11	Fri., 11/15	**10.6-.8** Scattering	**RE 10.b**
12	Mon., 11/18 Tues. 11/19 Wed.,11/20 Lab Fri., 11/22	**10.9-.10** Collision Complications: Inelastic, Relativistic, & Quantized **10.5, .11** Different Reference Frames **L10** Collisions (ballistic pendulum?) **11.1** Translational Angular Momentum **Quiz 10**	**RE 10.c** **EP9** *(Rep of Washinton's 3/2 visits Thurs night 7pm)* **RE 11.a; HW10:** Ch 10 Pr's 13*, 21, 30, "39",

Equipment

- Lab carts and track (not air track) & cart weights (black bars)
- Hover pucks (maybe)
- Collsion ppt
- 08_Rutherford_dist.py, and Scattering.exe

From Last Time

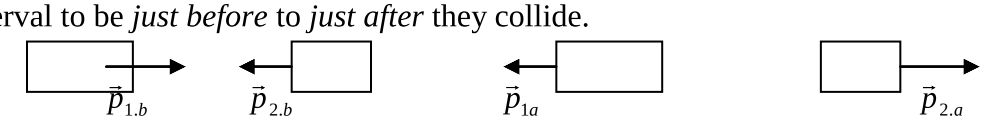
So, before the test, we'd started thinking about collisions
Collision – short, strong interactions; can neglect all other interactions during collision.

We'd started by looking at just 1-D collisions.

Review with ppt's questions

Ch. 9 Collisions: Exploring the Nucleus

- **Collision:** A relatively brief and strong interaction.
- **System = carts.**
  - Let's take our system to be the two carts on the air track, and our time interval to be *just before* to *just after* they collide.
- **System = carts.**
  - Let's take our system to be the two carts on the air track, and our time interval to be *just before* to *just after* they collide.
  -
  -
  - Then



$$\Delta\vec{p}_{system} = \vec{F}_{system\leftarrow net.ext}\Delta t$$

$$\Delta\vec{p}_1 + \Delta\vec{p}_2 = (\vec{F}_{1\leftarrow air} + \vec{F}_{1\leftarrow table.friction} + \vec{F}_{1\leftarrow table.normal} + \vec{F}_{1\leftarrow Earth.gravl} + \vec{F}_{2\leftarrow air} + \ldots)\Delta t$$

$$\Delta\vec{p}_1 + \Delta\vec{p}_2 \approx 0$$

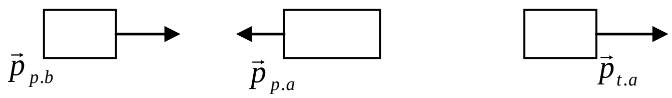
if Δt is if Δt is small enough.

  - **Demo: Maximally (Perfectly) Inelastic.** $\vec{v}_{big.f} = \vec{v}_{small.f}$.

$$m_{big}\vec{v}_{big.i} + m_{small}\vec{v}_{small.i} = (m_{big} + m_{small})\vec{v}_f$$

- $$\vec{v}_f = \frac{m_{big}\vec{v}_{big.i} + m_{small}\vec{v}_{small.i}}{(m_{big} + m_{small})}$$

- **Demo: Perfectly Elastic.** (collide carts and bounce magnetically). Now, in this case, $\vec{v}_{big.f} \neq \vec{v}_{small.f}$, we still need a second equation if we want to solve for two unknowns. What other relation holds through this collision? **Conservation of Energy.**
  $\Delta K_t + \Delta K_p = 0$
- $\dfrac{p_{t.a}^2 - p_{t.b}^2}{2m_t} + \dfrac{p_{p.a}^2 - p_{p.b}^2}{2m_p} = 0$
- $\vec{p}_{t.b} + \vec{p}_{pb} = \vec{p}_{.ta} + \vec{p}_{p.a}$
- **Putting these together when big was initially at rest, we go**



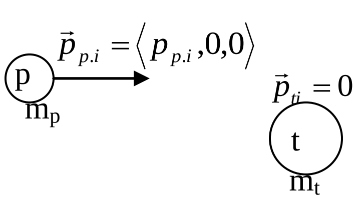
- **Projectile & Target.**
  - $p_{p.f} = p_{p.i}\left(\dfrac{m_p - m_t}{m_p + m_t}\right)$

    $p_{t.f} = p_{p.i}\left(\dfrac{2m_t}{m_p + m_t}\right)$

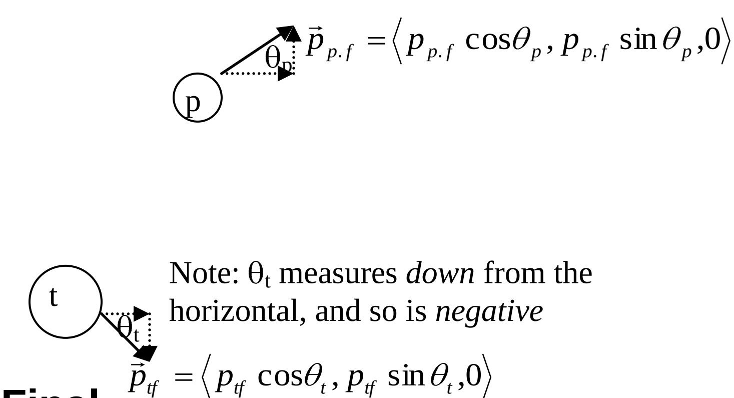
8.3 Scattering

- Last time, we looked at head-on collisions. We employed conservation of energy and conservation of momentum to predict the after-math of the collisions, and we got familiar with the range of possible out comes (depending upon the relative masses of the colliders and their relative velocities.)
- Now we're going to consider the more general case of an off-axis collision. Of course the result is the colliders bounce off at different *angles* – requiring 2-D description.
- Such collisions play a major role in particle physics where the behaviors of particles when they collide gives us important information about their properties such as charge, mass, and even internal structure.
- Today, we're going to walk the line between really getting into the math and staying qualitative.

Demo: hover pucks collide off-axis.
Demo: Alpha_on_alpha.py
Ppt.



Initial



Final

- **Conservation of Momentum:**
  - $$\vec{p}_{tot.i} = \vec{p}_{tot.f}$$
    $$\vec{p}_{pi} = \vec{p}_{pf} + \vec{p}_{tf}$$
    $$\hat{x}: \; p_{pi} = p_{pf}\cos\theta_p + p_{tf}\cos\theta_t$$
    $$\hat{y}: \; 0 = p_{pf}\sin\theta_p + p_{tf}\sin\theta_t = p_{pf}\sin\theta_p - | p_{tf}\sin\theta_t |$$
- **Conservation of Energy:**
  - Energy is a vector quantity, so we get the exact same, single, equation we did for a 1-D collision.
    $$\Delta E_{system} = \int_{r_i}^{r_f} \vec{F}_{net.ext} \cdot d\vec{r} \approx 0$$
  - $$\Delta K_{p.cm} + \Delta E_{p.\text{int}} + \Delta K_{t.cm} + \Delta E_{t.\text{int}} + \Delta U_{p,t} \approx 0$$
    $$\Delta K_{p.cm} + \Delta E_{p.\text{int}} + \Delta K_{t.cm} + \Delta E_{t.\text{int}} \approx 0$$
    - Let's say we choose our before and after at points of the same potential; most likely 0 potential, for the objects are no longer interacting.
  - **Elastic Collision.**
    - If the collision is *elastic*, we can say that the internal energies of the two objects are the same before and after the collision (they mightn't be *during* the collision, but we're not concerned with then.)
    - $$\frac{p_{pi}^2}{2m_p} = \frac{p_{pf}^2}{2m_p} + \frac{p_{tf}^2}{2m_t}$$
  - ***Three*** **Equations /** ***Four*** **Unknowns.**
    - **Three Equations.** Written this way, we have three equations (momentum x, momentum y, and energy)
    - **Four unknowns.** Four unknowns (magnitude of final momentum 1, magnitude of final momentum 2, direction of final momentum 1, and direction of final momentum 2).
    - **Fourth piece of info.**
      - We're going to focus a bit today on that 4th piece of information.
      - **Collision geometry.** If you're going to predict all of the final parameters, you're going to need one more piece of information, specifically, the *direction of the force of collision* – that gives the direction of the change in momentum for each object. Generally, you need to know something about the *collision itself*.
        - For our two pucks, that's a matter of geometry: what are the radii of the two pucks and how do they compare with the initial puck's path – this determines exactly where the two pucks collide.
        - For a collision of two charged particles, you need to consider the force law of the interaction.

- **Only 3 unknowns (know / measure one final)** Then again, in many situations, you are able to *measure* the initial and final momentum vectors (magnitude and direction) of *one* of the particles, and thus you can deduce the recoil of the other. For example: if you fire alpha particles at gold nuclei, you can't see the gold nuclei, but you *can* dictate the alpha particle's initial momentum and measure it's final momentum, and so deduce the gold nucleus' recoil.

- **Equal Masses.** Our two pucks are pretty much equal masses. We get an interesting result in this special case of $m_p = m_t = m$.
  - From conservation of energy:
    - $$\frac{p_{p.i}^2}{2m} = \frac{p_{p.f}^2}{2m} + \frac{p_{t.f}^2}{2m}$$
      $$p_{p.i}^2 = p_{p.f}^2 + p_{t.f}^2$$
  - From conservation of momentum:
    - $\vec{p}_{pi} = \vec{p}_{pf} + \vec{p}_{tf}$
    - Then, squaring it gives
      - $p_{pi}^2 = \vec{p}_{pi} \bullet \vec{p}_{pi} = (\vec{p}_{pf} + \vec{p}_{tf}) \bullet (\vec{p}_{pf} + \vec{p}_{tf})$
    - or
      - $p_{pi}^2 = p_{pf}^2 + p_{tf}^2 + 2\vec{p}_{pf} \bullet \vec{p}_{tf}$
  - Comparing this with the equation from the conservation of energy, clearly the last term must be 0. The dot product is $p_{pf}\, p_{tf} \cos\theta_{p-t}$.
  - **Q:** There are three factors here, so there are three ways their product can be zero. What's one?
    - projectile never hit target, so $p_{tf} = 0$
    - projectile hit target head on, so projectile stopped and target carried, so $p_{pf} = 0$
    - the angel between the two final momenta is $\pm 90°$

Demo: Try to collide pucks in the three ways (may need someone to help catch pucks)
- Notice that I was able to predict the angle *between* the two particles that collided, but I *couldn't* predict the direction they'd go relative to the initial direction.
- What was it about the collision that determined that?

8.3.1 Impact Parameter

- Which case we have (and exactly how the 90° is oriented) depends on the collision itself. For our pucks, it's simple geometry: the ratio of the projected center-to-center distance with the pucks' radii. This distance is referred to as the "impact parameter, b."
- if the impact parameter is larger than the sum of the two pucks' radii, then there's a miss:

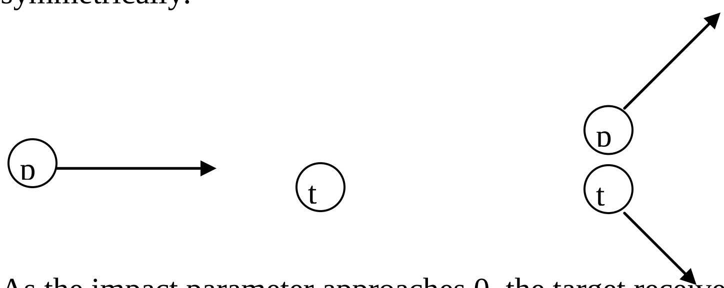
Miss
Big *b*

asymmetric large *b*

- If the impact parameter is just smaller than the combined radii, then the target puck barely moves and the projectile puck is barely deflected.

- If the impact parameter near ½ the combined radii, both pucks depart fairly symmetrically.

symmetric medium *b*



- As the impact parameter approaches 0, the target receives more of the momentum, until b = 0 and the projectile stops upon collision.

Head-on 0 *b*

- **Generally.** Say our colliders aren't equal mass; the picture's qualitatively very similar: the angle between the two out coming momenta depends upon the impact parameter.

8.4 Discovering the nucleus inside atoms

Turning this reasoning around, say you don't know how big your target object is, say an atomic nucleus, then observing how projectile objects, say alpha particles, scatter off of it tells you about the impact parameter and the size of the target.

- It was actually through scattering experiments, and impact parameter considerations, that the nucleus was discovered and its structure is probed.

8.4.1 The Rutherford experiment

- Alpha particles (He nuclei) make convenient, fairly massive, and + charged projectiles. Rutherford's assistants accelerated these at a target of thin gold foil and then observed where they hit a phosphorescent screen on either side of the foil.
- The interaction of an alpha particle with the nucleus is governed by

$F = \frac{1}{4\pi\varepsilon_o}\frac{q_\alpha q_{nuc}}{r^2_{\alpha-nuc}}$. In lab you'll have the opportunity to model this experiment.

Demo: 08_Rutherford_dist.py

For a material of uniform mass and + charge distribution, one would expect little significant scattering. Note the lack of small-angle scatters and the occasional backscatter.

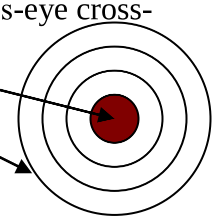
8.4.2 Computer modeling of the Rutherford Experiment

8.4.3 Distribution of scattering angles

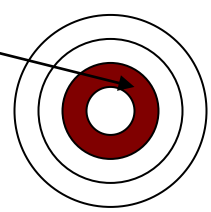
- Looking at the simulation, you can see that, given the range of possible 'impact parameters', there's a corresponding range of scattering angles. For hard objects, like pool balls, there's a pretty simple geometric relation between target & projectile size, impact parameter, and resulting scattering angle.
- In Rutherford's experiment, and others like it, the target can't actually be seen, indeed, the scattering is used to deduce the target's size. In this context, one fires a barrage of projectiles in the general direction of the target & the fraction that scatter at different angles tells you what fraction of your beam is taken up by the target.

8.4.3.1 Cross section

- In a hard collision, like that of pool balls, the projectile must be aimed within the geometric radius of the target in order for there to be any interaction / any scattering. Thus, by looking at the distribution of scattered projectiles, it's quite easy to figure out the size, or more specifically, the cross-sectional area of the target.
    - **Cross-section, Darts, and Probability.** Let's say I'm good enough at darts to be guarantee that I'll hit the board *somewhere* but it's completely random as to where. Take the bull's-eye to be the *target.* Then the probability that I get a bulls-eye is simply the ratio of the bull's-eye cross-sectional area to that of the whole board: $P_{bull} = \frac{\sigma_{bull}}{A_{board}}$.

      Similarly: $P_{10\,pts} = \frac{\sigma_{10\,pts}}{A_{board}}$ …

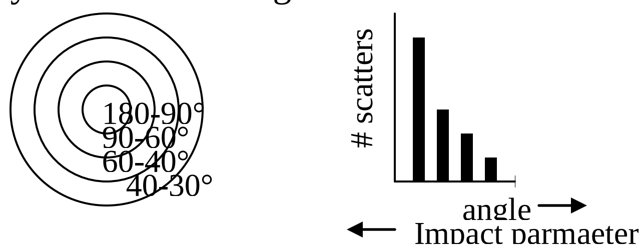
- **Cross-sections, alpha-particles, and scattering angles.** However, when we've got more long-range interactions, such as that of an alpha-particle with a gold nucleus, the alpha-particle needn't actually hit within the geometric radius of the nucleus to get deflected. We still use the language of "cross-section."
    - If we say the bull's-eye is like actually hitting the particle head-on, making the alpha-particle bounce back, then the next ring out is like coming quite close to the nucleus and deflecting sharply; the next ring out is coming not so close, and deflecting a little less sharply…
    - Again, the probability of each type of deflection is proportional to the corresponding ring's cross-section.

Demo: 08_Rutherford_dist.py rotate to look head on and see bulls eye

8.4.4 Conservation Laws Vs. Details of the interaction

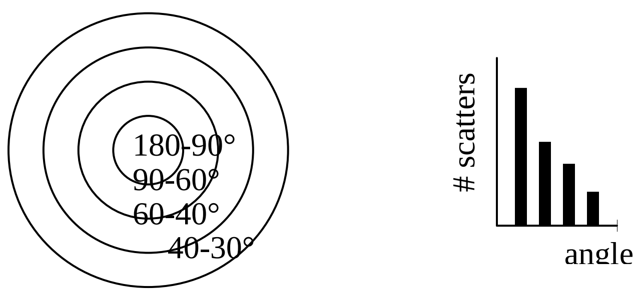
- **Cross-section / Scattering Angle & Force law.** Now imagine we had two targets, one is a gold nucleus, with its 79 protons, and one is a nucleus with twice as many protons. You shoot randomly at both with alpha-particles of the same energies. Let’s say that if you hit within these ranges of the center of a gold nucleus, you will be deflected by these angles:

Demo: 08_Rutherford_gold.py rotate to look head on and see bulls eye, see plot of Probability of scatter vs. angle.



- Since each ring out has a bigger area, the number of alpha-particles scattered at these different angles is greater for the bigger ring / smaller scattering angle. The distribution of scatters looks something like the plot beside it.

- Now imagine the nucleus with twice as many protons, at any given distance from this nucleus, the force is twice as strong as for the gold nucleus. Qualitativley, how should our ring of targets look?



Experience the same deflection at a greater distance / impact parameter

Demo: 08_Rutherford_zerconium.py rotate to look head on and see bulls eye, see plot of Probability of scatter vs. angle.

- **Different force law**
    - For that matter, say we’ve got a different force law, say the force dies off exponentially instead of like $1/r^2$. How would the target look differently?
        - Rings would be narrow near the center and quite broad outside.
    - How would the count of scatters vs. angle look?
        - The small angle scatters would be much more popular.

Moral: Scattering experiments can be used to discover the nature of the target and the force law by which it interacts with the projectile.