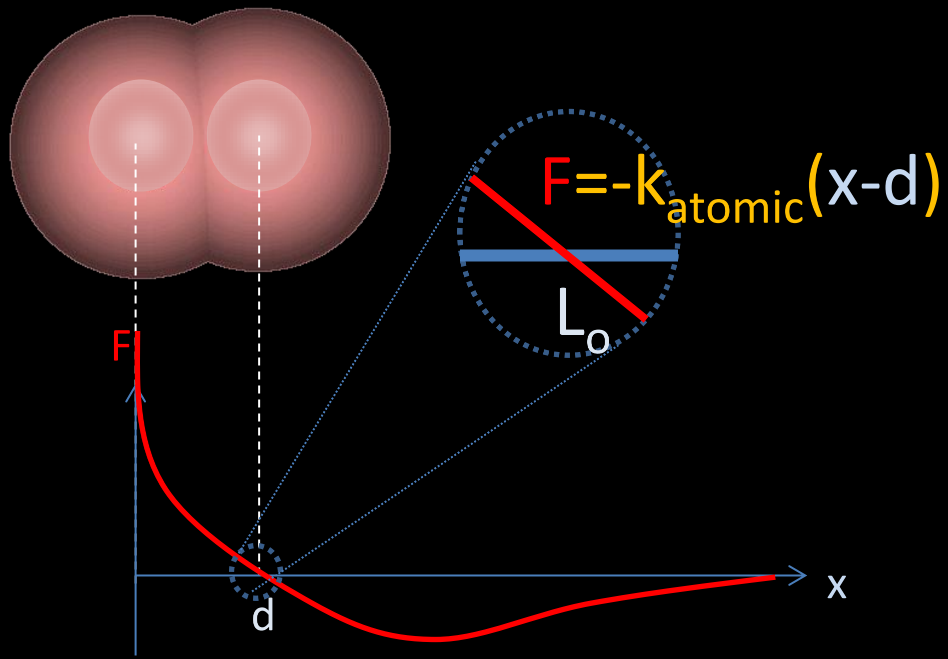
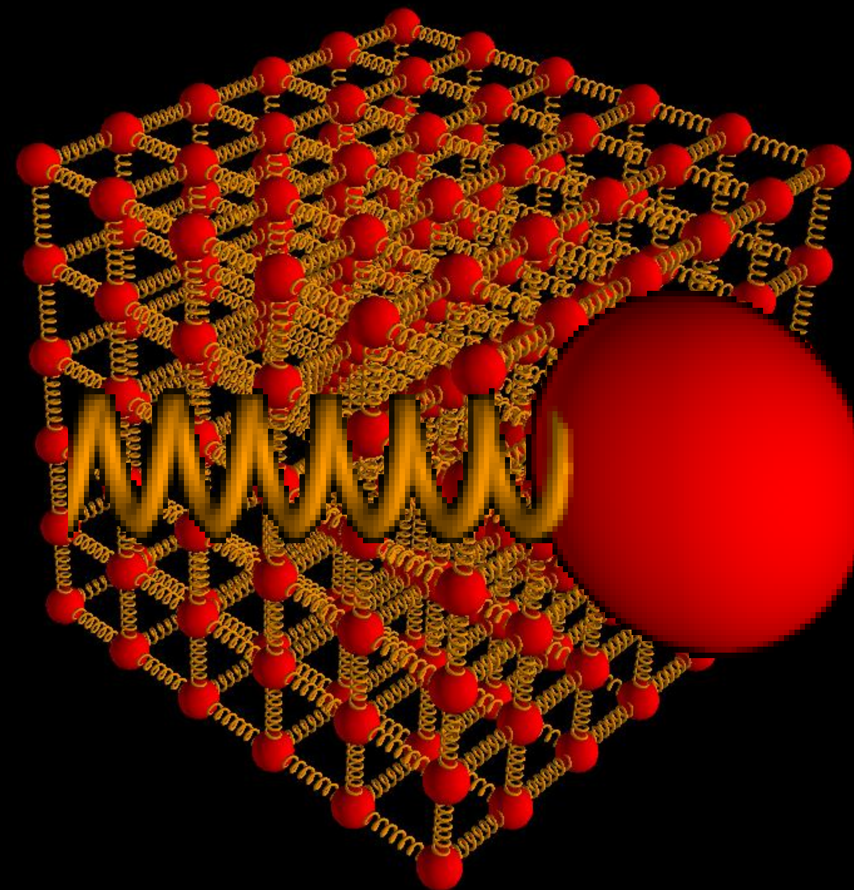
Fri	4.11-.12; .14-.15 Sound in Solids, Analytical Solutions Quiz 3	RE 4.c laptop, smartphone…
Mon. Lab . . Fri.	4.8, .13 Friction and Buoyancy & Suction Review Sessions (come having worked on sample test) Exam 1 (Ch 1-4)	RE 4.d EP 4, HW4: Ch 4 Pr’s 46, 50, 81, 88 & CP

Microscopic to Macroscopic Springs

Molecule



Solid



$$\vec{F} = -k_{spring} \left(\Delta \vec{L} \right) \Delta \vec{L}$$

Case Study in Three Modes of Exploration with Varying Force: Mass on Spring

Experimentation / Observation

Case Study in Three Modes of Exploration with Varying Force: Mass on Spring

Experimentation / Observation

Observations to Understand

- force, velocity, and position vary sinusoidally
- force and position vary in synch
- velocity varies out-of-synch
- Period's dependence
  - Mass - greater mass, slower
  - Stiffness - greater stiffness, faster
  - Amplitude – no effect !

Observations to Understand

- Changing gravity only changes center of oscillation

Computation / Simulation

$$\vec{F}_s = -k_s\left(\left|L\right| - \left|L_o\right|\right)\hat{L}$$

L_mag = mag(ball.pos)

L_hat = ball.pos/L_mag

F=-k*(L_mag-Lo)*L_hat

$$\vec{p}_f = \vec{p}_i + \vec{F}_s \Delta t$$

ball.p = ball.p + F*deltat

$$\vec{r}_f = \vec{r}_i + \frac{\vec{p}}{m}\Delta t$$

ball.pos = ball.pos + (ball.p/ball.m)*deltat

Finite changes to infinitesimal changes: derivatives

Case Study in Three Modes of Exploration with Varying Force: Mass on Spring

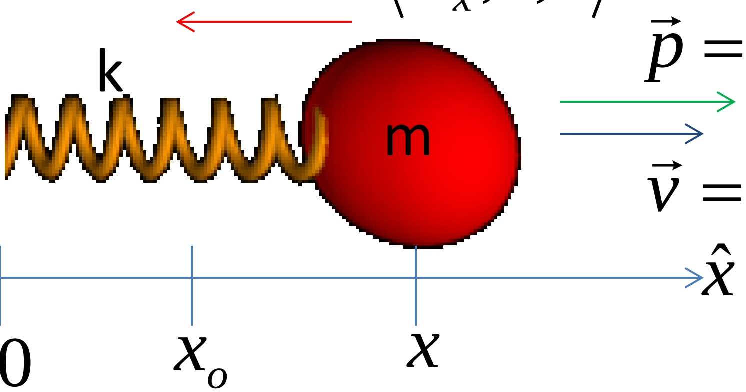
Theory / Analysis

System: Ball

$$\vec{F} = \langle F_x, 0, 0 \rangle$$

$$\vec{p} = \langle p_x, 0, 0 \rangle$$

$$\vec{v} = \langle v_x, 0, 0 \rangle$$



Guess from experiment and simulation

$$x(t) = X \cos\left(2\pi \frac{t}{T}\right) + x_o$$

Shorthand: $\omega \equiv \frac{2\pi}{T}$

$$x(t) = X \cos(\omega t) + x_o$$

$$F_x(t) = -k * [x(t) - x_o]$$

$$\frac{dp_x(t)}{dt} = -k * [x(t) - x_o]$$

$$\frac{d[mv_x(t)]}{dt} = -k * [x(t) - x_o]$$

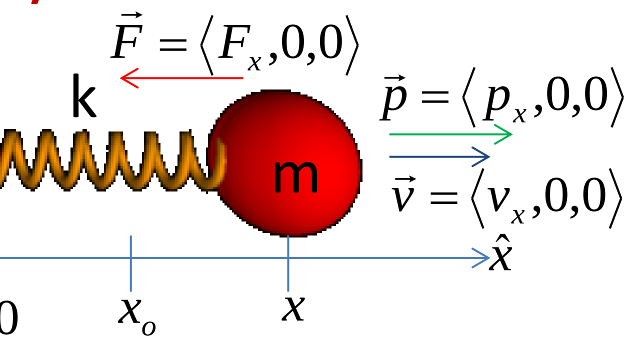
$$m \frac{d}{dt}\left[\frac{dx(t)}{dt}\right] = -k * [x(t) - x_o]$$

$$\frac{d^2 x(t)}{dt^2} = -\frac{k}{m} * [x(t) - x_o]$$

Case Study in Three Modes of Exploration with Varying Force: Mass on Spring

Theory / Analysis

System: Ball



$$F_x(t) = -k * [x(t) - x_o]$$

$$\frac{d^2 x(t)}{dt^2} = -\frac{k}{m} * [x(t) - x_o]$$

Guess

$$x(t) = X\cos(\omega t) + x_o$$

where: $\omega \equiv \frac{2\pi}{T}$

Plug in and see if guessed solution works

$$\frac{d^2}{dt^2}[X\cos(\omega t) + x_o] = -\frac{k}{m} * [X\cos(\omega t) + x_o - x_o]$$

$$\cancel{X}\frac{d^2}{dt^2}[\cos(\omega t)] = -\frac{k}{m} * [\cancel{X}\cos(\omega t)]$$

$$\frac{d}{dt}[-\omega\sin(\omega t)] = -\frac{k}{m}\cos(\omega t)$$

$$\cancel{-}\omega^2 \cancel{\cos(\omega t)} = \cancel{-}\frac{k}{m}\cancel{\cos(\omega t)}$$

$$\omega^2 = \frac{k}{m}$$

Our guess works if $\omega = \sqrt{\frac{k}{m}}$

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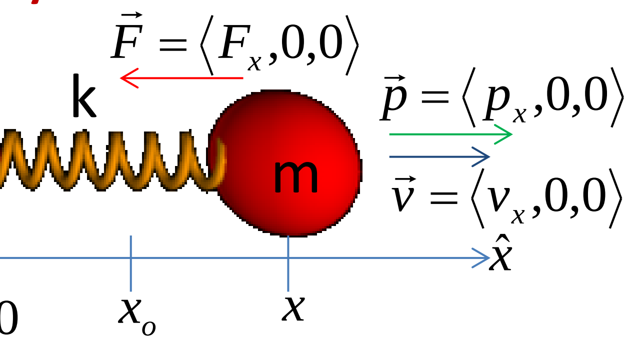
Theory / Analysis

System: Ball



$$F_x(t) = -k * [x(t) - x_o]$$

$$\frac{d^2 x(t)}{dt^2} = -\frac{k}{m} * [x(t) - x_o]$$

Solution

$$x(t) = X \cos(\omega t) + x_o$$

where: $\omega \equiv \frac{2\pi}{T} = \sqrt{\frac{k}{m}}$

Concisely tells us…

$$x(t) = X \cos(\omega t) + x_o$$

- **Sinusoidally oscillates**
- **About the equilibrium**
- **With a period that…**
  - **Shortens with greater stiffness**
  - **Lengthens with larger masses**

$$\omega \equiv \frac{2\pi}{T} = \sqrt{\frac{k}{m}} \Rightarrow T = 2\pi\sqrt{\frac{m}{k}}$$

  - **Doesn't care about amplitude**

Period dependence on: mass

Suppose the period of a spring-mass oscillator is 1 s. What will be the period if we double the mass?

a. T = 0.5 s
b. T = 0.7 s
c. T = 1.0 s
d. T = 1.4 s
e. T = 2.0 s

Period dependence on Stiffness:

Suppose the period of a spring-mass oscillator is 1 s. What will be the period if we double the spring stiffness? (We could use a stiffer spring, or we could attach the mass to two springs.)

a. T = 0.5 s
b. T = 0.7 s
c. T = 1.0 s
d. T = 1.4 s
e. T = 2.0 s

Period Dependence on Amplitude:

Suppose the period of a spring-mass oscillator is 1 s with an amplitude of 5 cm. What will be the period if we increase the amplitude to 10 cm, so that the total distance traveled in one period is twice as large?

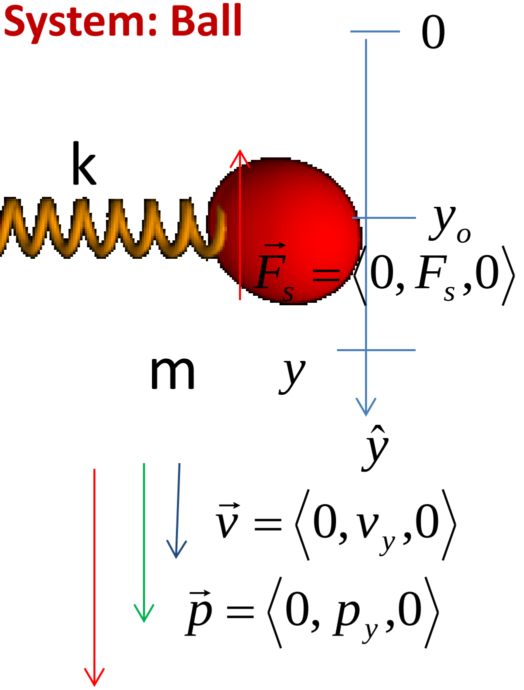
1) T = 0.5 s
2) T = 0.7 s
3) T = 1.0 s
4) T = 1.4 s
5) T = 2.0 s

Case Study in Three Modes of Exploration with Varying Force: Mass on Spring

Theory / Analysis

How does gravitational interaction change behavior?

System: Ball



$\vec{F}_E = \langle 0, mg, 0 \rangle$

Note: I've defined *down* as +y direction

So Earth's pull has + sign

$$\vec{F}_{net} = \vec{F}_s + \vec{F}_E = \langle 0, F_s + F_E, 0 \rangle$$

$$F_{net.y}(t) = -k * [y(t) - y_o] + mg$$

$$F_{net.y}(t) = -k * [y(t) - y_o] + \frac{k}{k} mg$$

$$F_{net.y}(t) = -k * [y(t) - y_o] + k \left[\frac{mg}{k} \right]$$

$$F_{net.y}(t) = -k * \left[y(t) - y_o - \frac{mg}{k} \right]$$

$$F_{net.y}(t) = -k * \left[y(t) - \left\{ y_o + \frac{mg}{k} \right\} \right]$$

$$F_{net.y}(t) = -k * [y(t) - y_o']$$

where $y_o' \equiv y_o + \frac{mg}{k}$

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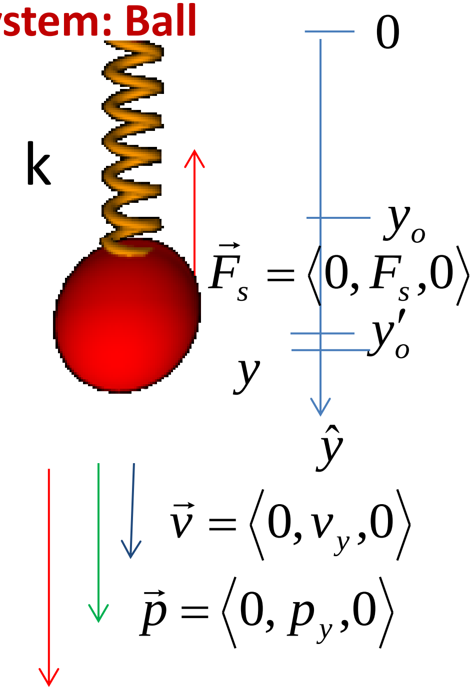
Theory / Analysis

How does gravitational interaction change behavior?

System: Ball



$$\vec{F}_E = \langle 0, mg, 0 \rangle$$

Note: I've defined *down* as +y direction

So Earth's pull has + sign

$$\vec{F}_{net} = \vec{F}_s + \vec{F}_E = \langle 0, F_s + F_E, 0 \rangle$$

$$F_{net.y}(t) = -k * [y(t) - y_o] + mg$$

.
.
.

$$F_{net.y}(t) = -k * [y(t) - y_o']$$ where $y_o' \equiv y_o + \frac{mg}{k}$

- Exact same form as for horizontal mass-spring, but shifted equilibrium

$$m \frac{d^2}{dt^2} y(t) = -k * [y(t) - y_o']$$

Solution:

$$y(t) = Y \cos(\omega t) + y_o'$$

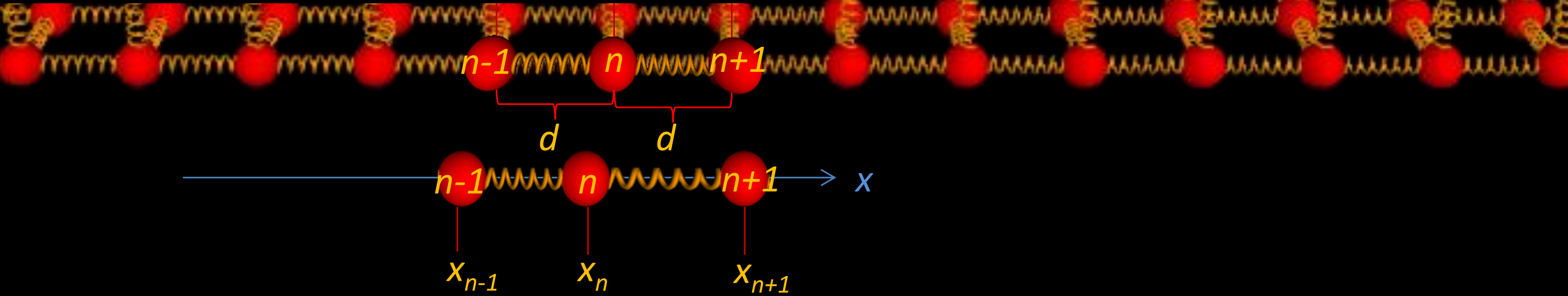
$$\omega \equiv \frac{2\pi}{T} = \sqrt{\frac{k}{m}} \Rightarrow T = 2\pi \sqrt{\frac{m}{k}}$$

Period dependence on *g*:

Suppose the period of a spring-mass oscillator is 1 s with an amplitude of 5 cm. What will be the period if we take the oscillator to a massive planet where g = 19.6 N/kg?

1) T = 0.5 s
2) T = 0.7 s
3) T = 1.0 s
4) T = 1.4 s
5) T = 2.0 s

Speed of Sound in a Solid: the logic



$$F_{n,net} = k_s(x_{n-1} + x_{n+1} - 2x_n)$$

$$\frac{dp_n}{dt} = k_s(x_{n-1} + x_{n+1} - 2x_n)$$

$$\frac{d(mv_n)}{dt} = k_s(x_{n-1} + x_{n+1} - 2x_n)$$

$$\frac{d\left(\frac{dx_n}{dt}\right)}{dt} = \frac{k_s}{m}(x_{n-1} + x_{n+1} - 2x_n)$$

$$\frac{\mathrm{d}^2 x_n}{\mathrm{d}t^2} = \frac{k_s}{m}(x_{n-1} + x_{n+1} - 2x_n)$$

$$\frac{\mathrm{d}^2 x_n}{\mathrm{d}t^2} = \frac{k_s}{m} d\left(\frac{(x_{n+1} - x_n)}{d} - \frac{(x_n - x_{n-1})}{d}\right)$$

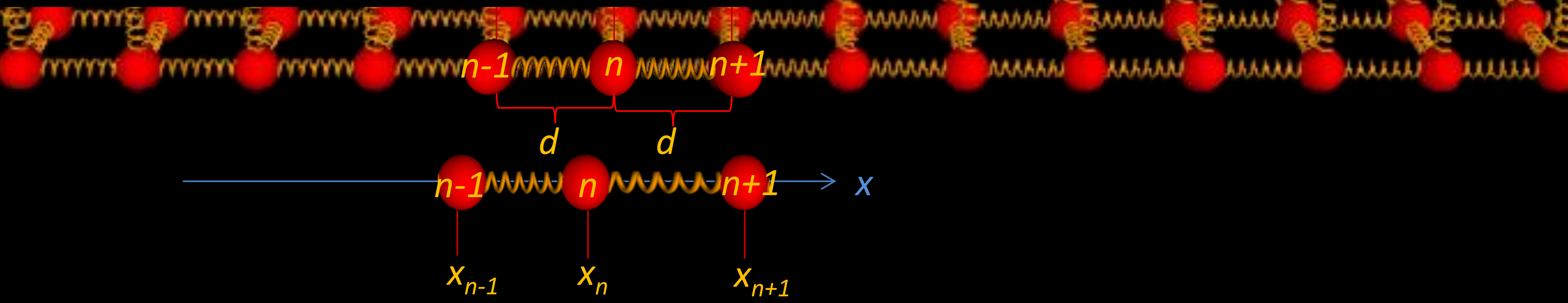
$$\frac{\mathrm{d}^2 \varepsilon_n}{\mathrm{d}t^2} \approx -\frac{k_s}{m} d^2 \frac{\left(\frac{\mathrm{d}x_{n+1}}{\mathrm{d}x} - \frac{\mathrm{d}x_n}{\mathrm{d}x}\right)}{d}$$

$$\frac{\mathrm{d}^2 x_n}{\mathrm{d}t^2} \approx -\frac{k_s}{m} d^2 \frac{\mathrm{d}^2 x_{n+1}}{\mathrm{d}x^2}$$

Speed of Sound in a Solid: the result

$$v = \sqrt{\frac{k_s}{m}}\, d$$

Speed of Sound in a Solid



$$v = \sqrt{\frac{k_s}{m}}\, d$$

Stiffer, for a given atomic displacement, greater force pulling it so greater velocity achieved.

More distance between atoms means further the distortion can propagate just through the light weight spring /bond without encountering the resistance of massive atoms.

More massive, more inertial resistance to applied force, less velocity achieved.

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