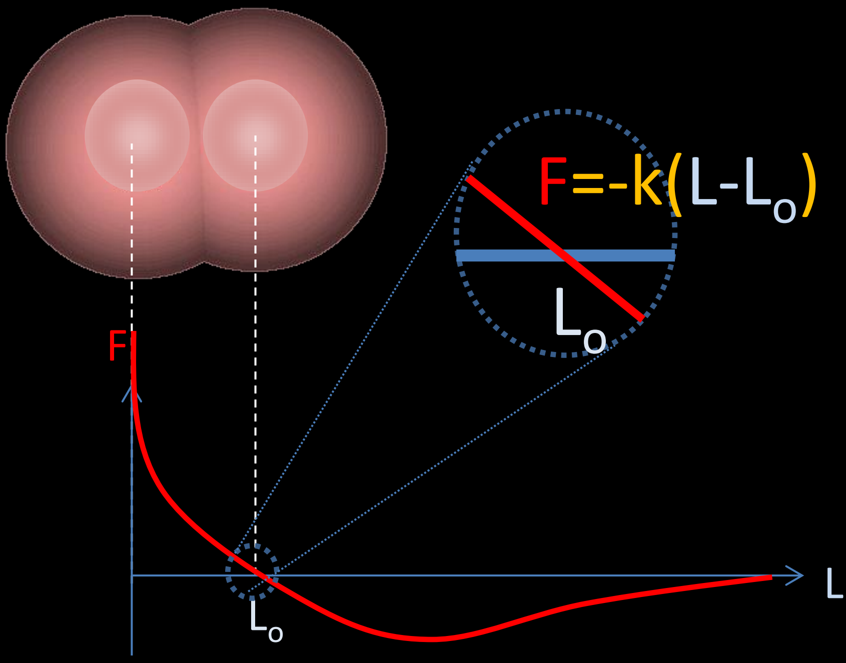
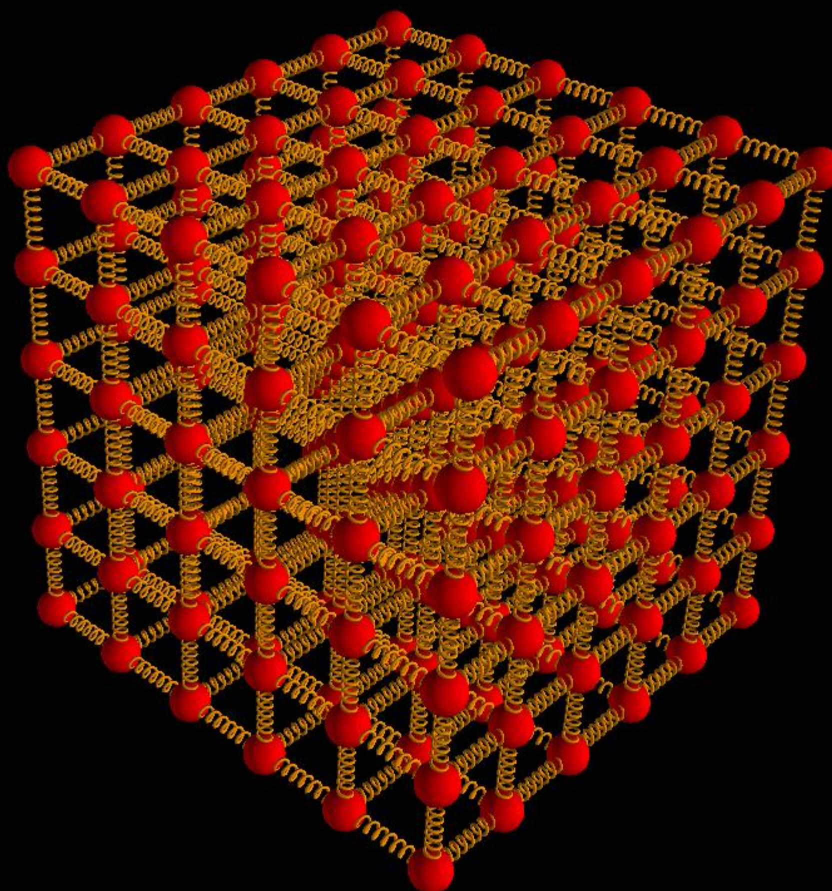
Wed. Lab Fri	4.6-.7, .9-.10 Stress, Strain, Young's Modulus, Compression, Sound *InStove @ noon* *Science Poster Session: Hedco7pm~9pm* L4: Young's Modulus & Speed of Sound (Read 4.11-.12) 4.11-.12; .14-.15 Sound in Solids, Analytical Solutions Quiz 3	RE 4.b RE 4.c
Mon. Tues. . . Fri.	4.8, .13 Friction and Buoyancy & Suction Exam 1 (Ch 1-4)	RE 4.d EP 4, HW4: Ch 4 Pr's 46, 50, 81, 88 & CP

Ball-Spring Model

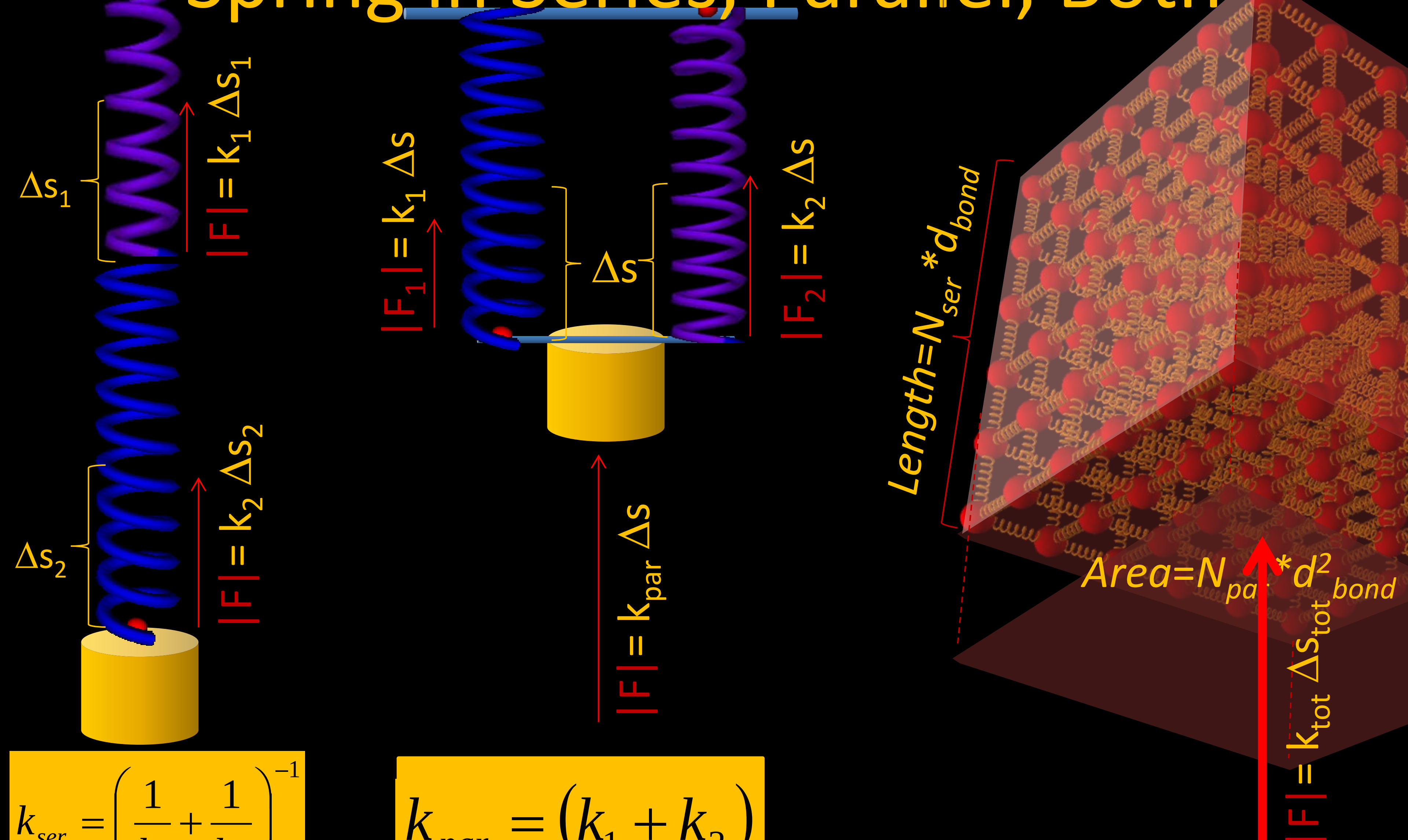
Molecule



Solid



Spring in Series, Parallel, Both



$$k_{ser} = \left(\frac{1}{k_1} + \frac{1}{k_2}\right)^{-1}$$

$$k_{par} = \left(k_1 + k_2\right)$$

$$k_{ser} = \frac{k_1}{N_{ser}}$$

N identical

$$k_{par} = N_{par} k_1$$

$$k_{tot} = \left(\frac{N_{par}}{N_{ser}} k_1\right) = \left(\frac{A}{dL} k_1\right)$$

Spring in Series & Parallel Rephrased
Stress, Strain, and Young's Modulus

$$F = k_{total} \Delta L$$

$$F = \left(k_{atomic} \left(\frac{A}{dL} \right) \right) \Delta L$$

From counting series and parallel

regroup

$$F = \left(\frac{k_{atomic}}{d} \right) \frac{A \Delta L}{L}$$

Microscopic details

Macroscopic measurables

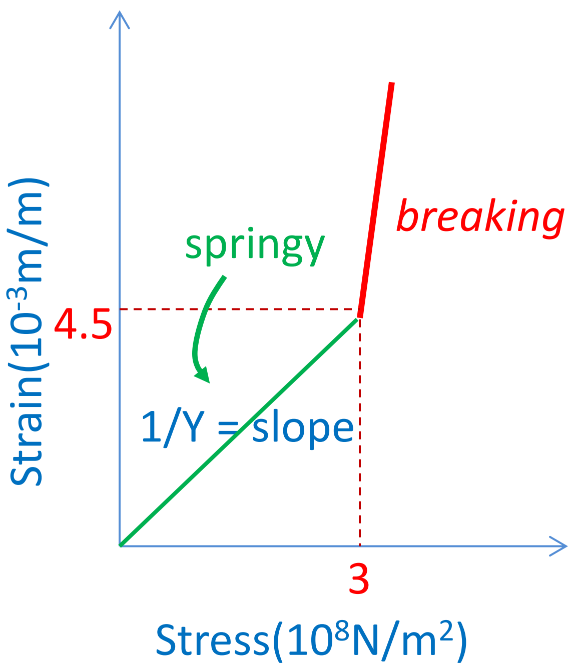
$$Y \equiv \text{Young's Modulus}$$

$$F = Y \frac{A \Delta L}{L}$$

$$\left(\frac{F}{A} \right) = Y \left(\frac{\Delta L}{L} \right)$$

Stress
force transmitted per area (pressure)

Strain
stretch per length (fractional stretch)



Two wires with equal lengths are made of pure copper. The diameter of wire A is twice the diameter of wire B.

When 6 kg masses are hung on the wires, wire B stretches more than wire A.

$Y = (F/A)/(DL/L) = k/d$

You make careful measurements and compute Young's modulus for both wires. What do you find?

1) $Y_A > Y_B$

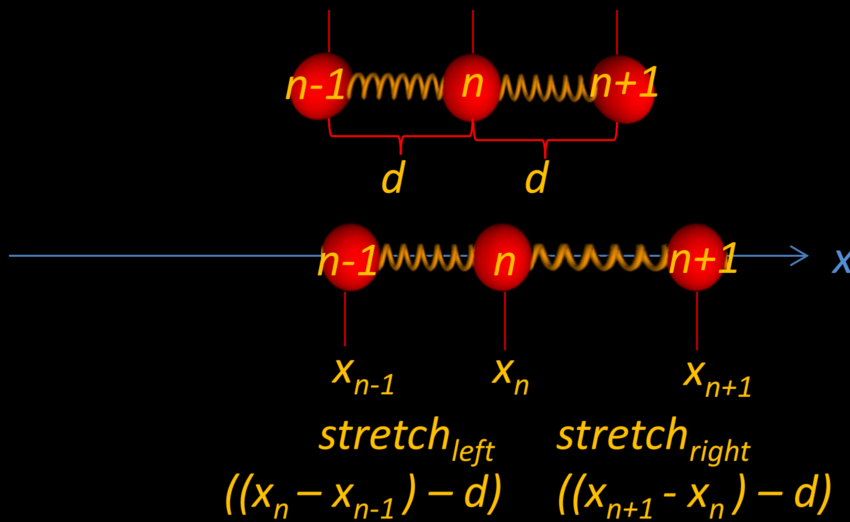
2) $Y_A = Y_B$

3) $Y_A < Y_B$

Example: You hang a heavy ball with a mass of 14 kg from a silver rod 2.6m long by 1.5 mm by 3.1mm. You measure a stretch of the rod, and find that the rod stretched 0.002898 m. Using these experimental data, what value of Young's modulus do you get?

The density of silver is 10.5 g/cm^3 and you can look up its atomic mass. What's the inter-atomic spring stiffness?

Speed of Sound in a Solid: the logic



$$F_{n,Left} = -k_s((x_n - x_{n-1}) - d))$$

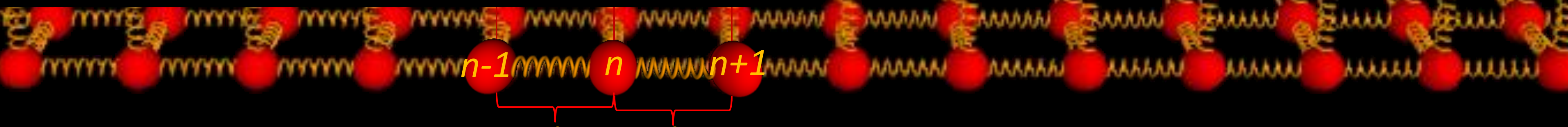
$$F_{n,Right} = k_s((x_{n+1} - x_n) - d))$$

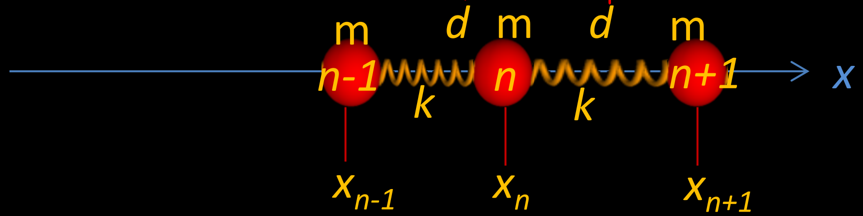
$$F_{n,net} = F_{n,Left} + F_{n,Right} = -k_s((x_n - x_{n-1}) - d)) + k_s((x_{n+1} - x_n) - d))$$

$$F_{n,net} = k_s(x_{n-1} + x_{n+1} - 2x_n)$$

Copmutational simulation

Speed of Sound in a Solid





$$F_{n,net} = k_s(x_{n-1} + x_{n+1} - 2x_n - x_{n-1})$$

Informed Guess at v's dependence

Stiffer, for a given atomic displacement, greater force pulling it so greater velocity achieved.

$$v = \sqrt{\frac{k_s}{m}}d$$

More distance between atoms means further the distortion can propagate just through the light weight spring /bond without encountering the resistance of massive atoms.

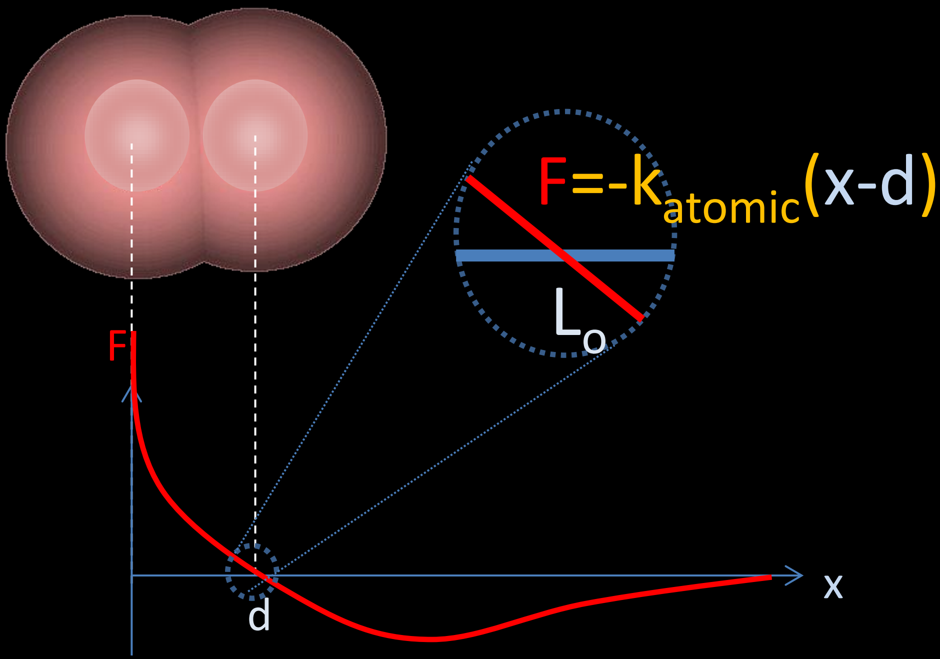
More massive, more inertial resistance to applied force, less velocity achieved.

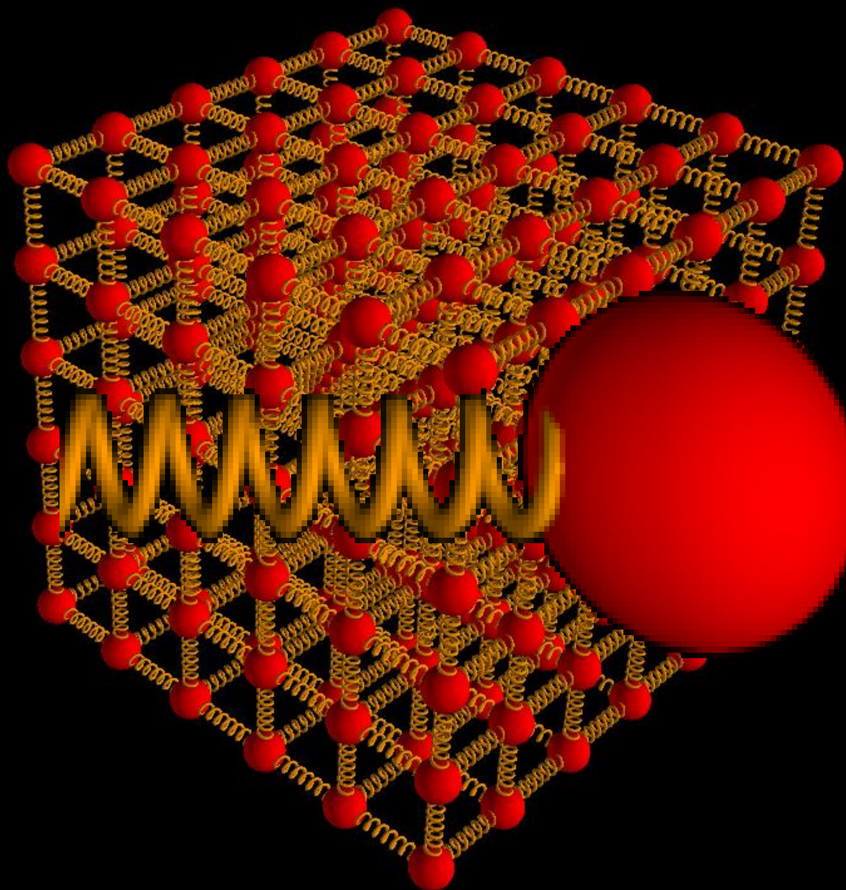
Example: The spring constant of aluminum is about 16 N/m. The typical separation of Al atoms was 2.6×10^{-10}m. Recall also that the atomic mass of aluminum is 27 g/mole. So what is the speed of sound in Aluminum?

Microscopic to Macroscopic Springs

Molecule

Solid





$$\vec{F} = -k_{spring}\left(\Delta\hat{\vec{L}}\right)\Delta\vec{L}$$

Case Study in Three Modes of Exploration with Varying Force: Mass on Spring

Experimentation / Observation

Case Study in Three Modes of Exploration with Varying Force: Mass on Spring

Experimentation / Observation

Observations to Understand

- force, velocity, and position vary sinusoidally
- force and position vary in synch
- velocity varies out-of-synch
- Period's dependence
  - Mass - greater mass, slower
  - Stiffness - greater stiffness, faster
  - Amplitude – no effect !

Observations to Understand

- Changing gravity only changes center of oscillation

Computation / Simulation

$$\vec{F}_s = -k_s\left(|L| - |L_o|\right)\hat{L}$$

L_mag = mag(ball.pos)
L_hat = ball.pos/L_mag
F=-k*(L_mag-Lo)*L_hat

$$\vec{p}_f = \vec{p}_i + \vec{F}_s \Delta t$$

ball.p = ball.p + F*deltat

$$\vec{r}_f = \vec{r}_i + \frac{\vec{p}}{m}\Delta t$$

ball.pos = ball.pos +
(ball.p/ball.m)*deltat

Finite changes to infinitesimal changes: derivatives

Case Study in Three Modes of Exploration with Varying Force: Mass on Spring

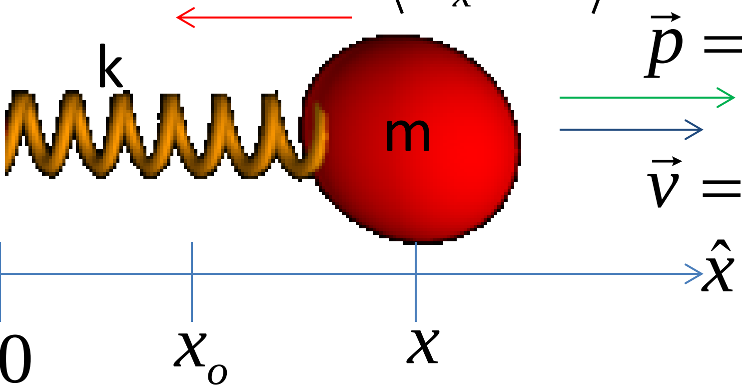
Theory / Analysis

System: Ball

$$\vec{F} = \langle F_x, 0, 0 \rangle$$

$$\vec{p} = \langle p_x, 0, 0 \rangle$$

$$\vec{v} = \langle v_x, 0, 0 \rangle$$



Guess from experiment and simulation

$$x(t) = X \cos\left(2\pi \frac{t}{T}\right) + x_o$$

Shorthand: $\omega \equiv \frac{2\pi}{T}$

$$x(t) = X \cos(\omega t) + x_o$$

$$F_x(t) = -k * [x(t) - x_o]$$

$$\frac{dp_x(t)}{dt} = -k * [x(t) - x_o]$$

$$\frac{d[mv_x(t)]}{dt} = -k * [x(t) - x_o]$$

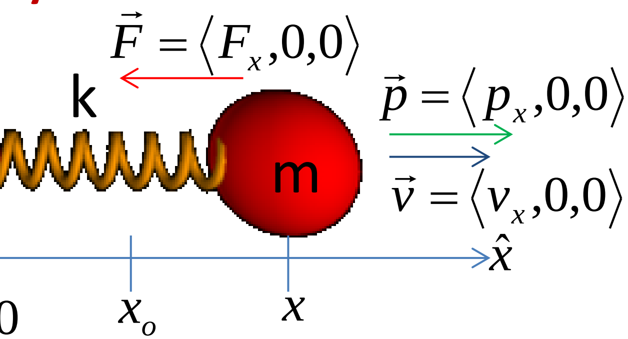
$$m \frac{d}{dt}\left[\frac{dx(t)}{dt}\right] = -k * [x(t) - x_o]$$

$$\frac{d^2 x(t)}{dt^2} = -\frac{k}{m} * [x(t) - x_o]$$

Case Study in Three Modes of Exploration with Varying Force: Mass on Spring

Theory / Analysis

System: Ball



$$F_x(t) = -k * [x(t) - x_o]$$

$$\frac{d^2 x(t)}{dt^2} = -\frac{k}{m} * [x(t) - x_o]$$

Guess

$$x(t) = X\cos(\omega t) + x_o$$

where: $\omega \equiv \frac{2\pi}{T}$

Plug in and see if guessed solution works

$$\frac{d^2}{dt^2}[X\cos(\omega t) + x_o] = -\frac{k}{m} * [X\cos(\omega t) + x_o - x_o]$$

$$\cancel{X}\frac{d^2}{dt^2}[\cos(\omega t)] = -\frac{k}{m} * [\cancel{X}\cos(\omega t)]$$

$$\frac{d}{dt}[-\omega\sin(\omega t)] = -\frac{k}{m}\cos(\omega t)$$

$$\cancel{-}\omega^2 \cancel{\cos(\omega t)} = \cancel{-}\frac{k}{m}\cancel{\cos(\omega t)}$$

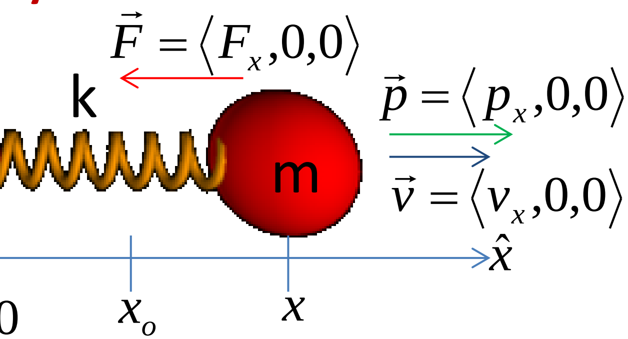
$$\omega^2 = \frac{k}{m}$$

Our guess works if $\omega = \sqrt{\frac{k}{m}}$

Case Study in Three Modes of Exploration with Varying Force: Mass on Spring

Theory / Analysis

System: Ball



$$F_x(t) = -k * [x(t) - x_o]$$

$$\frac{d^2 x(t)}{dt^2} = -\frac{k}{m} * [x(t) - x_o]$$

Solution

$$x(t) = X \cos(\omega t) + x_o$$

where: $\omega \equiv \frac{2\pi}{T} = \sqrt{\frac{k}{m}}$

Concisely tells us…

$$x(t) = X \cos(\omega t) + x_o$$

- **Sinusoidally oscillates**
- **About the equilibrium**
- **With a period that…**
  - **Shortens with greater stiffness**
  - **Lengthens with larger masses**

$$\omega \equiv \frac{2\pi}{T} = \sqrt{\frac{k}{m}} \Rightarrow T = 2\pi\sqrt{\frac{m}{k}}$$

- **Doesn't care about amplitude**

Period dependence on: mass

Suppose the period of a spring-mass oscillator is 1 s. What will be the period if we double the mass?

a. T = 0.5 s
b. T = 0.7 s
c. T = 1.0 s
d. T = 1.4 s
e. T = 2.0 s

Period dependence on Stiffness:

Suppose the period of a spring-mass oscillator is 1 s. What will be the period if we double the spring stiffness? (We could use a stiffer spring, or we could attach the mass to two springs.)

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Period Dependence on Amplitude:

Suppose the period of a spring-mass oscillator is 1 s with an amplitude of 5 cm. What will be the period if we increase the amplitude to 10 cm, so that the total distance traveled in one period is twice as large?

1) T = 0.5 s
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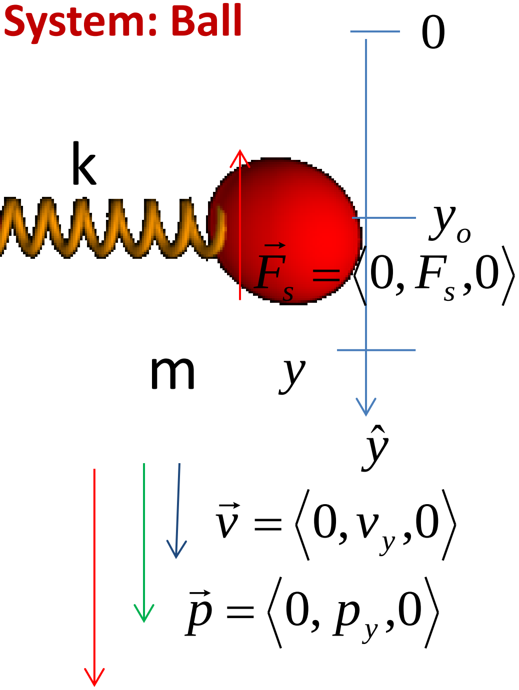
Case Study in Three Modes of Exploration with Varying Force: Mass on Spring

Theory / Analysis

How does gravitational interaction change behavior?

System: Ball



$\vec{F}_E = \langle 0, mg, 0 \rangle$

Note: I've defined *down* as +y direction

So Earth's pull has + sign

$$\vec{F}_{net} = \vec{F}_s + \vec{F}_E = \langle 0, F_s + F_E, 0 \rangle$$

$$F_{net.y}(t) = -k * [y(t) - y_o] + mg$$

$$F_{net.y}(t) = -k * [y(t) - y_o] + \frac{k}{k} mg$$

$$F_{net.y}(t) = -k * [y(t) - y_o] + k\left[\frac{mg}{k}\right]$$

$$F_{net.y}(t) = -k * \left[y(t) - y_o - \frac{mg}{k} \right]$$

$$F_{net.y}(t) = -k * \left[y(t) - \left\{ y_o + \frac{mg}{k} \right\} \right]$$

$$F_{net.y}(t) = -k * [y(t) - y'_o]$$

where $y'_o \equiv y_o + \dfrac{mg}{k}$

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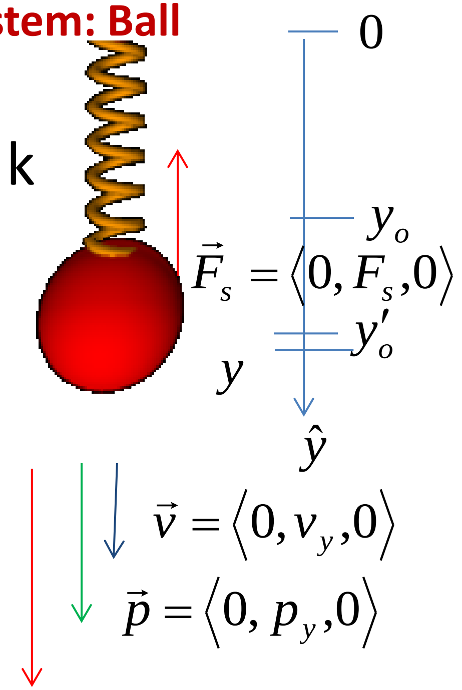

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$$\vec{F}_{net} = \vec{F}_s + \vec{F}_E = \langle 0, F_s + F_E, 0 \rangle$$

$$F_{net.y}(t) = -k * [y(t) - y_o] + mg$$

$$\vdots$$

$$F_{net.y}(t) = -k * [y(t) - y_o'] \quad \text{where } y_o' \equiv y_o + \frac{mg}{k}$$

- Exact same form as for horizontal mass-spring, but shifted equilibrium

$$m \frac{d^2}{dt^2} y(t) = -k * [y(t) - y_o']$$

Solution:

$$y(t) = Y \cos(\omega t) + y_o'$$

$$\omega \equiv \frac{2\pi}{T} = \sqrt{\frac{k}{m}} \Rightarrow T = 2\pi \sqrt{\frac{m}{k}}$$

Period dependence on *g*:

Suppose the period of a spring-mass oscillator is 1 s with an amplitude of 5 cm. What will be the period if we take the oscillator to a massive planet where g = 19.6 N/kg?

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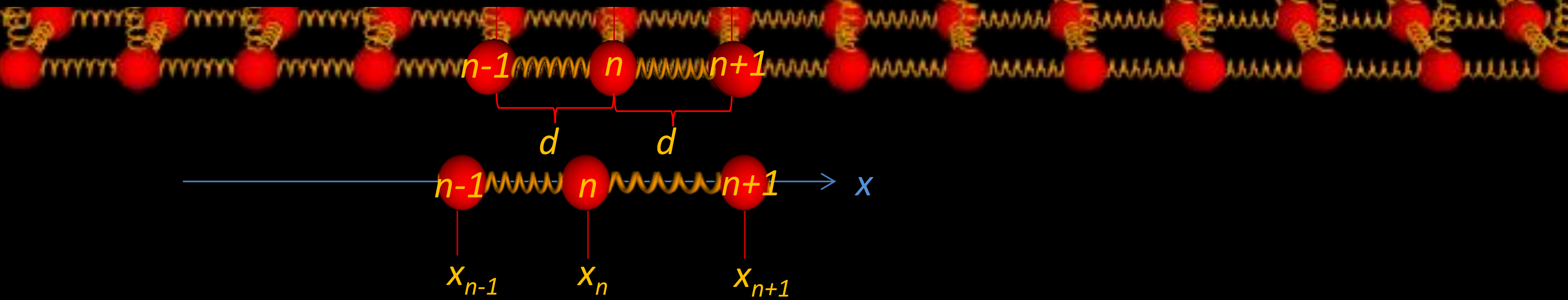
Speed of Sound in a Solid: the logic



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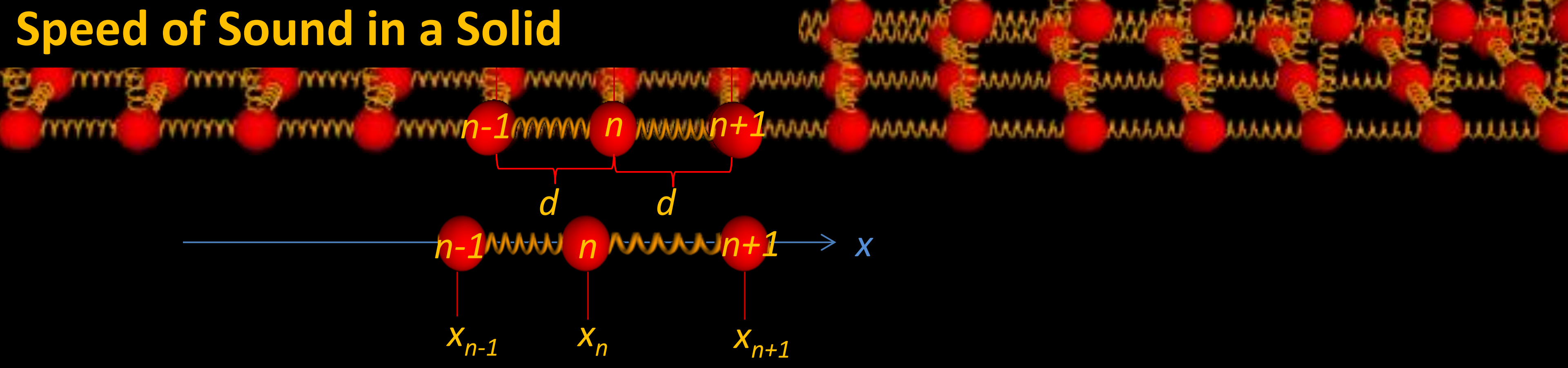
$$\frac{d\left(\frac{dx_n}{dt}\right)}{dt} = \frac{k_s}{m}(x_{n-1} + x_{n+1} - 2x_n)$$

$$\frac{\mathrm{d}^2 x_n}{\mathrm{d}t^2} = \frac{k_s}{m}(x_{n-1} + x_{n+1} - 2x_n)$$

Speed of Sound in a Solid: the result

$$v = \sqrt{\frac{k_s}{m}}d$$

Speed of Sound in a Solid



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