Fri.	2.6 – .8 Constant Force, time estimates, Models	RE 2.c
Mon. Tues	3.1 – .5, .14-.15 Fundamental Forces, Gravitation	RE 3.a EP 2, HW2: Ch 2 Pr's 40, 57, 63, 67 & CP

- **Non-constant Force** – spring intro. continued
- **Constant force** – gravitation (near Earth)
- **Time estimates for collisions**

example with the spring over here, I hang 4.9 N (0.5 kg) weight from it, and it stretches by _______m. So, what's it's stiffness?

Q 2.5 c

A spring is 12 cm (0.12 m) long when relaxed. Its stiffness is 30 N/m. You push on the spring, compressing it so its length is now 10 cm (0.10 m).

What is the magnitude of the force the spring now exerts on your hand?

a) 0.6 N
b) 3 N
c) 3.6 N
d) 30 N

Three ways to Explore

Experiment: Observe Motion

Compute: Simulate Motion (with force and momentum visualized)

while $t < t_{max}$:

$$\vec{F}_{net \to object} \Leftarrow -k_s * \left(\left| \vec{r}_{object} \right| - L_{eq} \right) \hat{r}$$

$$\vec{p}_{object} \Leftarrow \vec{p}_{object} + \vec{F}_{net \to object} \Delta t$$

$$\vec{r}_{object} \Leftarrow \vec{r}_{object} + \frac{\vec{p}_{object}}{m_{object}} \Delta t$$

$$t \Leftarrow t + \Delta t$$

Analytical: (for a later chapter) build and solve ‘equations of motion.’

2.6 Constant Force –near-Earth gravitation

A ball is initially on the ground, and you kick it with initial velocity < 3,7,0> m/s. At this speed air resistance is negligible. Assume the usual coordinate system.

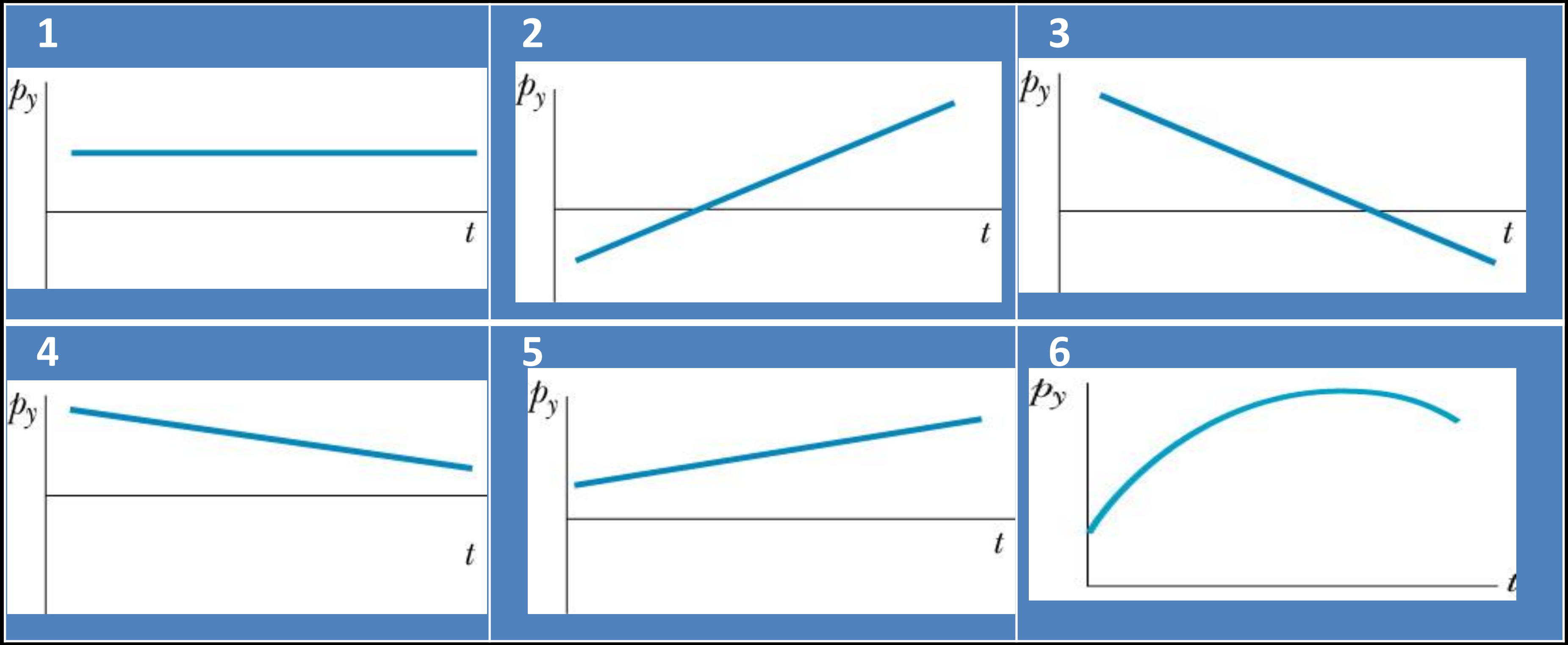
Which component(s) of the ball's momentum will change in the *next* half second (after the ball’s left your foot)?

1) p_x
2) p_y
3) p_z
4) p_x & p_y
5) p_y & p_z
5) p_z & p_x
7) p_x, p_y, & p_z

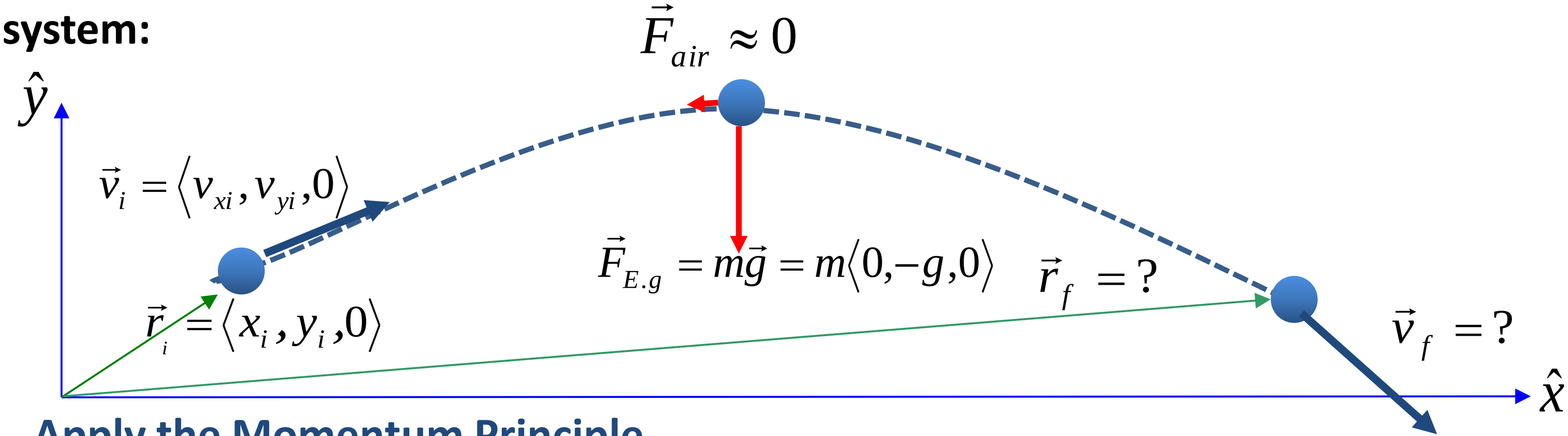
The initial momentum of the ball was < 1.5, 3.5, 0 > kg*m/s.
The final momentum of the ball is < 1.5, 1.05, 0 > kg*m/s.

Therefore...

Q2.6.c: Which graph correctly shows p_y for the ball during this 0.5 s?



You throw a ball so that just after it leaves your hand at location $\vec{r}_i = \langle x_i, y_i, 0 \rangle$ it has velocity $\vec{v}_i = \langle v_{xi}, v_{yi}, 0 \rangle$. Now that it has left your hand, (and we're neglecting air resistance) the net force all the time is $\vec{F}_{net} \approx \vec{F}_{E.g} = \langle 0, -mg, 0 \rangle$. *What are the velocity and position of the ball after time Δt?*



Apply the Momentum Principle

$$\vec{p}_f = \vec{p}_i + \vec{F}_{net.avg}\Delta t$$

$$\vec{p}_f = \vec{p}_i + \vec{F}_{E.g}\Delta t$$ Presumably v<<c

$$m\vec{v}_f = m\vec{v}_i + \vec{F}_{E.g}\Delta t$$

$$m\vec{v}_f = m\vec{v}_i + m\vec{g}\Delta t$$

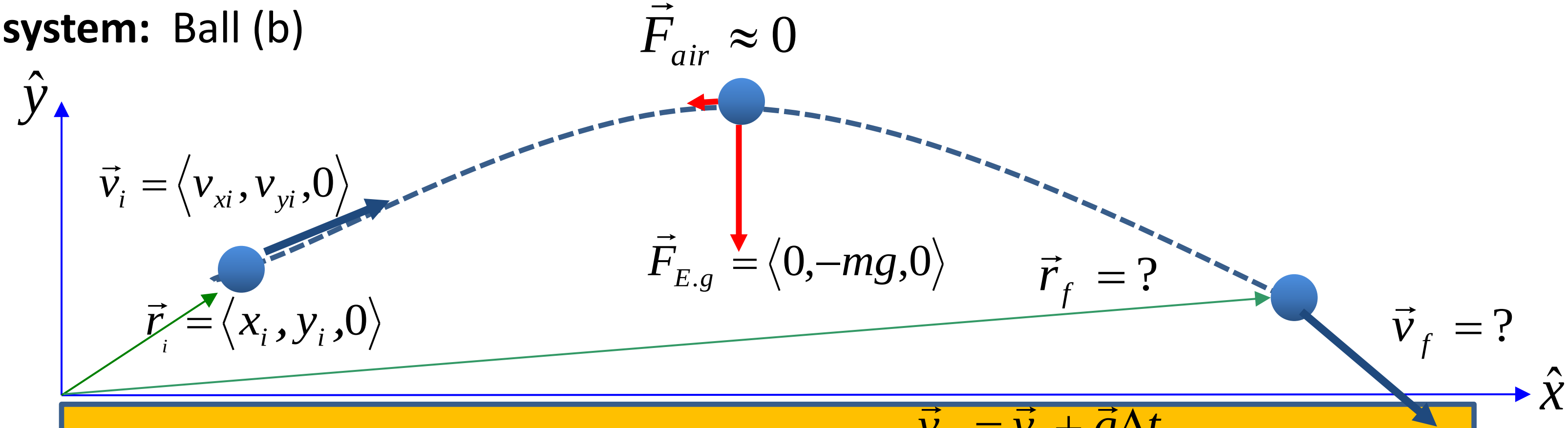
$$\vec{v}_f = \vec{v}_i + \vec{g}\Delta t$$

$$\langle v_{xf}, v_{yf}, 0 \rangle = \langle v_{xi}, v_{yi}, 0 \rangle + \langle 0, -g, 0 \rangle \Delta t$$

$\hat{x}$: $v_{xf} = v_{xi}$

$\hat{y}$: $v_{yf} = v_{yi} - g\Delta t$

system: Ball (b)



Apply the Momentum Principle

$$\vec{v}_f = \vec{v}_i + \vec{g}\Delta t$$

$\hat{x}:\quad v_{xf} = v_{xi}$ $\qquad$ $\hat{y}:\quad v_{yf} = v_{yi} - g\Delta t$

Apply Position Update

$$\vec{r}_f = \vec{r}_i + \vec{v}_{avg}\Delta t$$

$$\vec{v}_{avg} = \frac{\vec{v}_i + \vec{v}_f}{2} = \frac{\vec{v}_i + \left(\vec{v}_i + \vec{g}\Delta t\right)}{2} = \vec{v}_i + \tfrac{1}{2}\vec{g}\Delta t$$

$$\vec{r}_f = \vec{r}_i + \left(\vec{v}_i + \tfrac{1}{2}\vec{g}\Delta t\right)\Delta t$$

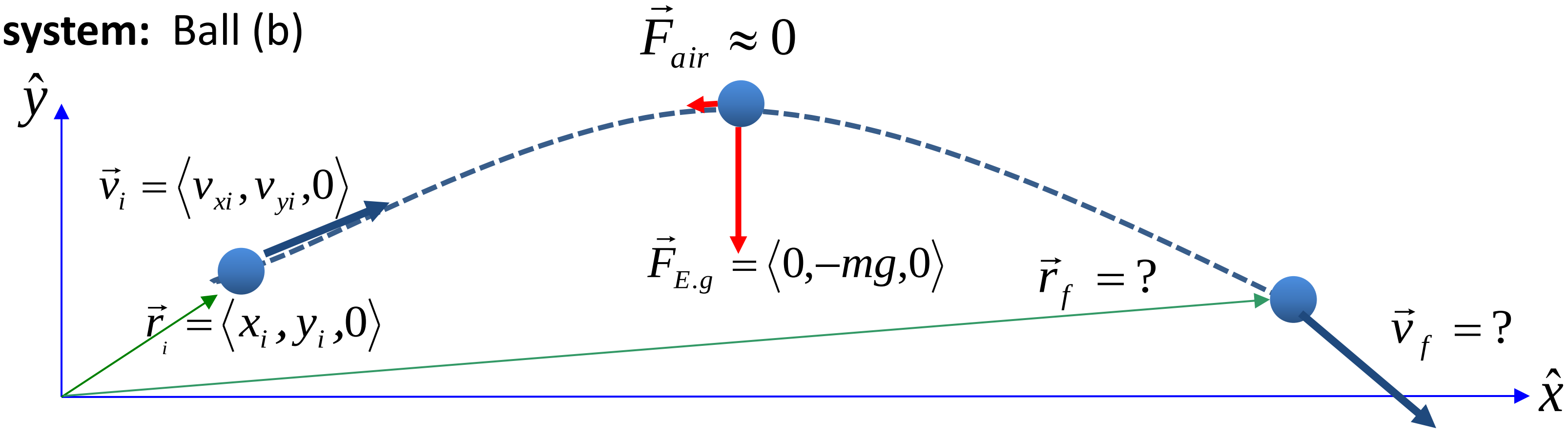
$$\vec{r}_f = \vec{r}_i + \vec{v}_i\Delta t + \tfrac{1}{2}\vec{g}\left(\Delta t\right)^2$$

$$\langle x_f, y_f, 0 \rangle = \langle x_i, y_i, 0 \rangle + \langle v_{ix}, v_{iy}, 0 \rangle \Delta t + \tfrac{1}{2}\langle 0, -g, 0 \rangle \left(\Delta t\right)^2$$

$\hat{x}\qquad x_f = x_i + v_{ix}\Delta t$

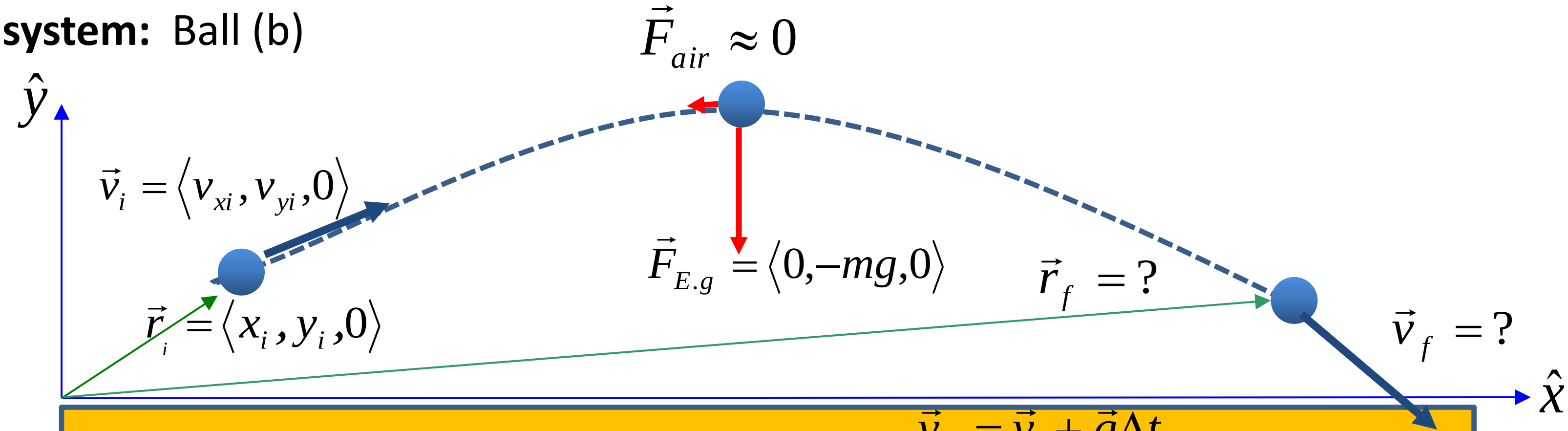
$\hat{y}\qquad y_f = y_i + v_{iy}\Delta t - \tfrac{1}{2} g\left(\Delta t\right)^2$

system: Ball (b)



Initially the velocity of the ball is < 3 , 7 ,0 > m/s. After 0.5 s, the ball's velocity is < 3, 2.1, 0 > m/s. What is the best choice for the y-component of the ball's average velocity during this interval?	1) 2.10 m/s 2) 4.55 m/s 3) 4.90 m/s 4) 7.00 m/s 5) 9.10 m/s

system: Ball (b)



Apply the Momentum Principle

$$\vec{v}_f = \vec{v}_i + \vec{g}\Delta t$$

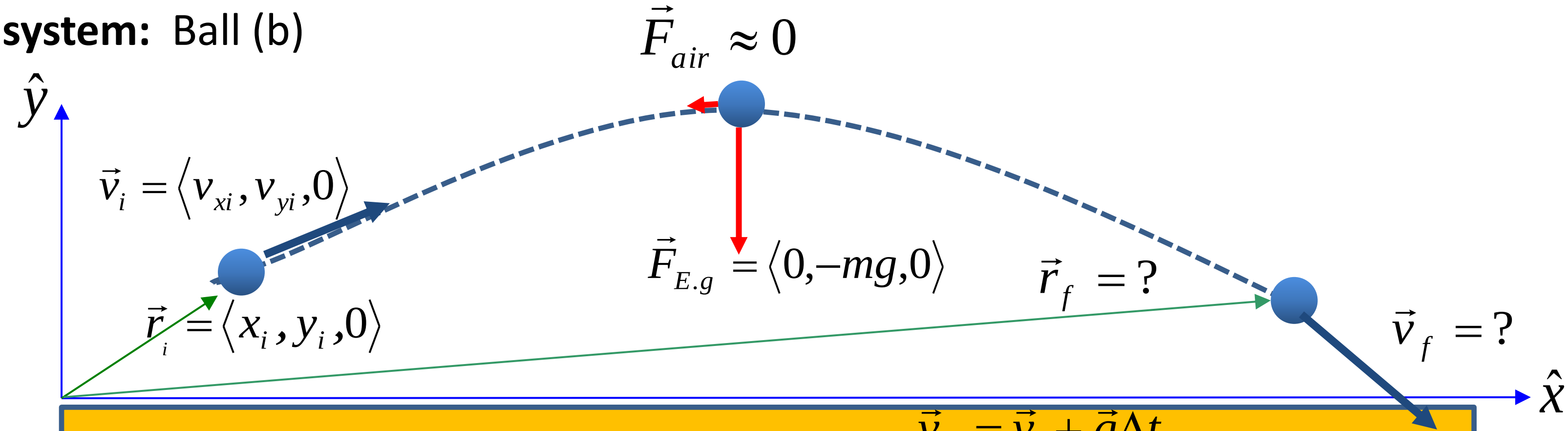
$\hat{x}$: $v_{xf} = v_{xi}$ $\quad$ $\hat{y}$: $v_{yf} = v_{yi} - g\Delta t$

Apply Position Update

$$\vec{r}_f = \vec{r}_i + \vec{v}_i \Delta t + \tfrac{1}{2}\vec{g}(\Delta t)^2$$

$\hat{x}$: $x_f = x_i + v_{ix}\Delta t$ $\quad$ $\hat{y}$: $y_f = y_i + v_{iy}\Delta t - \tfrac{1}{2} g(\Delta t)^2$

system: Ball (b)



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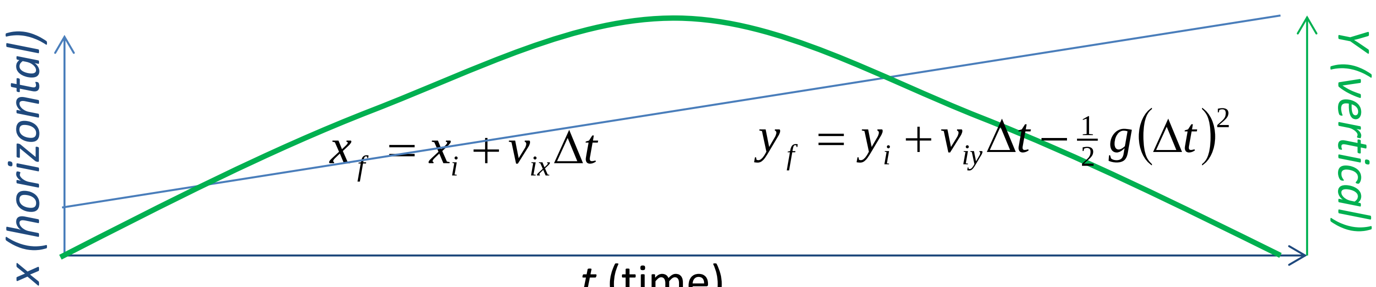
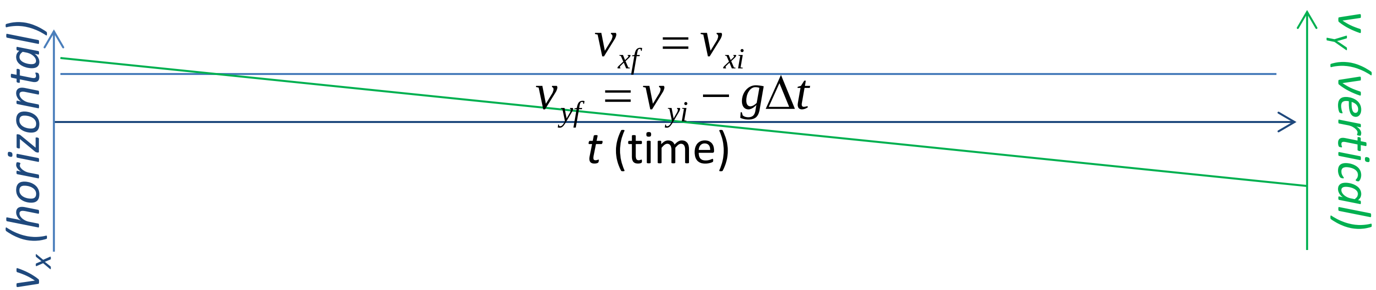
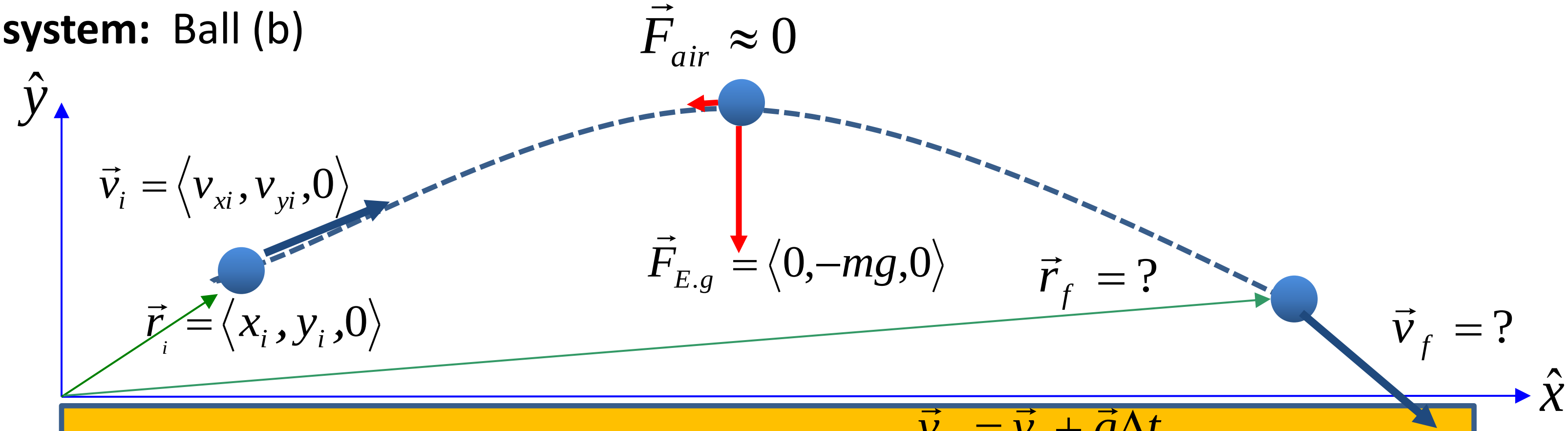
$\hat{x}$: $x_f = x_i + v_{ix}\Delta t$ $\quad$ $\hat{y}$: $y_f = y_i + v_{iy}\Delta t - \tfrac{1}{2} g(\Delta t)^2$

Graphical Representations





system: Ball (b)



Apply the Momentum Principle

$$\vec{v}_f = \vec{v}_i + \vec{g}\Delta t$$

$$\hat{x}: \quad v_{xf} = v_{xi} \qquad \hat{y}: \quad v_{yf} = v_{yi} - g\Delta t$$

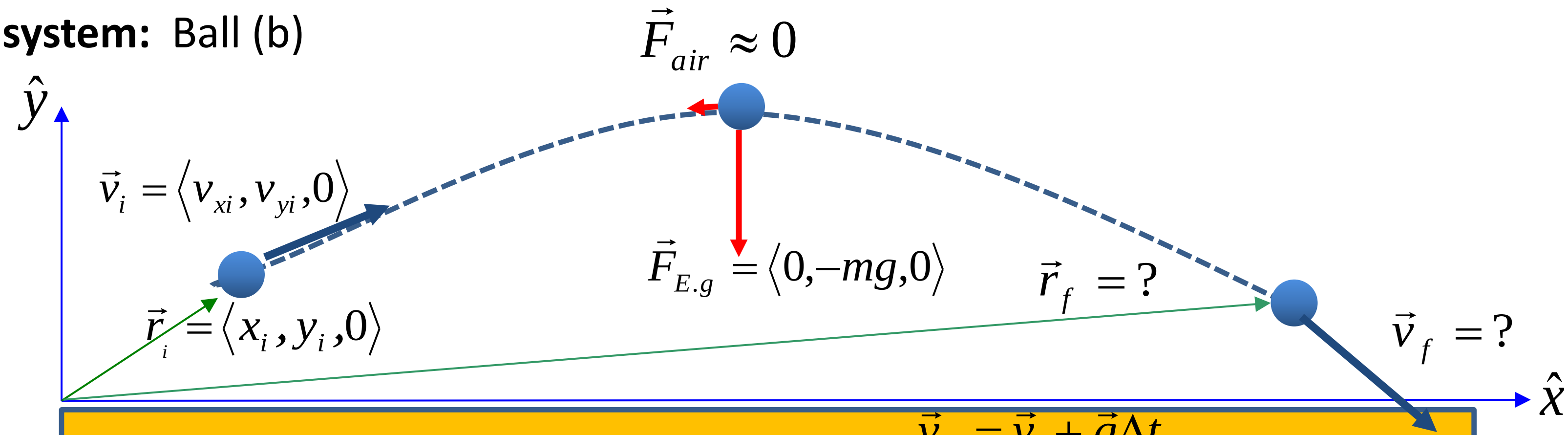
Apply Position Update

$$\vec{r}_f = \vec{r}_i + \vec{v}_i \Delta t + \tfrac{1}{2}\vec{g}(\Delta t)^2$$

$$\hat{x}: \quad x_f = x_i + v_{ix}\Delta t \qquad \hat{y}: \quad y_f = y_i + v_{iy}\Delta t - \tfrac{1}{2} g(\Delta t)^2$$

Example: If you throw a ball horizontally off of a 10-m high cliff at 5 m/s, how far from the base of the cliff will it hit the ground?

system: Ball (b)



Apply the Momentum Principle

$$\vec{v}_f = \vec{v}_i + \vec{g}\Delta t$$

$\hat{x}$: $v_{xf} = v_{xi}$ $\quad$ $\hat{y}$: $v_{yf} = v_{yi} - g\Delta t$

Apply Position Update

$$\vec{r}_f = \vec{r}_i + \vec{v}_i \Delta t + \tfrac{1}{2}\vec{g}(\Delta t)^2$$

$\hat{x}$: $x_f = x_i + v_{ix}\Delta t$ $\quad$ $\hat{y}$: $y_f = y_i + v_{iy}\Delta t - \tfrac{1}{2} g(\Delta t)^2$

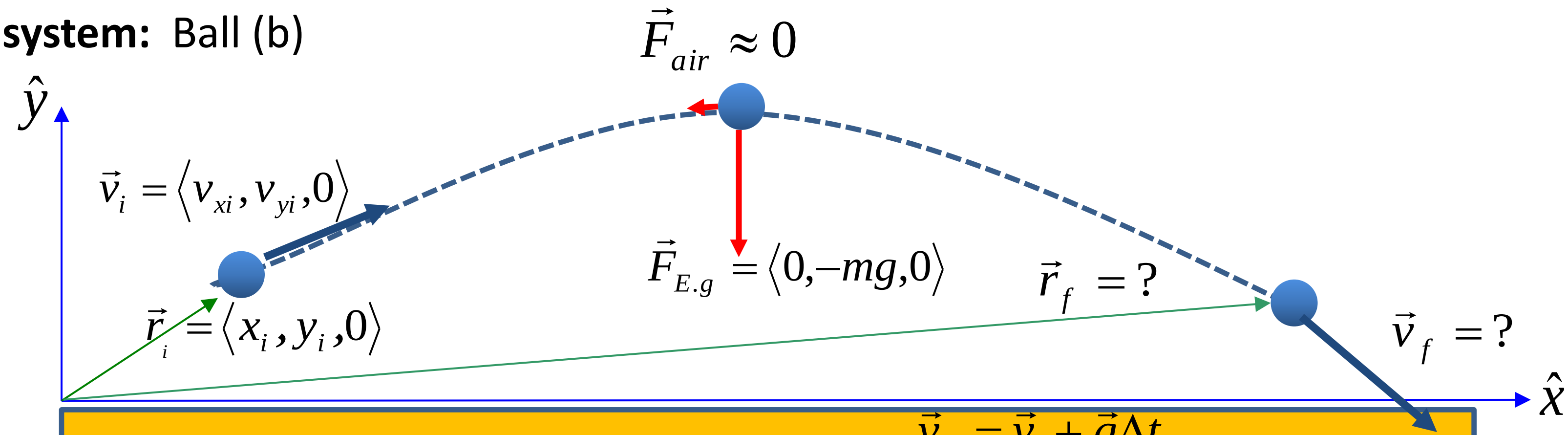
Example: At the start of a football game, the kickoff has an initial speed of 22 m/s at 40° above horizontal.

a) How long does the ball stay in the air?

b) How far away does the ball hit the ground?

c) What is the maximum height that the ball reaches?

system: Ball (b)



Apply the Momentum Principle

$$\vec{v}_f = \vec{v}_i + \vec{g}\Delta t$$

$\hat{x}$: $v_{xf} = v_{xi}$ $\quad\quad$ $\hat{y}$: $v_{yf} = v_{yi} - g\Delta t$

Apply Position Update

$$\vec{r}_f = \vec{r}_i + \vec{v}_i\Delta t + \tfrac{1}{2}\vec{g}(\Delta t)^2$$

$\hat{x}$: $x_f = x_i + v_{ix}\Delta t$ $\quad\quad$ $\hat{y}$: $y_f = y_i + v_{iy}\Delta t - \tfrac{1}{2}g(\Delta t)^2$

Exercise: You have probably seen a basketball player throw the ball to a teammate at the other end of the court, 30 m away.

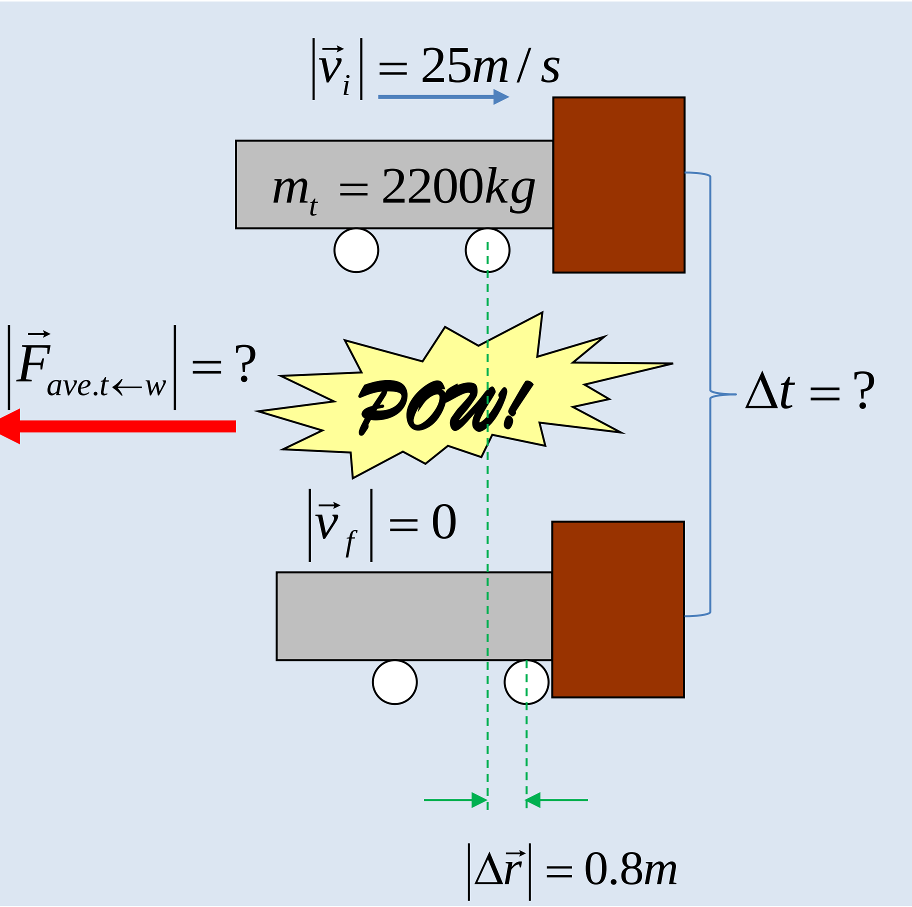
a) Estimate a reasonable initial angle for such a throw, and then determine the corresponding initial speed.

b) For your chosen angle, how long does it take for the ball to go the length of the court?

c) What is the highest point along the trajectory, relative to the thrower's hand?

Example of Estimating collision force

Say a 2200 kg truck, going 25m/s, hits a brick wall and comes to a dead stop. In the process, the truck's hood crumples back 0.8 m. Estimate the magnitude of the average force of the collision.



$$\vec{F}_{avg.t\leftarrow w} = \frac{\Delta\vec{p}}{\Delta t}$$

$$\left|\vec{F}_{avg.t\leftarrow w}\right| \approx m_t \frac{\left|\vec{v}_f - \vec{v}_i\right|}{\Delta t} \quad (\vec{v}_f \to 0)$$

$$\left|\vec{F}_{avg.t\leftarrow w}\right| \approx m_t \frac{\left|\vec{v}_i\right|}{\Delta t}$$

$$\Delta t = \left|\frac{\Delta\vec{r}}{\vec{v}_{avg}}\right|$$

$$\left|\vec{v}_{avg}\right| \approx \left|\frac{\vec{v}_f + \vec{v}_i}{2}\right| = \left|\frac{\vec{v}_i}{2}\right| \quad (\vec{v}_f \to 0)$$

$$\Delta t = \left|\frac{2\Delta\vec{r}}{\vec{v}_i}\right|$$

$$\left|\vec{F}_{avg.t\leftarrow w}\right| \approx m_t \frac{\left|\vec{v}_i\right|^2}{2\left|\Delta\vec{r}\right|}$$

$$\left|\vec{F}_{avg.t\leftarrow w}\right| \approx (2200kg)\cdot\frac{(25m/s)^2}{2\cdot(0.8m)^2}$$

$$\left|\vec{F}_{avg.t\leftarrow w}\right| \approx 1.1\times10^6 N$$

A 5-kg lead ball is dropped from rest at a height of 10 m. The ball leaves a 5 cm deep dent in the ground.

a) How fast is the ball traveling just before hitting the ground?

b) What is the approximate force exerted by the ground on the ball while it is stopping?

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