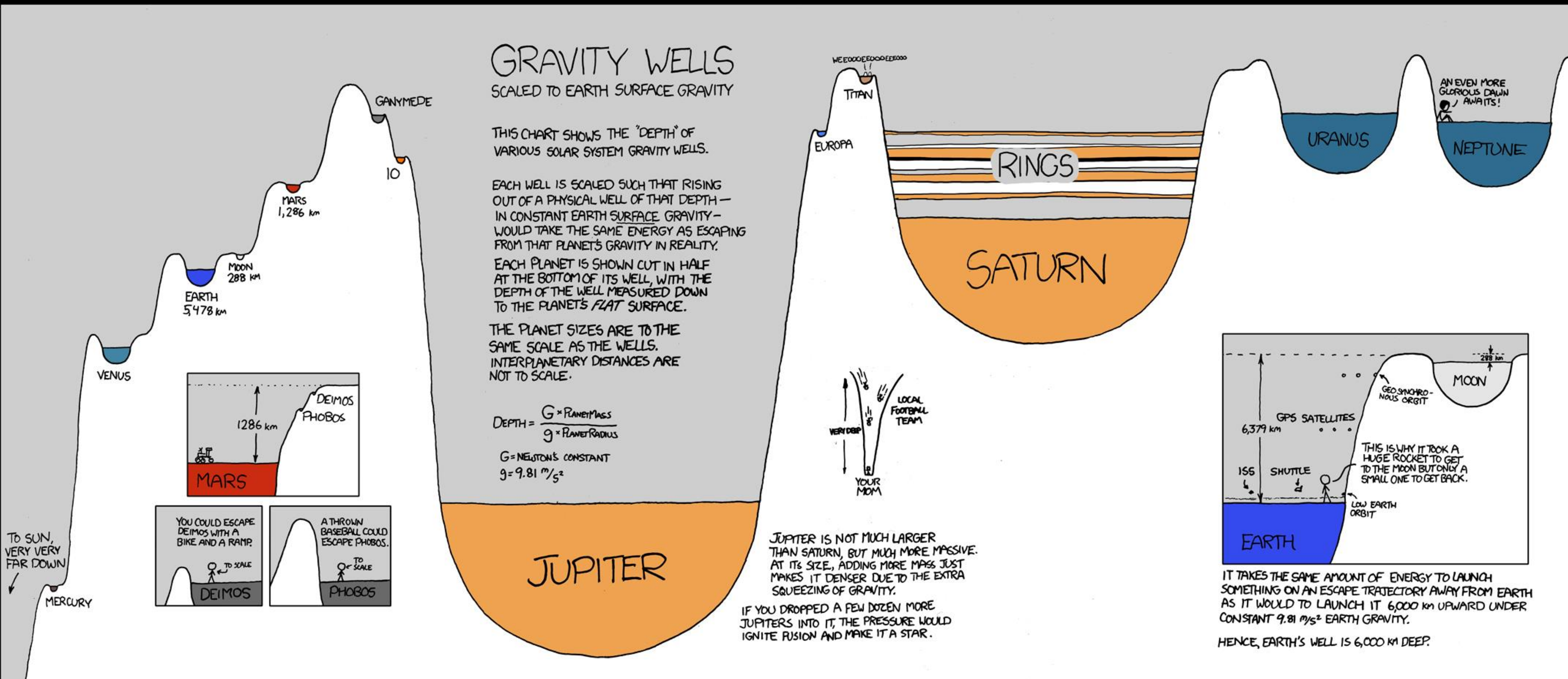


04_potential_energy_well.py

The system is a comet and a star. In which case(s) will the comet escape from the star and never return?



1) A	2) B	3) C	4) A,B	5) B,C	6) A,B,C



GRAVITY WELLS
SCALED TO EARTH SURFACE GRAVITY
THIS CHART SHOWS THE "DEPTH" OF VARIOUS SOLAR SYSTEM GRAVITY WELLS.
EACH WELL IS SCALED SUCH THAT RISING OUT OF A PHYSICAL WELL OF THAT DEPTH — IN CONSTANT EARTH SURFACE GRAVITY — WOULD TAKE THE SAME ENERGY AS ESCAPING FROM THAT PLANET'S GRAVITY IN REALITY.
EACH PLANET IS SHOWN CUT IN HALF AT THE BOTTOM OF ITS WELL, WITH THE DEPTH OF THE WELL MEASURED DOWN TO THE PLANET'S FLAT SURFACE.
THE PLANET SIZES ARE TO THE SAME SCALE AS THE WELLS. INTERPLANETARY DISTANCES ARE NOT TO SCALE.
DEPTH = $\frac{G \times \text{PLANET MASS}}{g \times \text{PLANET RADIUS}}$
G = NEWTON'S CONSTANT
g = 9.81 m/s²
TO SUN, VERY VERY FAR DOWN
MERCURY
VENUS
EARTH
5,478 km
MOON
288 km
MARS
1,286 km
GANYMEDE
IO
1286 km
DEIMOS
PHOBOS
MARS
YOU COULD ESCAPE DEIMOS WITH A BIKE AND A RAMP.
TO SCALE
DEIMOS
A THROWN BASEBALL COULD ESCAPE PHOBOS.
TO SCALE
PHOBOS
JUPITER
WEEEEEEEEEEEEEEEEE
TITAN
EUROPA
RINGS
SATURN
LOCAL FOOTBALL TEAM
VERY DEEP
YOUR MOM
JUPITER IS NOT MUCH LARGER THAN SATURN, BUT MUCH MORE MASSIVE. AT ITS SIZE, ADDING MORE MASS JUST MAKES IT DENSER DUE TO THE EXTRA SQUEEZING OF GRAVITY.
IF YOU DROPPED A FEW DOZEN MORE JUPITERS INTO IT, THE PRESSURE WOULD IGNITE FUSION AND MAKE IT A STAR.
URANUS
AN EVEN MORE GLORIOUS DAWN AWAITS!
NEPTUNE
288 km
MOON
GEOSYNCHRONOUS ORBIT
GPS SATELLITES
6,379 km
THIS IS WHY IT TOOK A HUGE ROCKET TO GET TO THE MOON BUT ONLY A SMALL ONE TO GET BACK.
ISS
SHUTTLE
LOW EARTH ORBIT
EARTH
IT TAKES THE SAME AMOUNT OF ENERGY TO LAUNCH SOMETHING ON AN ESCAPE TRAJECTORY AWAY FROM EARTH AS IT WOULD TO LAUNCH IT 6,000 km UPWARD UNDER CONSTANT 9.81 m/s² EARTH GRAVITY.
HENCE, EARTH'S WELL IS 6,000 km DEEP.

Force as negative gradient (3-D slope) of Potential Energy

small change in potential

$$dU_{1,2} = -\vec{F}_{1\to2} \cdot d\vec{r}_{1\to2} = -\left(F_{1\to2.x}dx + F_{1\to2.y}dy + F_{1\to2.z}dz\right)$$

Say only moves in the x direction, then

$$dU_{1,2} = -F_{1\to2.x}dx \quad \text{so} \quad -\frac{dU_{1,2}}{dx} = F_{1\to2.x}$$

Similarly, if only moves in the y direction, then

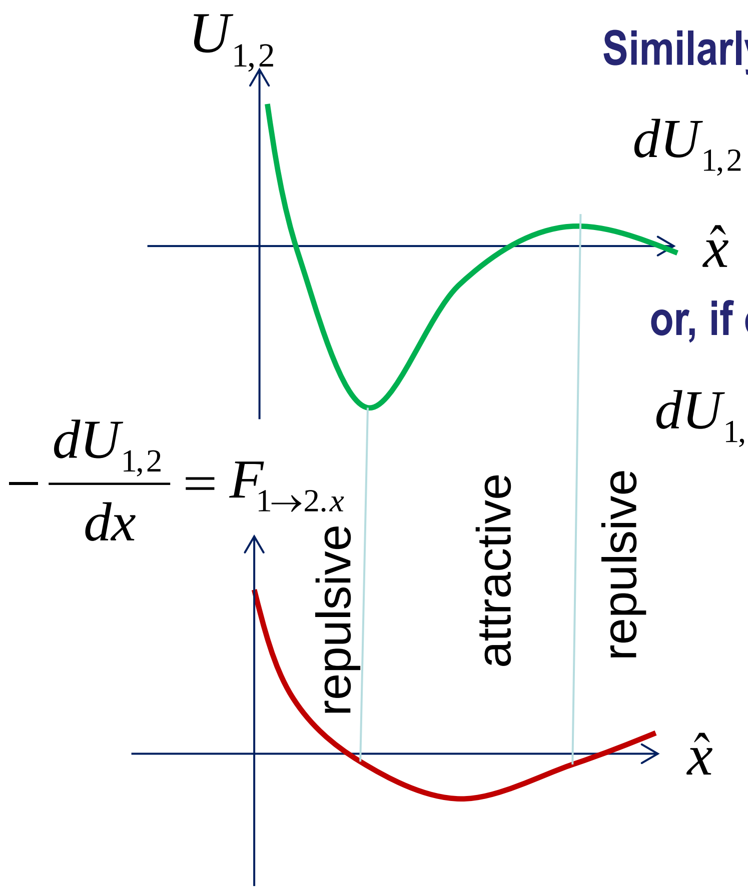
$$dU_{1,2} = -F_{1\to2.y}dy \quad \text{so} \quad -\frac{dU_{1,2}}{dy} = F_{1\to2.y}$$

or, if only moves in the z direction, then

$$dU_{1,2} = -F_{1\to2.z}dz \quad \text{so} \quad -\frac{dU_{1,2}}{dz} = F_{1\to2.z}$$

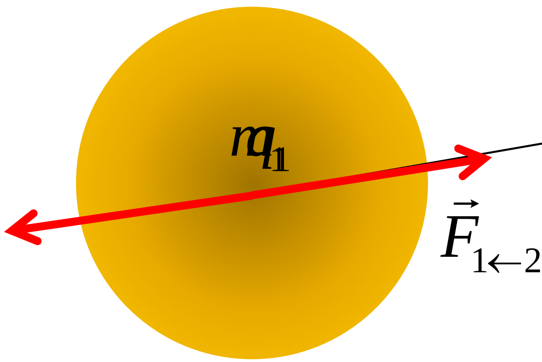
Moving in all directions,

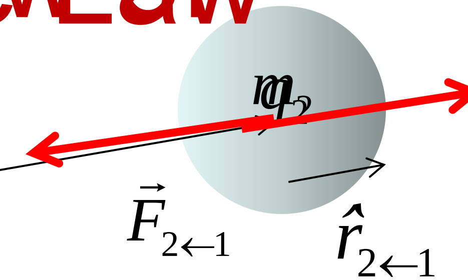
$$\vec{F}_{1\to2} = \left\langle F_{1\to2.x}, F_{1\to2.y}, F_{1\to2.z} \right\rangle = -\left\langle \frac{\partial U_{1,2}}{\partial x_{1\to2}}, \frac{dU_{1,2}}{dy_{1\to2}}, \frac{dU_{1,2}}{dz_{1\to2}} \right\rangle$$



Coulomb's Electric Force Law

Newton's Universal Law of Gravitation





$\vec{r}_{2\leftarrow 1}$

$$\vec{F}_{2\leftarrow 1} = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{\left|\vec{r}_{2\leftarrow 1}\right|^2}\hat{r}_{2\leftarrow 1}$$

$$\vec{F}_{2\leftarrow 1} = -G\frac{m_1 m_2}{\left|\vec{r}_{2\leftarrow 1}\right|^2}\hat{r}_{2\leftarrow 1}$$

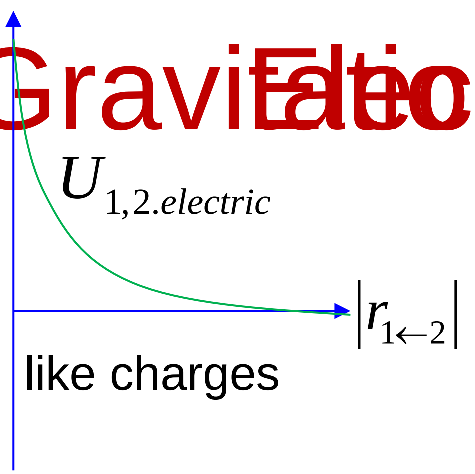
$$\frac{1}{4\pi\varepsilon_o} = 9\times 10^{9}\,\frac{N\cdot m^2}{(C)^2}$$

$$G = 6.679\times 10^{-11}\,\frac{N\cdot m^2}{(kg)^2}$$

$$\hat{r}_{2\leftarrow 1} = \frac{\vec{r}_{2\leftarrow 1}}{\left|\vec{r}_{2\leftarrow 1}\right|}$$

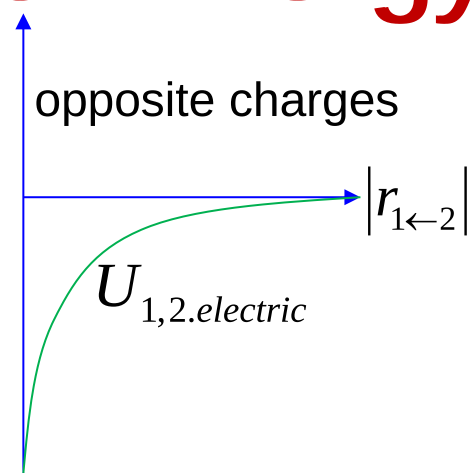
Electric Potential Energy

Gravitational Potential Energy



$$U_{1,2} = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{\left|r_{1\leftarrow 2}\right|}$$

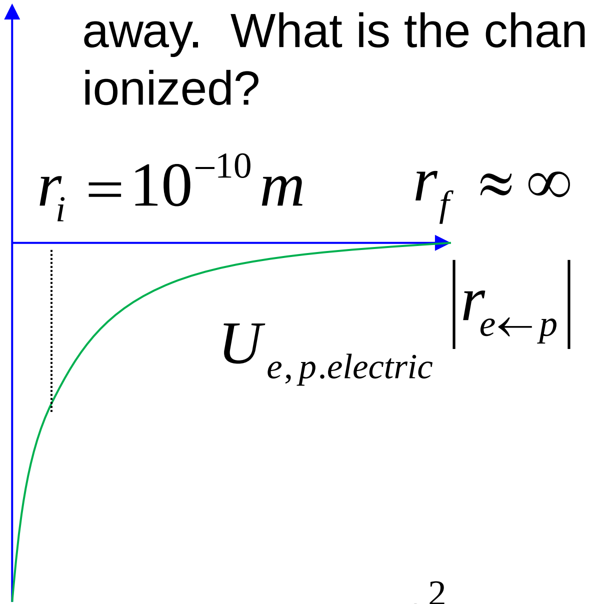
$$U_{1,2} = -G\frac{m_1 m_2}{\left|r_{1\leftarrow 2}\right|}$$



Example: Ionize Hydrogen. In a hydrogen atom the electron averages around 10^{-10} m from the proton. When a hydrogen atom is ionized, the electron is stripped away. What is the change in electric potential energy when such an atom is ionized?

System= electron + proton

Active environment = none



$$U_{e,p.elect} = \frac{1}{4\pi\varepsilon_o}\frac{q_e q_p}{|r_{e\leftarrow p}|}$$

$$U_{e,p.elect} = \frac{1}{4\pi\varepsilon_o}\frac{-e^2}{|r_{1\leftarrow 2}|}$$

$$\Delta U_{e,p,elct} = \frac{1}{4\pi\varepsilon_o}\frac{-e^2}{|r_f|} - \frac{1}{4\pi\varepsilon_o}\frac{-e^2}{|r_i|} = \frac{e^2}{4\pi\varepsilon_o}\left(\frac{1}{|r_i|} - \frac{1}{|r_f|}\right)$$

$\frac{1}{|r_f|} \to 0$, $r_f \to \infty$

$$\Delta U_{e,p,elct} = \frac{1}{4\pi\left(8.85\times10^{-12}\frac{C^2}{Nm^2}\right)}\left(\frac{\left(1.6\times10^{-19}C\right)^2}{10^{-10}m}\right)$$

$$\Delta U_{e,p,elct} = 2.3\times10^{-18} J$$

Or in eV's (divide by electron charge)

$$= 2.3\times10^{-18} J \frac{1e}{1.6\times10^{-19}C} = 14 eV$$

Comparison:
Electric vs. Gravitational

$$\frac{U_{e,p.elect}}{U_{e,p.grav}} = \frac{\frac{1}{4\pi\varepsilon_o}\frac{-e^2}{|r_{1\leftarrow 2}|}}{-G\frac{m_e m_p}{|r_{1\leftarrow 2}|}}$$

$$\frac{U_{e,p.elect}}{U_{e,p.grav}} = \frac{1}{4\pi\varepsilon_o G}\frac{e^2}{m_e m_p}$$

$$\frac{9\times10^9\frac{Nm^2}{C^2}\left(1.6\times10^{-19}C\right)^2}{\left(6.7\times10^{-11}\frac{Nm^2}{kg^2}\right)\left(9\times10^{-31}kg\right)\left(1.7\times10^{-27}kg\right)}$$

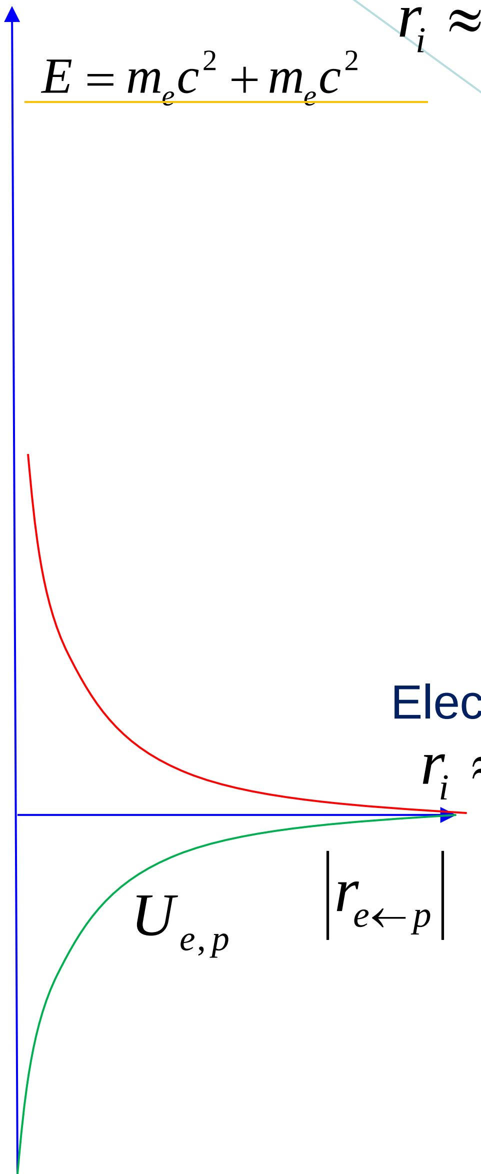
$$\frac{U_{e,p.elect}}{U_{e,p.grav}} = 5.6\times10^{39}$$

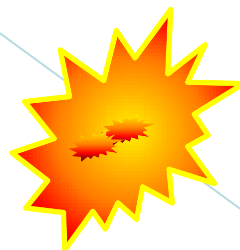
Return to Rest Energy and Mass

Pair (electron and positron) Annihilation

$-e$

$r_i \approx \infty$





initial final

$$e^- + e^+ \rightarrow \gamma + \gamma$$

Electron and positron Two photons (light pulses)

$$E = 2m_e c^2 = 2E_\gamma$$

$$\left(0.511 MeV / c^2\right) c^2 = E_\gamma$$

$$0.511 MeV = E_\gamma$$

$+e$

-e

Return to Rest Energy and Mass

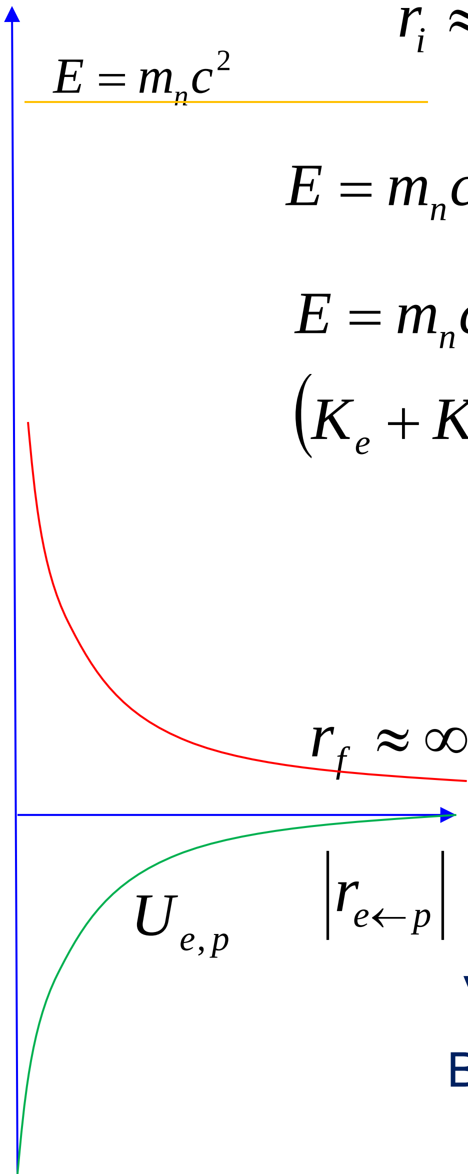
Neutron Decay

initial final

$$n^0 \rightarrow p^+ + e^- + \bar{\nu}_e$$

neutron Proton, electron, and neutrino

$r_i \approx \infty$



Nearly massless

Finally infinitely far apart

$$E = m_n c^2 = m_e c^2 + m_p c^2 + m_\nu c^2 + K_e + K_p + K_\nu + U_{e,p} + U_{e,\nu} + U_{\nu,p}$$

$$E = m_n c^2 = m_e c^2 + m_p c^2 + K_e + K_p + K_\nu$$

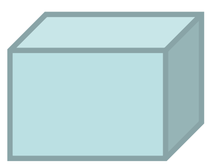
$$(K_e + K_p + K_\nu) = m_n c^2 - (m_e c^2 + m_p c^2)$$

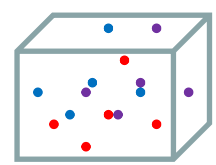
$$= 939.6MeV - (0.511MeV + 938.3MeV) = 0.79MeV$$

p+ e- n ν_e

Mass as Energy and Energy as Mass

Box o' decaying Neutrons





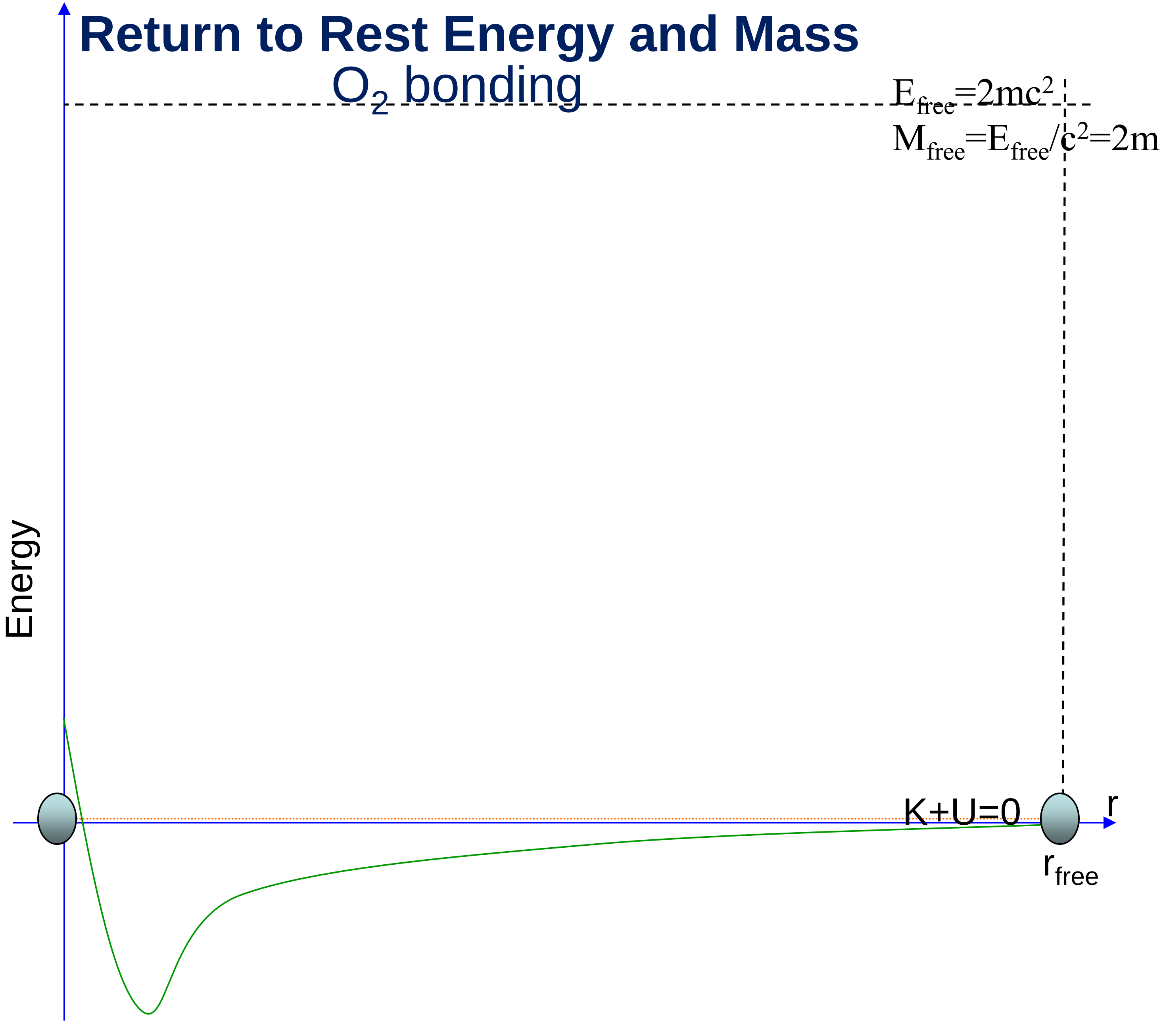
$$E = E_{rest} = m_{box} c^2 = \sum_{all.particles} ((m_n + m_e + m_p + m_\nu)c^2 + K_e + K_p + K_\nu + U_{e,p})$$

Viewed from outside Peaking inside

Box's mass *includes* internal kinetic and potential energies

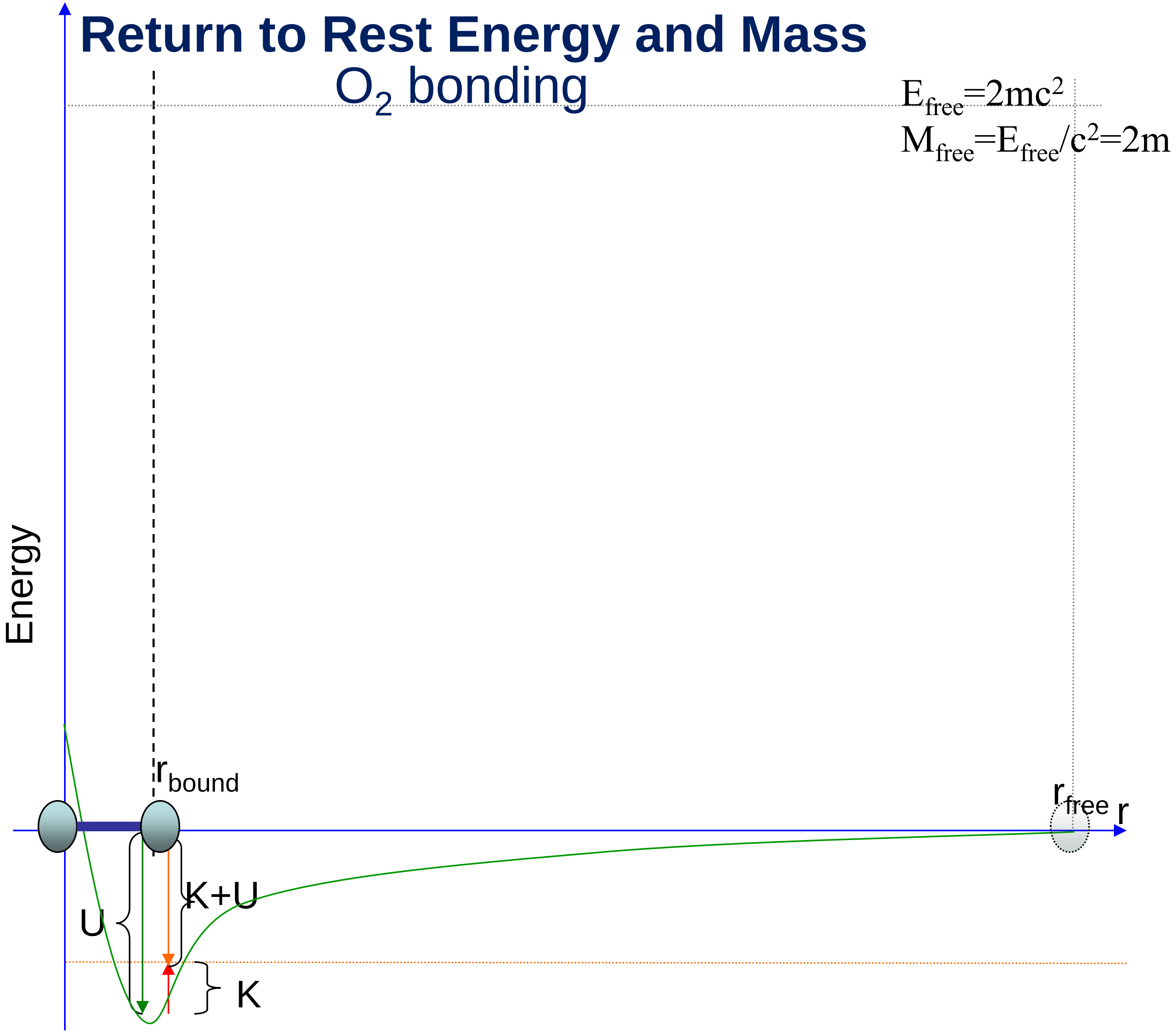
Return to Rest Energy and Mass

O_2 bonding



Return to Rest Energy and Mass

O_2 bonding



Return to Rest Energy and Mass

O_2 bonding

$E_{free}=2mc^2$

$M_{free}=E_{free}/c^2=2m$

K+U

$E_{bound}=2mc^2+(K+U)$

$M_{bound}=E_{bound}/c^2=2m+(K+U)/c^2$

Note: would have shed excess energy by emitting photon / light pulse

Energy / Mass difference

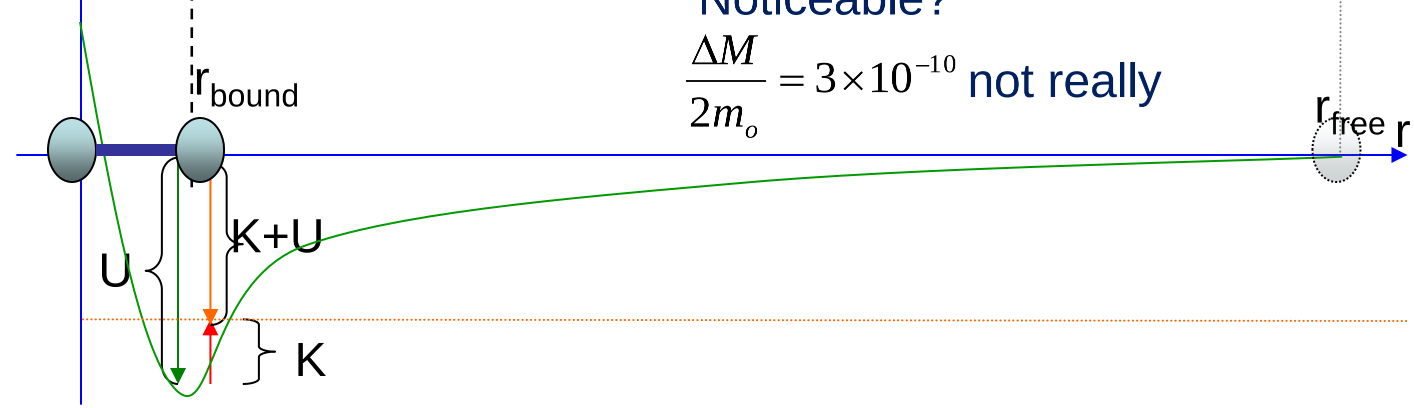
$$\Delta M = M_{free} - M_{bound} = \left(E_f - E_b\right)/c^2$$

$$\Delta M = \left(2mc^2 - \left(2mc^2 + (K+U)_{bound}\right)\right)/c^2$$

$$\Delta M = -(K+U)_{bound}/c^2 = 5eV/c^2 = 9\times10^{-36}kg$$

Noticeable?

$$\frac{\Delta M}{2m_o} = 3\times10^{-10}$$ not really



Return to Rest Energy and Mass

Nuclear Binding: Iron nucleus

If an iron nucleus were disintegrated, how much K + U energy would be consumed /produced?

initial final

$$Fe_{56}^{26} \rightarrow 26p^{+} + 30n$$

Iron nucleus Protons and neutrons

Useful info

$M_{Fe.nuc} = 52107 MeV/c^2$

$m_n = 939.9 MeV/c^2$

$m_p = 938.3 MeV/c^2$

Noticeable?

$$\frac{\Delta mc^2}{m_{Fe}c^2} = 0.009 \approx 1\%$$

yes

$$E_i = E_f$$

$$E_{r.Fe} = \sum_{all.particles} (E_r + K) + \sum_{all.pairs} U$$

$$m_{Fe}c^2 = 26 \cdot m_p c^2 + 30 \cdot m_n c^2 + \left(\sum_{all.particles} K + \sum_{all.pairs} U \right)$$

$$m_{Fe}c^2 - \left(26 \cdot m_p c^2 + 30 \cdot m_n c^2\right) = \left(\sum_{all.particles} K + \sum_{all.pairs} U \right)$$

$$52107 MeV - \left(26 \cdot (939.9 MeV) + 30 \cdot (938.3 MeV)\right) = \left(\sum_{all.particles} K + \sum_{all.pairs} U \right)$$

$$-482 MeV = \left(\sum_{all.particles} K + \sum_{all.pairs} U \right)$$

Rest *and* Electric-Potential *and* Kinetic

A U-235 nucleus is struck by a slow-moving neutron, so that the merge and become U-236, with mass $M_{U\text{-}236}$ This nucleus is unstable to falling apart – fission. One way it could do so is to first slosh into something of a dumbbell shape, now most of the into two symmetric nuclei, Pd-118, with mass $M_{Pd\text{-}118}$, each has ½ the original number of protons, i.e., $q_{Pd} = 46e$. Having fallen apart, the two palladium nuclei no longer experience a Strong interaction holding them together, just the Electric repulsion of each other's protons. Subsequently, they accelerate away.

a) What's the final speed of one of the Pd atoms, when they have sped far, far apart?

b) What is the distance between the Pd atoms just after fission?

Mon.	6.14-.17 Electric and Rest Energy	RE 6.e
Tues.		EP6, HW6: Pr's 58, 59, 91, 99(a-c), 105(a-c)
Wed.	7.1-.4 Macroscopic Energy Quiz 6	RE 7.a bring laptop, smartphone, pad,…
Lab	L6 Work and Energy	laptop
Fri.	7.5-.9 Energy Transfer	RE 7.b