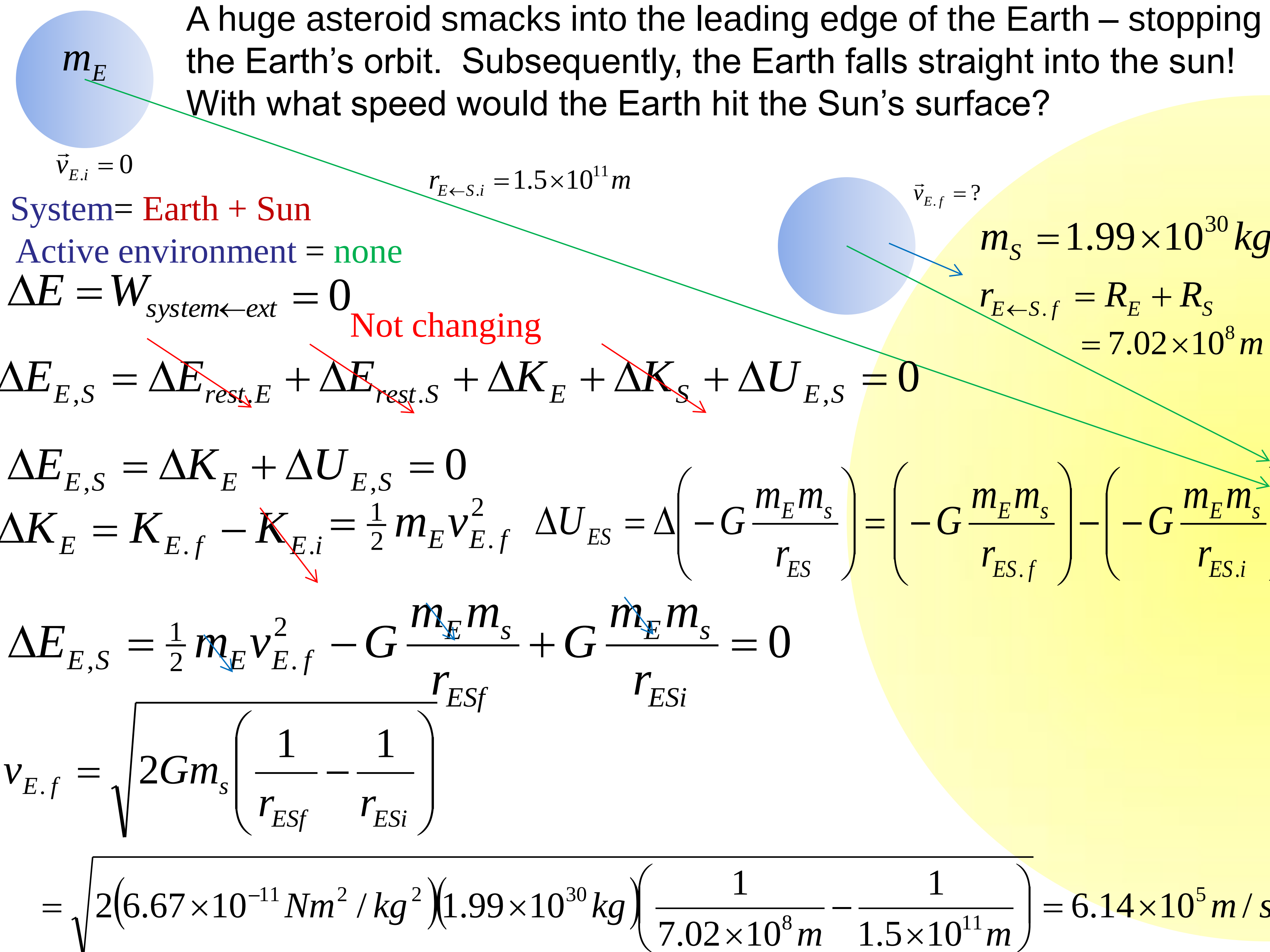
Fri.	6.11, 14-17 Visualizing Electric and Rest Energy	RE 6.d,e
Mon. Tues.	Things Engineers and Physicists Do	EP6, HW6: Ch 6 Pr’s 58, 59, 91, 99(a-c), 105(a-c)

A huge asteroid smacks into the leading edge of the Earth – stopping the Earth's orbit. Subsequently, the Earth falls straight into the sun! With what speed would the Earth hit the Sun's surface?



System= Earth + Sun

Active environment = none

$m_S = 1.99\times10^{30} kg$

$r_{E\leftarrow S.f} = R_E + R_S$
$= 7.02\times10^{8} m$

$$\Delta E = W_{system\leftarrow ext} = 0$$

Not changing

$$\Delta E_{E,S} = \cancel{\Delta E_{rest.E}} + \cancel{\Delta E_{rest.S}} + \Delta K_E + \cancel{\Delta K_S} + \Delta U_{E,S} = 0$$

$$\Delta E_{E,S} = \Delta K_E + \Delta U_{E,S} = 0$$

$$\Delta K_E = K_{E.f} - \cancel{K_{E.i}} = \tfrac{1}{2} m_E v_{E.f}^2 \quad \Delta U_{ES} = \Delta\left(-G\frac{m_E m_s}{r_{ES}}\right) = \left(-G\frac{m_E m_s}{r_{ES.f}}\right) - \left(-G\frac{m_E m_s}{r_{ES.i}}\right.$$

$$\Delta E_{E,S} = \tfrac{1}{2}\cancel{m_E} v_{E.f}^2 - G\frac{\cancel{m_E} m_s}{r_{ESf}} + G\frac{\cancel{m_E} m_s}{r_{ESi}} = 0$$

$$v_{E.f} = \sqrt{2Gm_s\left(\frac{1}{r_{ESf}} - \frac{1}{r_{ESi}}\right)}$$

$$= \sqrt{2\left(6.67\times10^{-11} Nm^2/kg^2\right)\left(1.99\times10^{30} kg\right)\left(\frac{1}{7.02\times10^{8} m} - \frac{1}{1.5\times10^{11} m}\right)} = 6.14\times10^{5} m/s$$

System: comet + star

Surroundings: negligible

As a comet travels away from a star, how do the kinetic energy and potential energy of the system change?

K	U
a) increase	decrease
b) increase	increase
c) decrease	increase
d) decrease	decrease
e) no change	no change

Force as negative gradient (3-D slope) of Potential Energy

small change in potential

$$dU_{1,2} = -\vec{F}_{1\to2}\cdot d\vec{r}_{1\to2} = -\left(F_{1\to2.x}dx + F_{1\to2.y}dy + F_{1\to2.z}dz\right)$$

Say only moves in the x direction, then

$$dU_{1,2} = -F_{1\to2.x}dx \quad \text{so} \quad -\frac{dU_{1,2}}{dx} = F_{1\to2.x}$$

Similarly, if only moves in the y direction, then

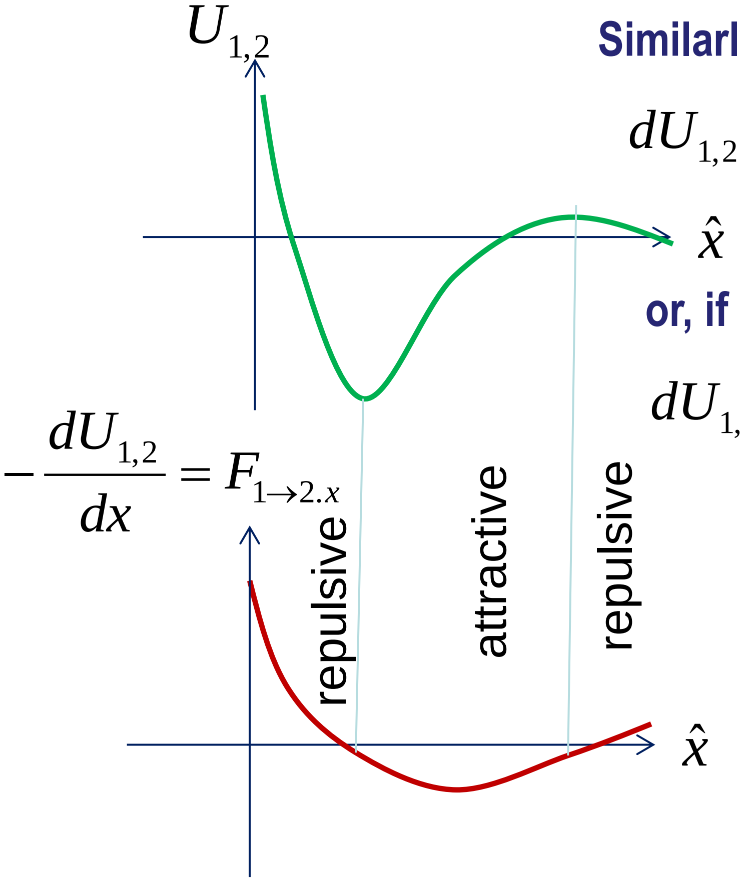
$$dU_{1,2} = -F_{1\to2.y}dy \quad \text{so} \quad -\frac{dU_{1,2}}{dy} = F_{1\to2.y}$$

or, if only moves in the z direction, then

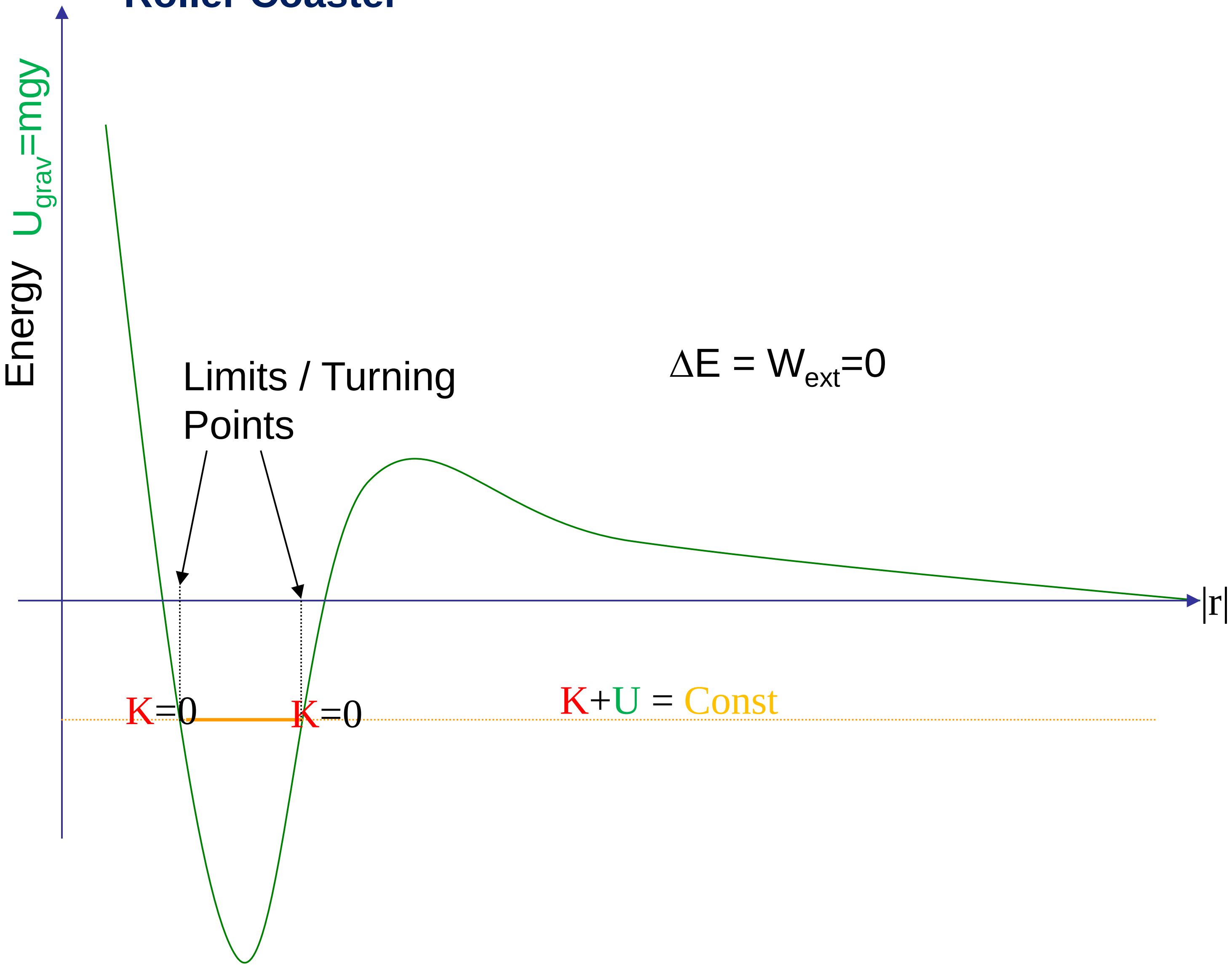
$$dU_{1,2} = -F_{1\to2.z}dz \quad \text{so} \quad -\frac{dU_{1,2}}{dz} = F_{1\to2.z}$$

Moving in all directions,

$$\vec{F}_{1\to2} = \left\langle F_{1\to2.x}, F_{1\to2.y}, F_{1\to2.z}\right\rangle = -\left\langle \frac{\partial U_{1,2}}{\partial x_{1\to2}}, \frac{dU_{1,2}}{dy_{1\to2}}, \frac{dU_{1,2}}{dz_{1\to2}}\right\rangle$$



Conceptual Understanding from Energy Diagrams
Roller Coaster



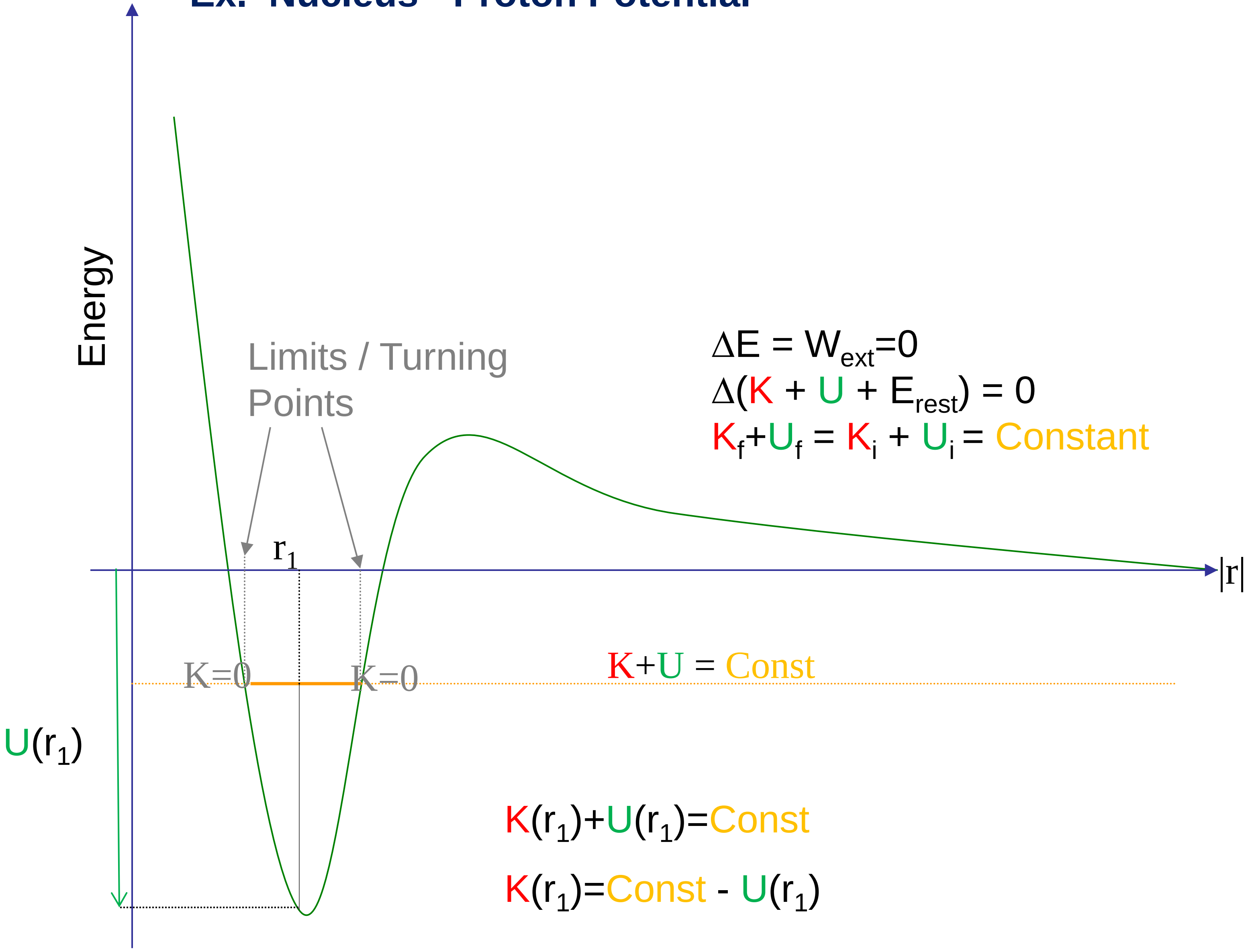
Limits / Turning Points

$\Delta E = W_{ext} = 0$

K+U = Const

04_potential_energy_well.py

Conceptual Understanding from Energy Diagrams
Ex. Nucleus - Proton Potential



$\Delta E = W_{ext} = 0$

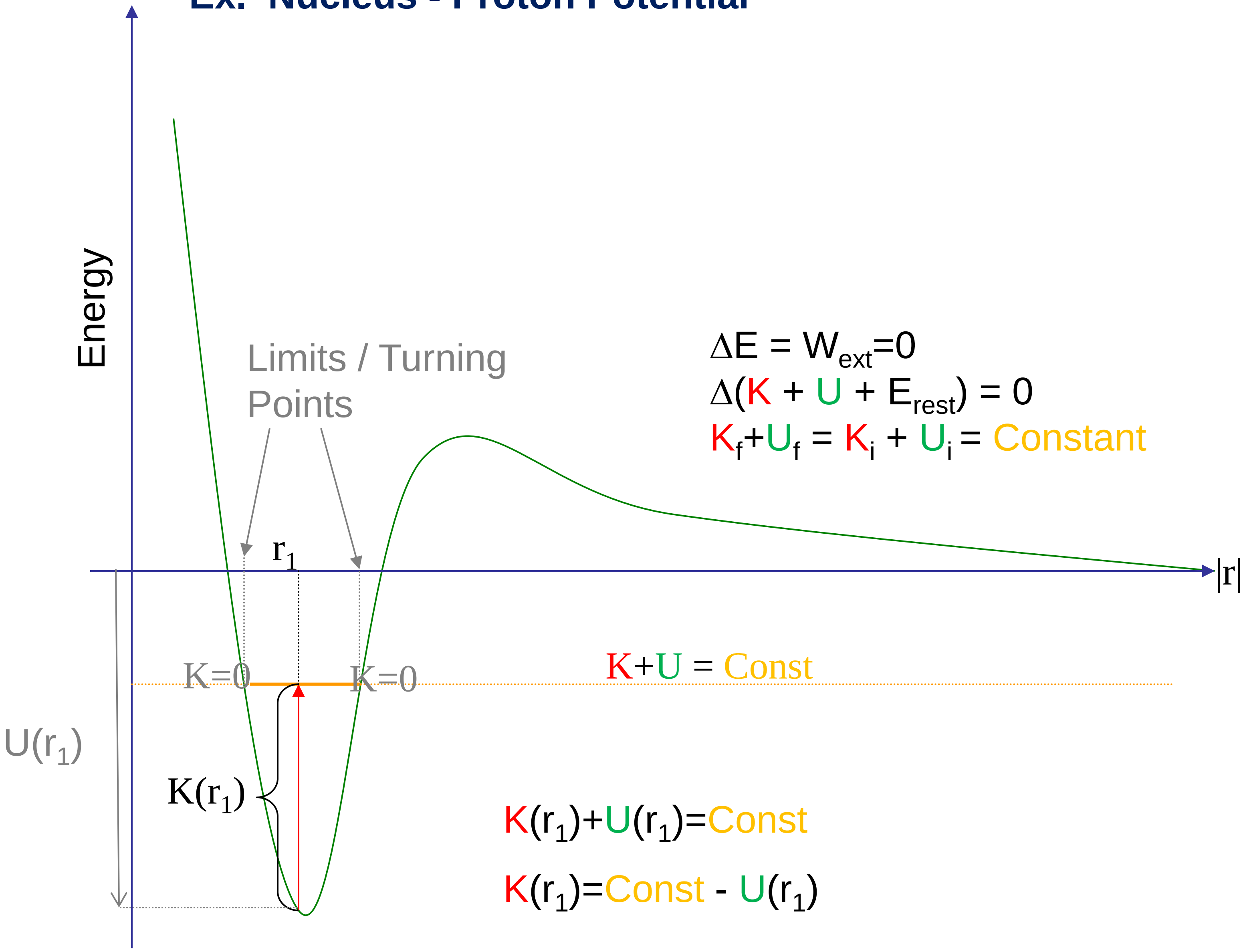
$\Delta(K + U + E_{rest}) = 0$

$K_f + U_f = K_i + U_i = \text{Constant}$

$K(r_1)+U(r_1)=\text{Const}$

$K(r_1)=\text{Const} - U(r_1)$

Conceptual Understanding from Energy Diagrams
Ex. Nucleus - Proton Potential



$\Delta E = W_{ext}=0$

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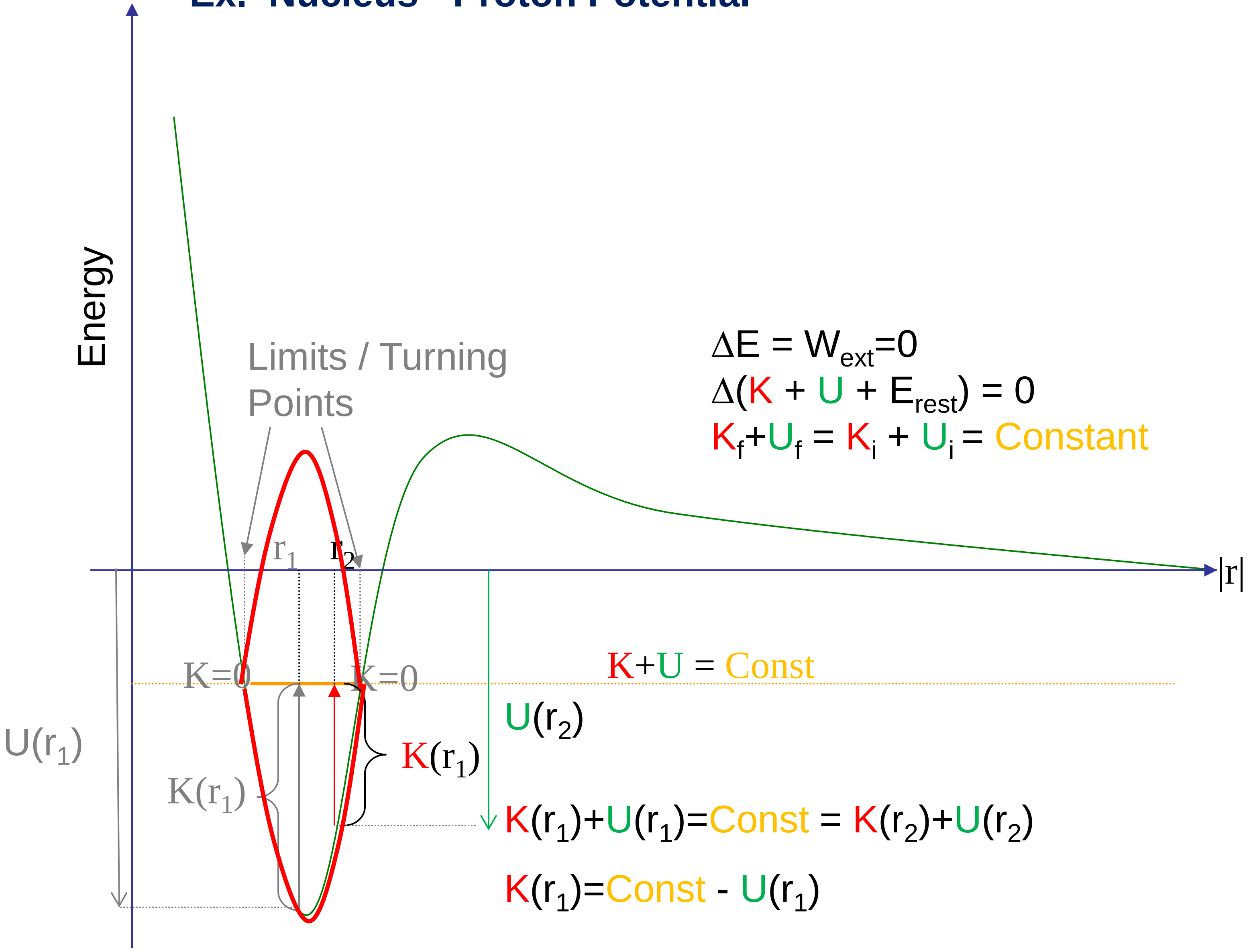
Conceptual Understanding from Energy Diagrams
Ex. Nucleus - Proton Potential



$\Delta E = W_{ext} = 0$

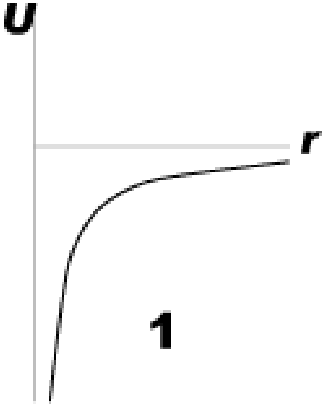
$\Delta(K + U + E_{rest}) = 0$

$K_f + U_f = K_i + U_i = \text{Constant}$

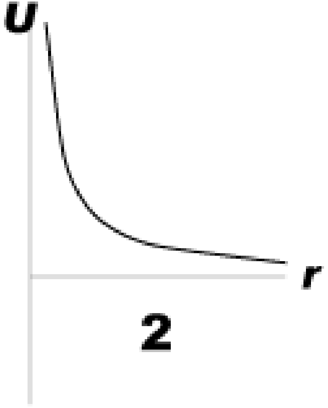
$K(r_1) + U(r_1) = \text{Const} = K(r_2) + U(r_2)$

$K(r_1) = \text{Const} - U(r_1)$

Which of the following graphs of *U* vs *r* represents the gravitational potential energy, $U = -GMm/r$?



1



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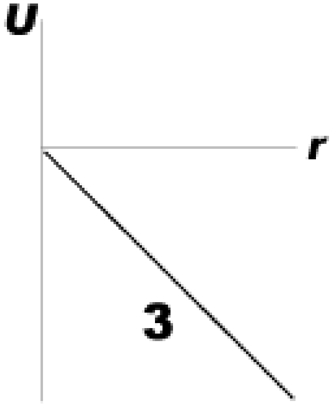

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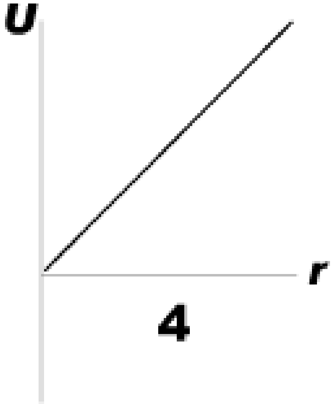

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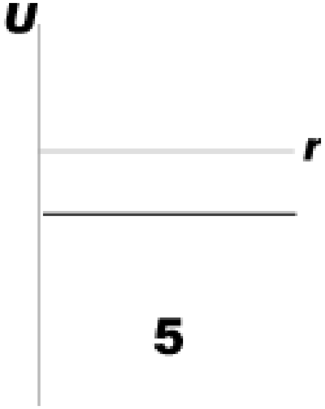
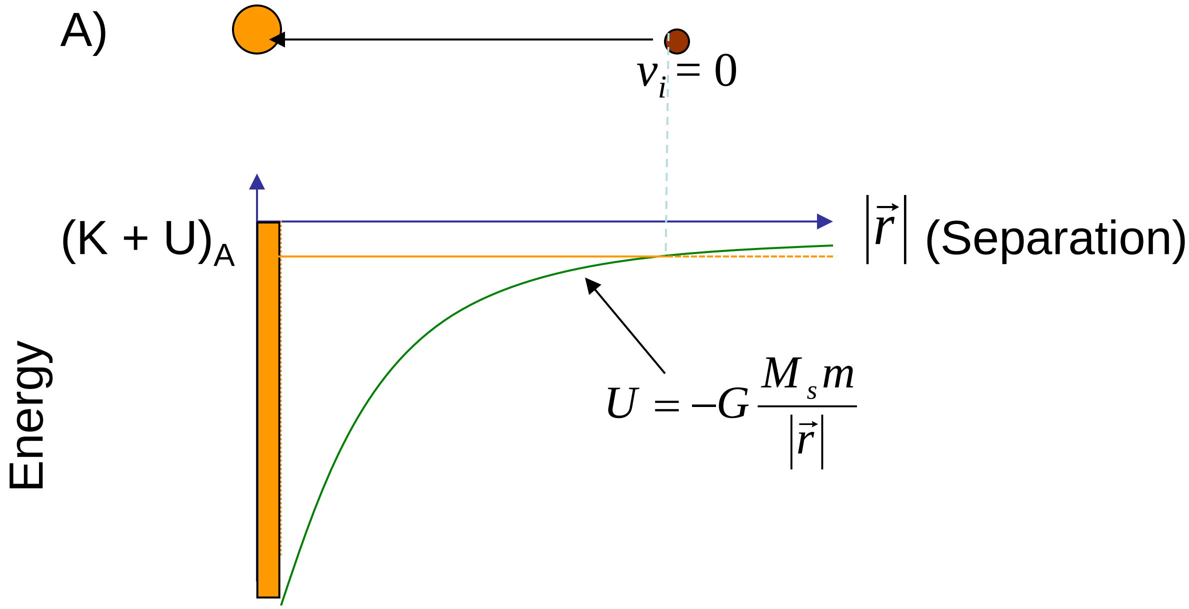


5

Energy Diagrams

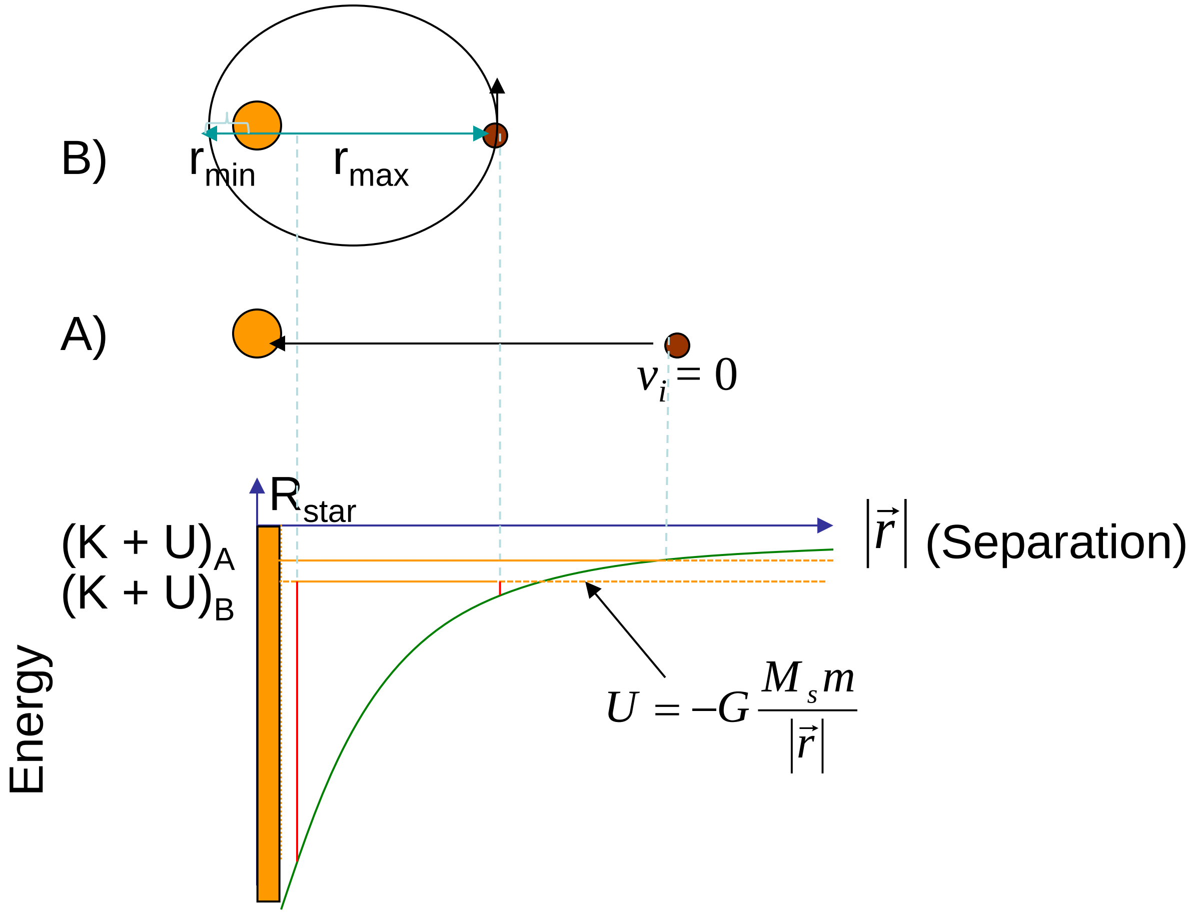
Ex. Gravitational, Bound



$$U = -G\frac{M_s m}{|\vec{r}|}$$

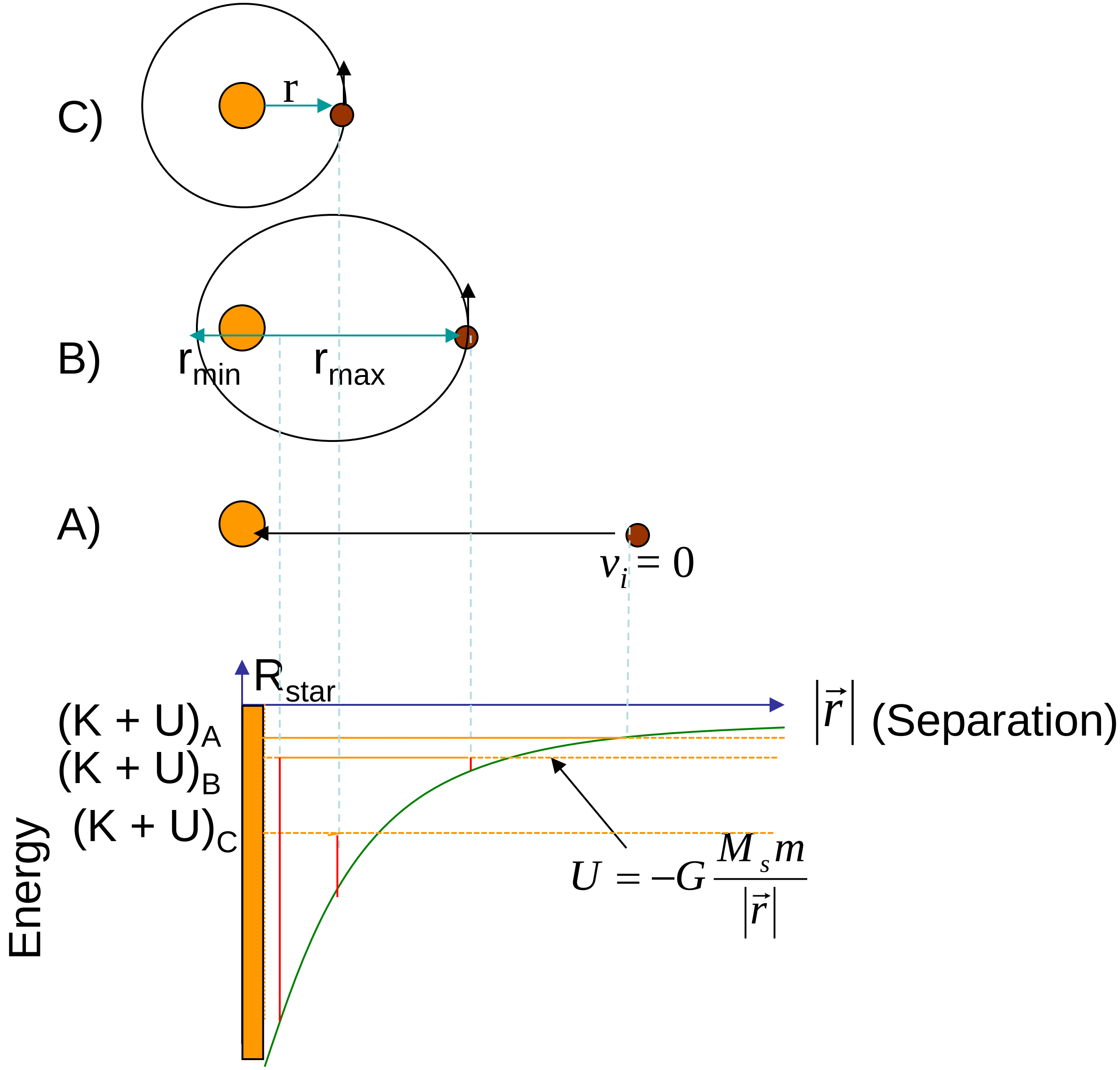
Energy Diagrams

Ex. Gravitational, Bound



Energy Diagrams

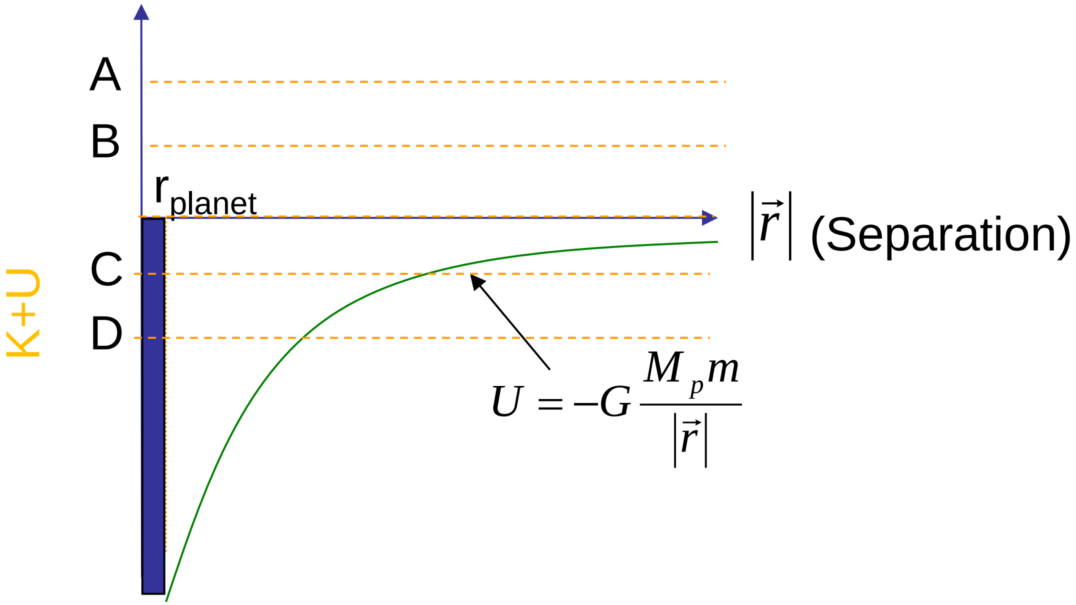
Ex. Gravitational, Bound



Energy Diagrams

Ex. Gravitational, Un-Bound / Escape

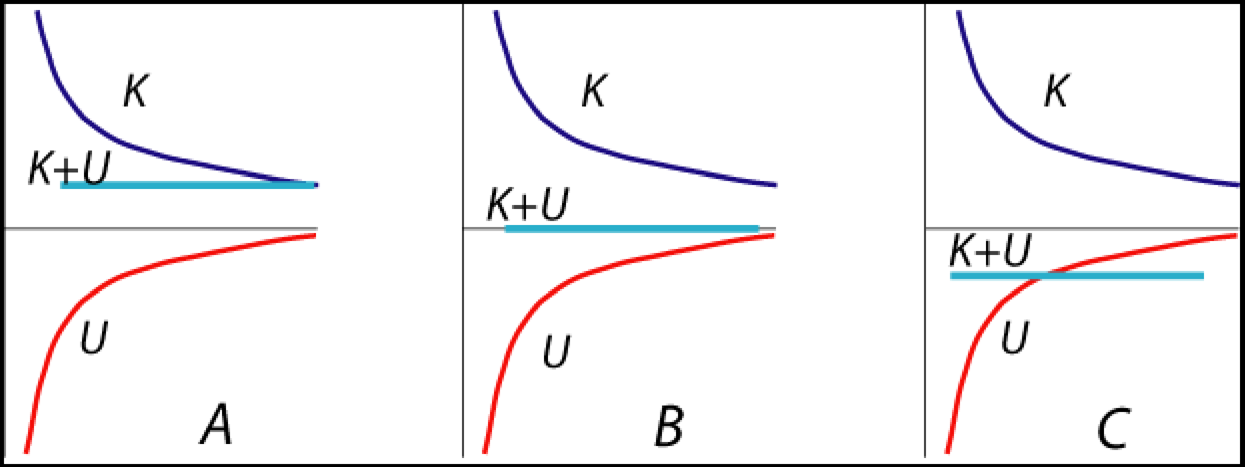
Consider an rocket launching from a planet's surface, which of the following represent *un-bound* systems (so the rocket could get away and never fall back to the planet)?



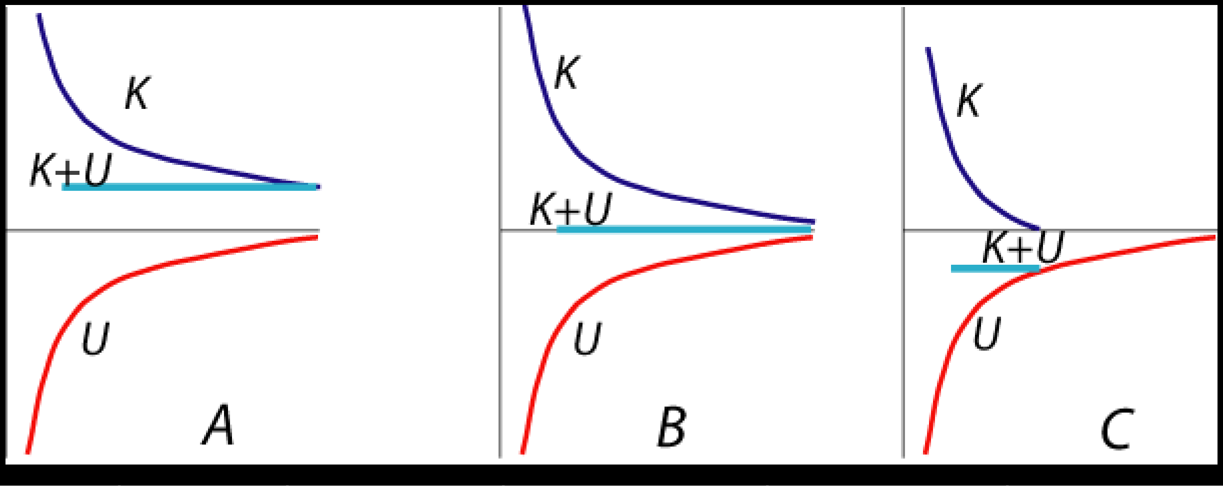
1. A
2. B
3. A & B
4. C
5. D
6. C & D
7. A,B,C, & D

Special Case: K + U = 0 and *Escape Speed*

In which graph does the cyan line correctly represent the sum of kinetic energy plus potential energy?



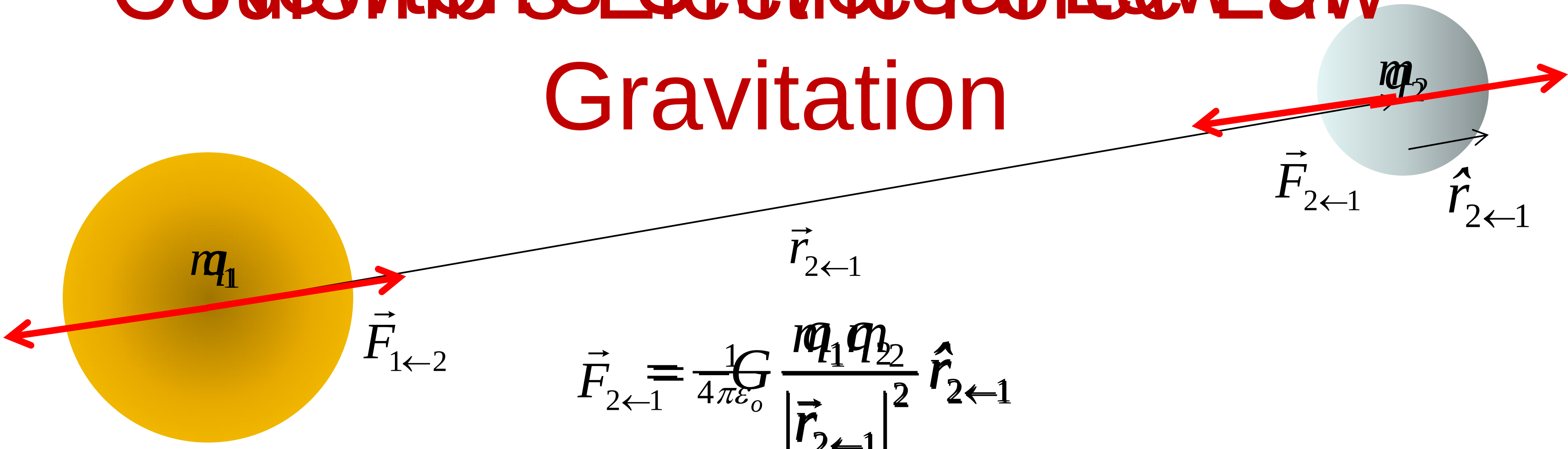
The system is a comet and a star. In which case(s) will the comet escape from the star and never return?



1) A	2) B	3) C	4) A,B	5) B,C	6) A,B,C

Newton's Universal Law of Gravitation

Coulomb's Electric Force Law



$$\vec{F}_{2\leftarrow 1} = -G\frac{m_1 m_2}{\left|\vec{r}_{2\leftarrow 1}\right|^2}\hat{r}_{2\leftarrow 1}$$

$$\vec{F}_{2\leftarrow 1} = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{\left|\vec{r}_{2\leftarrow 1}\right|^2}\hat{r}_{2\leftarrow 1}$$

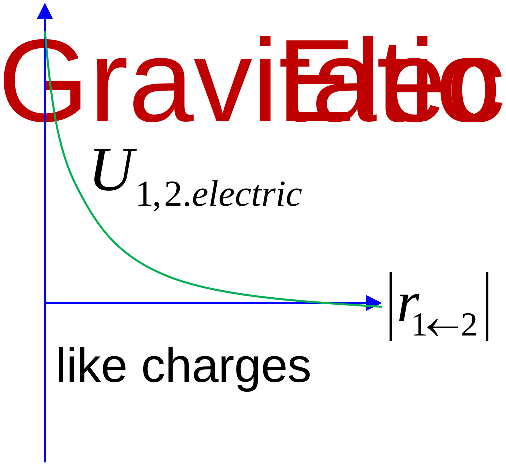
$$G = 6.67\times 10^{-11}\,\frac{N\cdot m^2}{(kg)^2}$$

$$\frac{1}{4\pi\varepsilon_o} = 9\times 10^{9}\,\frac{N\cdot m^2}{(C)^2}$$

$$\hat{r}_{2\leftarrow 1} = \frac{\vec{r}_{2\leftarrow 1}}{\left|\vec{r}_{2\leftarrow 1}\right|}$$

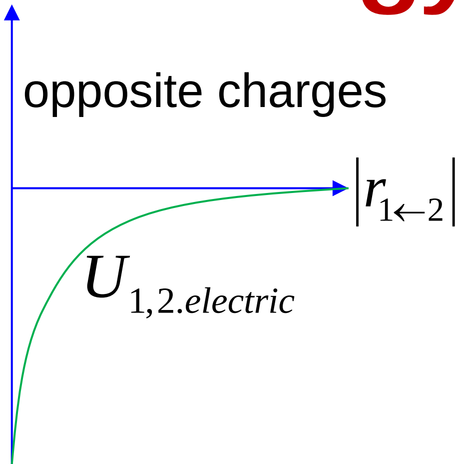
Gravitational Potential Energy

Electric Potential Energy



$$U_{1,2} = -G\frac{m_1 m_2}{\left|r_{1\leftarrow 2}\right|}$$

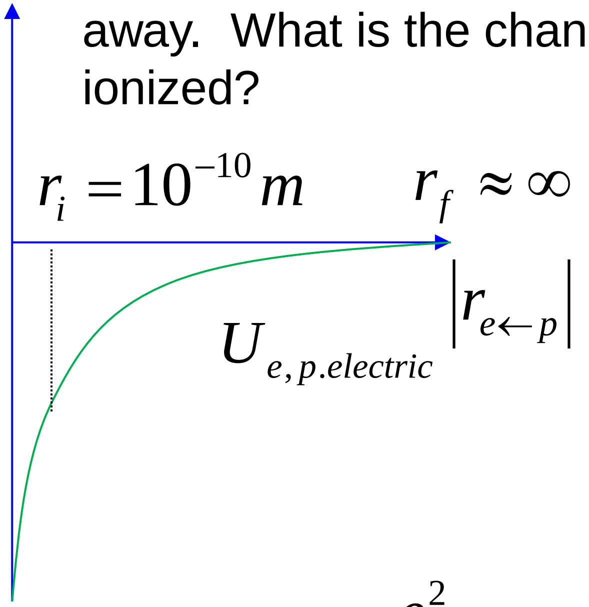
$$U_{1,2} = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{\left|r_{1\leftarrow 2}\right|}$$



Example: Ionize Hydrogen. In a hydrogen atom the electron averages around 10^{-10} m from the proton. When a hydrogen atom is ionized, the electron is stripped away. What is the change in electric potential energy when such an atom is ionized?

System= electron + proton

Active environment = none



$$U_{e,p.elect} = \frac{1}{4\pi\varepsilon_o}\frac{q_e q_p}{|r_{e\leftarrow p}|}$$

$$U_{e,p.elect} = \frac{1}{4\pi\varepsilon_o}\frac{-e^2}{|r_{1\leftarrow 2}|}$$

$$\Delta U_{e,p,elct} = \frac{1}{4\pi\varepsilon_o}\frac{-e^2}{|r_f|} - \frac{1}{4\pi\varepsilon_o}\frac{-e^2}{|r_i|} = \frac{e^2}{4\pi\varepsilon_o}\left(\frac{1}{|r_i|} - \cancelto{0}{\frac{1}{|r_f|}}\right) \leftarrow \infty$$

$$\Delta U_{e,p,elct} = \frac{1}{4\pi\left(8.85\times10^{-12}\,\frac{C^2}{Nm^2}\right)}\left(\frac{\left(1.6\times10^{-19}C\right)^2}{10^{-10}m}\right)$$

$$\Delta U_{e,p,elct} = 2.3\times10^{-18}\,J$$

Or in eV's (divide by electron charge)

$$= 2.3\times10^{-18}\,J\,\frac{1e}{1.6\times10^{-19}\,C} \quad = 14eV$$

Comparison:
Electric vs. Gravitational

$$\frac{U_{e,p.elect}}{U_{e,p.grav}} = \frac{\frac{1}{4\pi\varepsilon_o}\frac{-e^2}{|r_{1\leftarrow 2}|}}{-G\frac{m_e m_p}{|r_{1\leftarrow 2}|}}$$

$$\frac{U_{e,p.elect}}{U_{e,p.grav}} = \frac{1}{4\pi\varepsilon_o G}\frac{e^2}{m_e m_p}$$

$$\frac{9\times10^9\,\frac{Nm^2}{C^2}\left(1.6\times10^{-19}C\right)^2}{\left(6.7\times10^{-11}\,\frac{Nm^2}{kg^2}\right)\left(9\times10^{-31}kg\right)\left(1.7\times10^{-27}kg\right)}$$

$$\frac{U_{e,p.elect}}{U_{e,p.grav}} = 5.6\times10^{39}$$

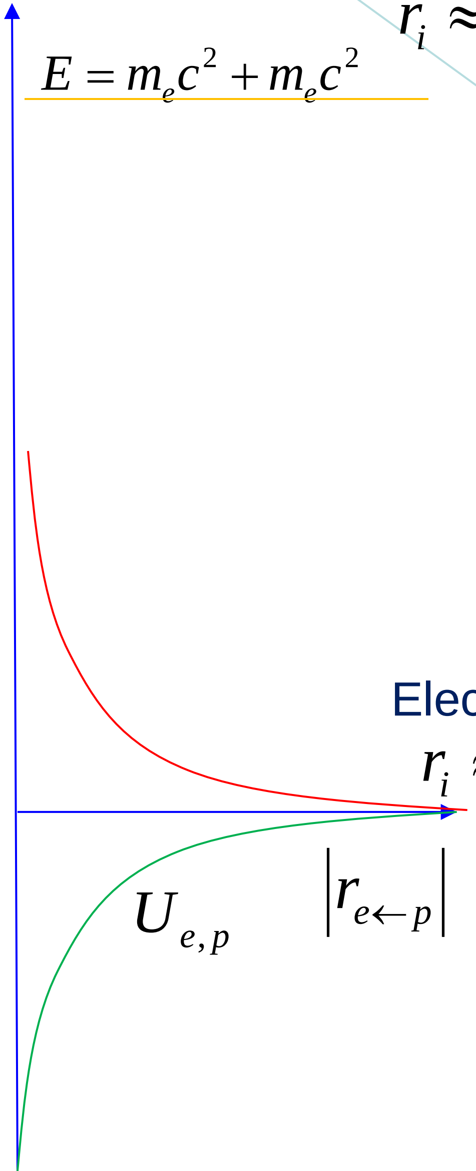
Return to Rest Energy and Mass

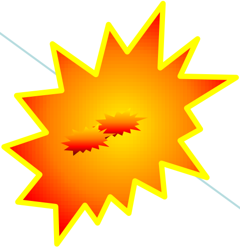
Pair (electron and positron) Annihilation

-e

$r_i \approx \infty$

Application Note:
Positron Electron Tomography (PET) Scans





initial final

$$e^- + e^+ \rightarrow \gamma + \gamma$$

Electron and positron Two photons (light pulses)

$$E = 2m_e c^2 = 2E_\gamma$$

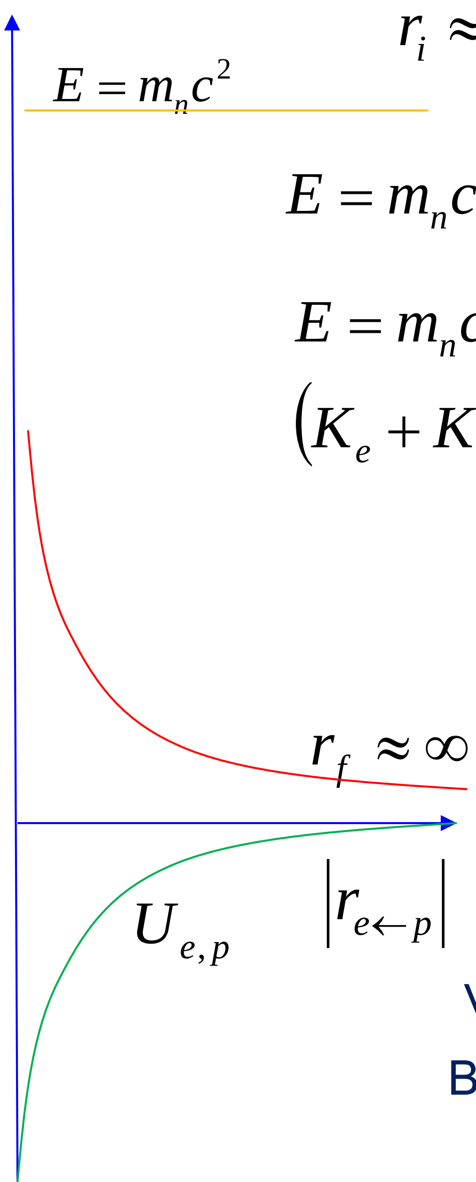
$$\left(0.511 MeV / c^2\right) c^2 = E_\gamma$$

$$0.511 MeV = E_\gamma$$

+e

Return to Rest Energy and Mass

Neutron Decay



initial final

$$n^0 \rightarrow p^+ + e^- + \bar{\nu}_e$$

neutron Proton, electron, and neutrino

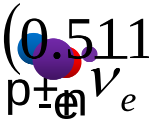
Nearly massless Finally infinitely far apart

$$E = m_n c^2 = m_e c^2 + m_p c^2 + m_\nu c^2 + K_e + K_p + K_\nu + U_{e,p} + U_{e,\nu} + U_{\nu,p}$$

$$E = m_n c^2 = m_e c^2 + m_p c^2 + K_e + K_p + K_\nu$$

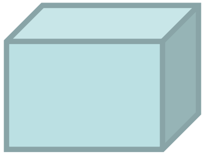
$$\left(K_e + K_p + K_\nu \right) = m_n c^2 - \left(m_e c^2 + m_p c^2 \right)$$

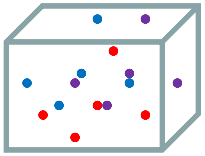
$$= 939.6 MeV - \left(0.511 MeV + 938.3 MeV \right) = 0.79 MeV$$



Mass as Energy and Energy as Mass

Box o' decaying Neutrons





$$E = E_{rest} = m_{box} c^2 \quad = \sum_{all.particles} \left(\left(m_e + m_p + m_\nu \right) c^2 + K_e + K_p + K_\nu + U_{e,p} \right)$$

Viewed from outside Peaking inside

Box's mass *includes* internal kinetic and potential energies

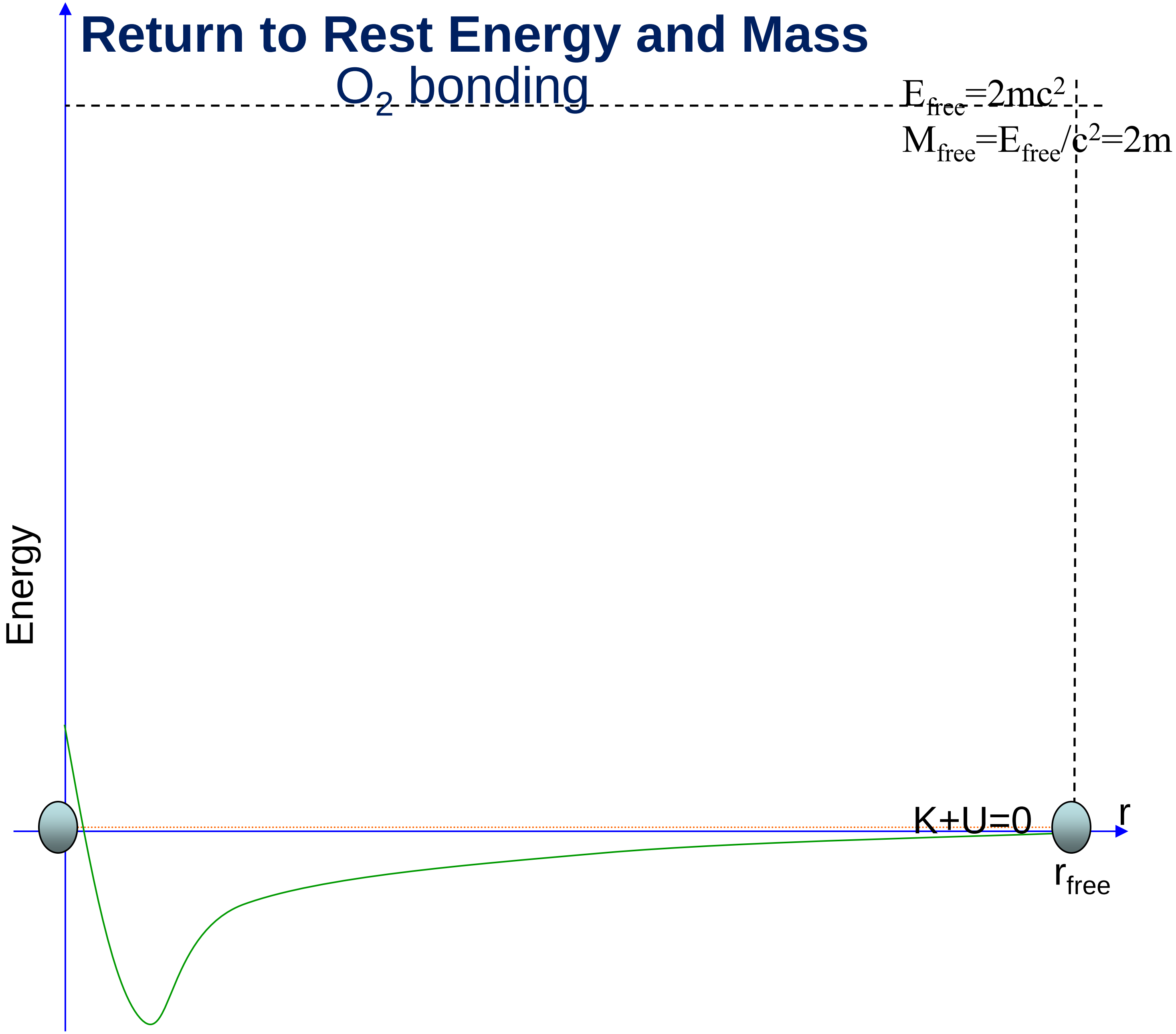
What *is* Mass

Quantification of…

- Gravitational ‘charge’
- Inertia
- Internal Energy

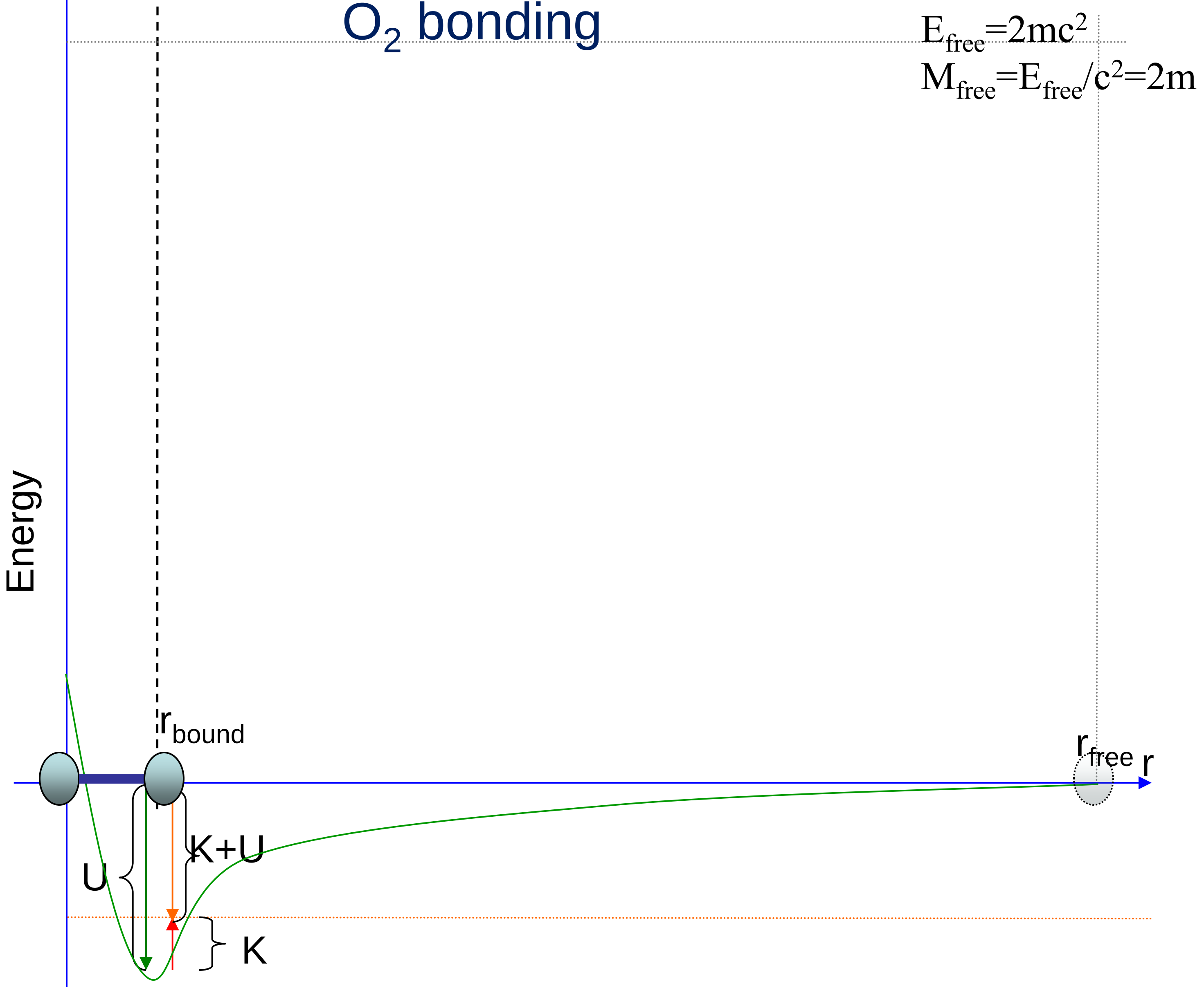
Return to Rest Energy and Mass

O_2 bonding



Return to Rest Energy and Mass

O_2 bonding

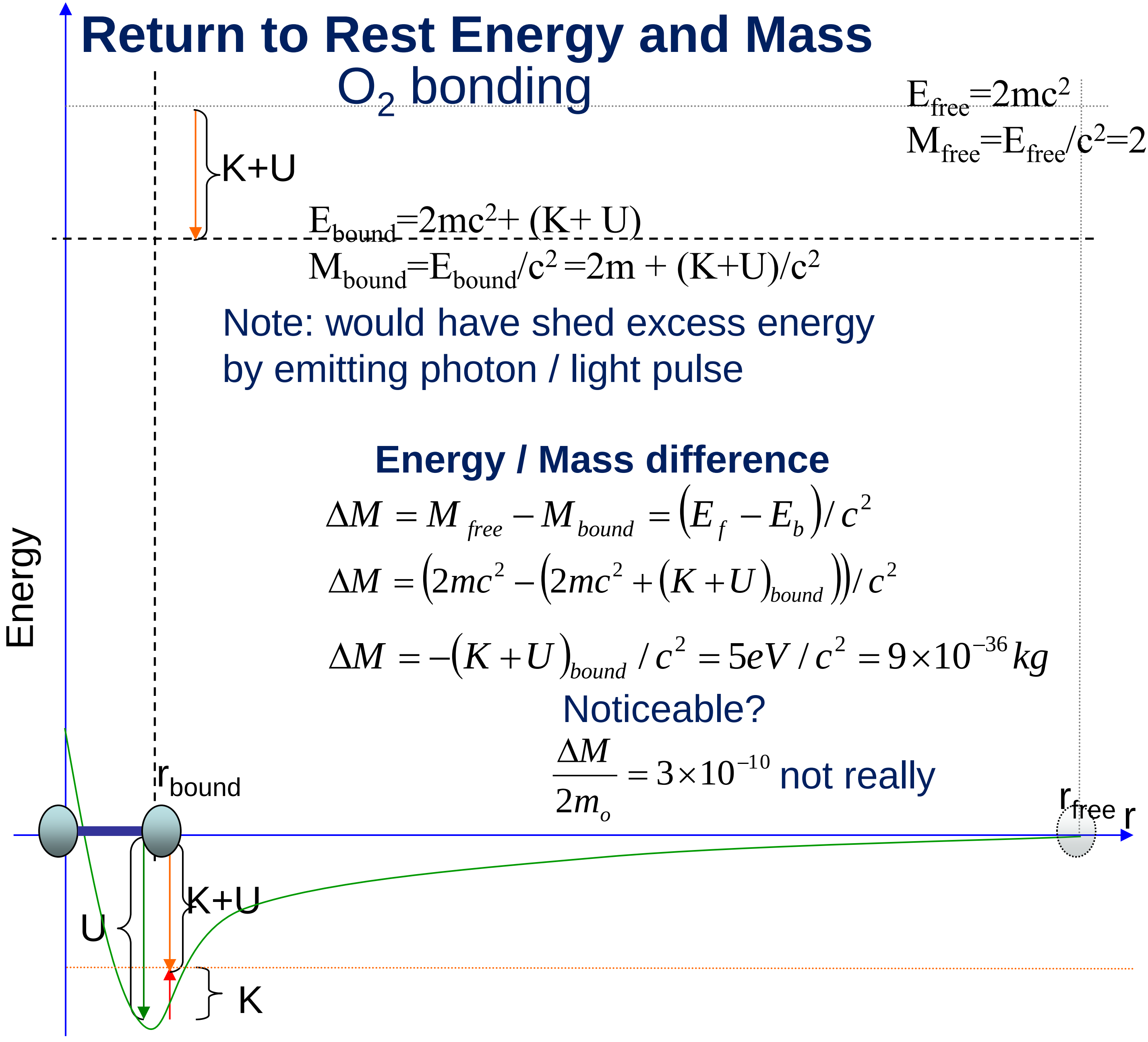


$E_{free}=2mc^2$

$M_{free}=E_{free}/c^2=2m$

Return to Rest Energy and Mass

O_2 bonding



$E_{free}=2mc^2$
$M_{free}=E_{free}/c^2=2m$

$E_{bound}=2mc^2+(K+U)$
$M_{bound}=E_{bound}/c^2=2m+(K+U)/c^2$

Note: would have shed excess energy by emitting photon / light pulse

Energy / Mass difference

$$\Delta M = M_{free} - M_{bound} = \left(E_f - E_b\right)/c^2$$

$$\Delta M = \left(2mc^2 - \left(2mc^2 + \left(K+U\right)_{bound}\right)\right)/c^2$$

$$\Delta M = -\left(K+U\right)_{bound}/c^2 = 5eV/c^2 = 9\times 10^{-36}kg$$

Noticeable?

$$\frac{\Delta M}{2m_o} = 3\times 10^{-10}$$ not really

Return to Rest Energy and Mass

Nuclear Binding: Iron nucleus

If an iron nucleus were disintegrated, how much K + U energy would be consumed /produced?

initial final

$$Fe_{56}^{26} \rightarrow 26p^{+} + 30n$$

Iron nucleus Protons and neutrons

Useful info

$M_{Fe.nuc} = 52107 MeV/c^2$

$m_n = 939.9 MeV/c^2$

$m_p = 938.3 MeV/c^2$

Noticeable?

$$\frac{\Delta mc^2}{m_{Fe}c^2} = 0.009 \approx 1\%$$

yes

$$E_i = E_f$$

$$E_{r.Fe} = \sum_{all.particles}(E_r + K) + \sum_{all.pairs} U$$

$$m_{Fe}c^2 = 26 \cdot m_p c^2 + 30 \cdot m_n c^2 + \left(\sum_{all.particles} K + \sum_{all.pairs} U \right)$$

$$m_{Fe}c^2 - \left(26 \cdot m_p c^2 + 30 \cdot m_n c^2\right) = \left(\sum_{all.particles} K + \sum_{all.pairs} U \right)$$

$$52107 MeV - \left(26 \cdot (939.9 MeV) + 30 \cdot (938.3 MeV)\right) = \left(\sum_{all.particles} K + \sum_{all.pairs} U \right)$$

$$-482 MeV = \left(\sum_{all.particles} K + \sum_{all.pairs} U \right)$$

Rest *and* Electric-Potential *and* Kinetic

A U-235 nucleus is struck by a slow-moving neutron, so that the merge and become U-236, with mass $M_{U\text{-}236}$ This nucleus is unstable to falling apart – fission. One way it could do so is to first slosh into something of a dumbbell shape, now most of the into two symmetric nuclei, Pd-118, with mass $M_{Pd\text{-}118}$, each has ½ the original number of protons, i.e., $q_{Pd} = 46e$. Having fallen apart, the two palladium nuclei no longer experience a Strong interaction holding them together, just the Electric repulsion of each other's protons. Subsequently, they accelerate away.

a) What's the final speed of one of the Pd atoms, when they have sped far, far apart?

b) What is the distance between the Pd atoms just after fission?

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