

This is a closed book, closed notes exam. Calculators are permitted, but no saved equations may be used. Please let me know if a necessary equation does not appear below. Point assignments are noted throughout the exam.

You must provide your own Equation Sheet. Hand it in with your exam. It can contain no pictures beyond a circuit component icons and a triangle for trig relations. It can contain no words beyond unit names. It can contain no redundant equations. Any of these will result in 0 points on relevant exam questions.

**Units**

$$B \quad \text{Tesla} = \frac{N}{\text{Amp} \cdot m} \qquad \text{Emf} \quad \text{Volt} = \frac{J}{\text{Coul}} \qquad L, M \quad \text{Henry} = \Omega \cdot s$$

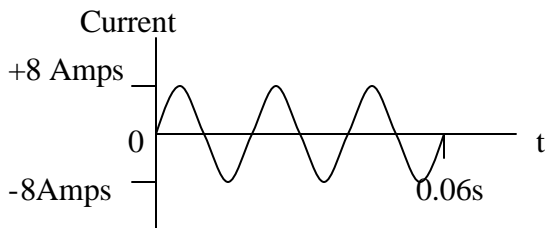
$$C \quad \text{Farad} = \frac{\text{sec}}{\Omega} \qquad I \quad \text{Amp} = \frac{\text{Coul}}{\text{sec}} \qquad F \quad \text{Weber} = \text{Tesla} \cdot m^2$$

$$R, Z, X_C, X_L \quad \text{Ohm} = \Omega = \frac{\text{Volt}}{\text{Amp}} = \frac{J \cdot s}{\text{Coul}^2}$$

**Useful Constants**

$$e = 1.60 \times 10^{-19} \text{C}, \epsilon_0 = 8.85 \times 10^{-12} \text{C}^2 / (\text{N} \cdot \text{m}^2), m_0 = 4\pi \times 10^{-7} \text{T} \cdot \text{m} / \text{Amp s}$$

- 1) (5 pts.) Below, the current through a circuit element is plotted as a function of time. What is the rms value of this current?



$$I_o = 8 \text{ Amps} = \text{Amplitude of Current oscillation}$$

$$I_{rms} = \sqrt{\frac{1}{2}} I_o = \sqrt{\frac{1}{2}} \cdot 8 \text{ Amps} = \boxed{5.65 \text{ Amps}}$$

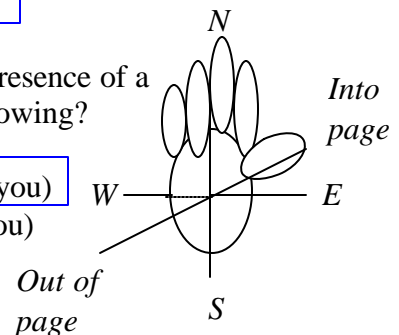
- 2) (5 pts.) What is the average power dissipated across a circuit with a  $16 \Omega$  impedance that is plugged into a wall socket providing  $emf_o = 170 \text{ Volts}$ ?

$$\langle P \rangle = \frac{1}{2} I_o Emf_o, \quad Emf_o = I_o R \Rightarrow I_o = \frac{Emf_o}{R}$$

$$\langle P \rangle = \frac{1}{2} \frac{Emf_o}{R} Emf_o = \frac{1}{2} \frac{Emf_o^2}{R} = \frac{1}{2} \frac{(170 \text{ Volts})^2}{16 \Omega} = \boxed{903 \text{ Watts}}$$

- 3) (5 pts.) A current flowing feels a force pushing it East in the presence of a magnetic field pointing North. What direction is the current flowing?

- a. East                      c. West                      **e. Into the page (away from you)**  
 b. North                    d. South                    f. Out of the page (toward you)



- 4) (5 pts.) A proton is moving with a speed of  $1.0 \times 10^6$  m/s when it encounters a magnetic field that is at  $60.0^\circ$  to the velocity. As a result it feels a force of  $1.0 \times 10^{-9}$  N. How strong is the field?

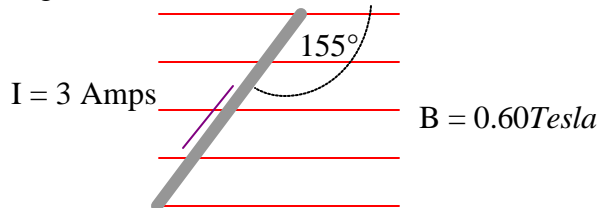
$$F = qvB \sin \theta \Rightarrow B = \frac{F}{qv \sin \theta} = \frac{1.0 \times 10^{-9} \text{ N}}{1.60 \times 10^{-19} \text{ Coul} \cdot 1.0 \times 10^6 \text{ m/s} \cdot \sin 60^\circ} = 7,200 \text{ T} = 7.2 \text{ kT}$$

- 5) (5 pts.) A doubly ionized (i.e. missing two electrons) particle with a mass of  $6.64 \times 10^{-27}$  kg is accelerated from rest through a potential difference of  $2.45 \times 10^6$  Volts. How strong a field must it enter to have a radius of curvature of 0.01 m?

An atom 'missing two electrons' will have two unpaired positive charges, so  $q = 2e$ .

$$m = \frac{qr^2 B^2}{2\Delta V} \Rightarrow B = \sqrt{\frac{m2\Delta V}{qr^2}} = \sqrt{\frac{6.64 \times 10^{-27} \text{ kg} \cdot 2 \cdot 2.45 \times 10^6 \text{ Volts}}{2 \cdot 1.60 \times 10^{-19} \text{ Coul} \cdot (0.01 \text{ m})^2}} = 32 \text{ T}$$

- 6) (5 pts.) From the illustration, determine the magnitude of the magnetic force exerted on the wire 0.05 m long wire.

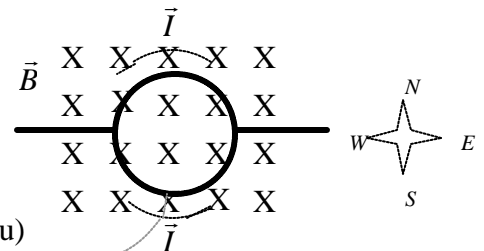


$$F = ILB \sin \theta_{I,B}$$

$$F = 3 \text{ Amps} \cdot 0.05 \text{ m} \cdot 0.60 \text{ T} \cdot \sin 155^\circ$$

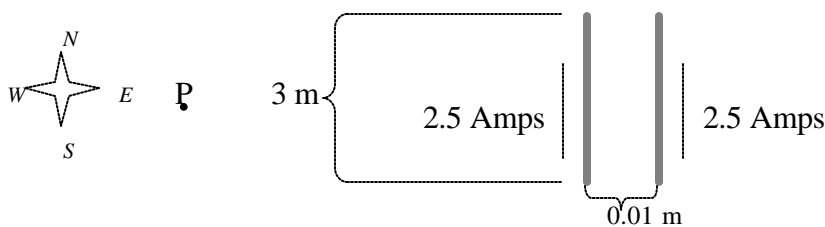
$$F = 3.8 \times 10^{-2} \text{ N}$$

- 7) (5 pts.) In the presence of a magnetic field directed into the page, current is passing about a coil as illustrated. What is the direction of the magnetic force exerted on this point at the **bottom** of the coil?



- a. East  
 b. North  
 c. West  
 d. South  
 e. Into the page (away from you)  
 f. Out of the page (toward you)

- 8) (5 pts.) Below is a picture of two parallel, current-carrying wires of length and separation shown. What is the *magnitude* of the force exerted by one on the other?



$$F = ILB \sin \theta_{I,B}$$

$$B = \frac{\mu_0 I}{2pr}$$

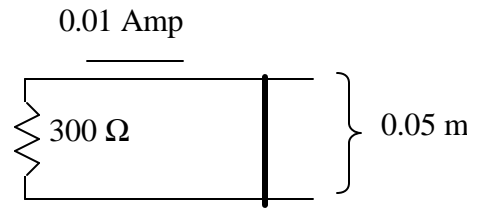
Right Hand rule says that the field due to the right wire points out of the page at (perpendicular to) the left wire:  $\theta = 90^\circ$ .

$$F = IL \frac{\mu_0 I}{2pr} \sin \theta_{I,B} = L \frac{\mu_0 I^2}{2pr} \sin \theta_{I,B} = 3 \text{ m} \frac{4\pi \times 10^{-7} \frac{\text{T}\cdot\text{m}}{\text{A}} \cdot (2.5 \text{ A})^2}{2\pi \cdot 0.01 \text{ m}} = 3.75 \times 10^{-4} \text{ N}$$

- 9) (5 pts.) In the previous picture, what is the *direction* of the net magnetic field at point "P", which is in the plane of the two wires?

- a. East  
 b. North  
 c. West  
 d. South  
 e. Into the page (away from you)  
 f. Out of the page (toward you)

- 10) A conducting bar moves along two conducting rails that are joined by a resistor. If there is a uniform, 0.2 Tesla, magnetic field pointing straight into the page everywhere and a 0.01 Amp, clockwise-flowing, current is induced,



- a. (5 pts.) what is the *magnitude* of the induced emf?

$$emf = IR = 0.01Amp \cdot 300\Omega = \boxed{3Volts}$$

- b. (5 pts.) is the bar moving left or right? **Left** following the right hand rule, if the bar is moved left (thumb), in the presence of a field pointing into the page (fingers), then the charged particles on the bar get pushed down the page (palm), this push drives a current down the bar... clockwise around the circuit as observed.

- 11) (5 pts.) The Earth's magnetic field of  $4.95 \times 10^{-5}$  Tesla passes through the classroom doorway at an angle of  $50^\circ$  off below horizontal. If the door is 1.50 m tall and 0.90 m wide, what is the magnitude of the magnetic flux through it?

$$\Phi = AB \cos \mathbf{q}_{B \perp A} = 1.5m \cdot 0.9m \cdot 4.95 \times 10^{-5} T \cdot \cos 50^\circ = \boxed{4.25 \times 10^{-5} Weber}$$

- 12) (5 pts.) A magnetic field is directed perpendicular to the plane of a  $0.15 \text{ m} \times 0.30 \text{ m}$  rectangular coil of 120 loops of wire. To induce an average *Emf* of magnitude 1.2Volts in the coil, the magnetic field is increased from 0.1 Tesla to 1.5 Tesla. Over what time interval must this change take place?

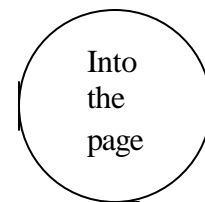
$$Emf = N \frac{\Delta \Phi}{\Delta t} = N \frac{\Delta(AB \cos \mathbf{q}_{B \perp A})}{\Delta t} \Rightarrow \Delta t = N \frac{\Delta(AB \cos \mathbf{q}_{B \perp A})}{Emf} = N \frac{AB_f \cos \mathbf{q}_{B \perp A} - AB_i \cos \mathbf{q}_{B \perp A}}{Emf}$$

$$\Delta t = 120 \frac{0.15m \cdot 0.30m \cdot 1.5T \cos 0^\circ - 0.15m \cdot 0.30m \cdot 0.1T \cos 0^\circ}{1.2Volts} = \boxed{6.3s}$$

Note: the angle is measured between the magnetic field and the *perpendicular* to the area. Since the field itself is perpendicular to the area, the angle is  $0^\circ$ .

- 13) (5 pts.) Given the direction of current flow indicated to the right, illustrate the direction the associated magnetic field points both *inside* and *outside* the current loop.

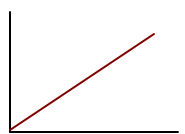
Out of the page



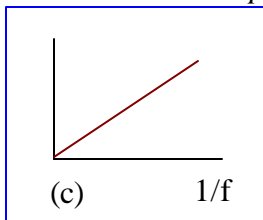
- 14) (5 pts.) Which of the following graphs illustrates how *capacitive reactance* varies with frequency?



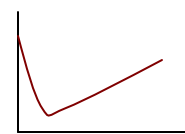
(a)



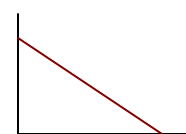
(b)



(c)



(d)



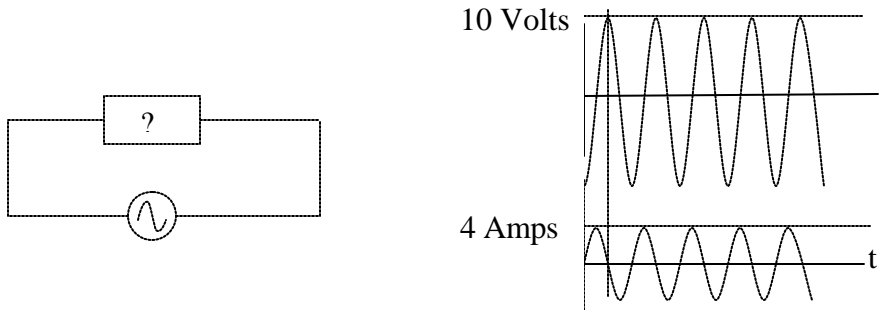
(e)

$$X_c = \frac{1}{2\pi fC} = \frac{1}{2\pi C} \left( \frac{1}{f} \right) \text{ so it is linear in } 1/f.$$

- 15) (5 pts.) The reactance of a capacitor at 100 Hz is 40 Ω. Determine the capacitance.

$$X_c = \frac{1}{2\pi f C} \Rightarrow C = \frac{1}{2\pi f X_c} = \frac{1}{2\pi \cdot 100\text{Hz} \cdot 40\Omega} = 3.98 \times 10^{-5} \text{ F}$$

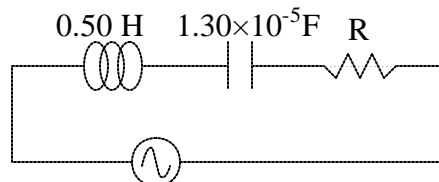
- 16) (5 pts.) The plots below are of the voltage and current around a single component in an AC circuit. Which is it, a capacitor or a resistor? Capacitor – the current and voltage across a capacitor are out of phase as is illustrated.



- 17) (5 pts.) Regardless of which of the three the component above is, what is the value of the impedance?

$$Emf_o = I_o Z \Rightarrow Z = \frac{Emf_o}{I_o} = \frac{10\text{Volts}}{4\text{Amps}} = 2.5\Omega$$

- 18) (5 pts.) If driven at 50 Hz, what resistor must be inserted into this circuit to give a total impedance of 100 Ω?



$$Z = \sqrt{R^2 + (X_L - X_C)^2} \Rightarrow Z^2 = R^2 + (X_L - X_C)^2 \Rightarrow R^2 = Z^2 - (X_L - X_C)^2 \Rightarrow R = \sqrt{Z^2 - (X_L - X_C)^2}$$

$$R = \sqrt{Z^2 - (X_L - X_C)^2}$$

$$X_c = \frac{1}{2\pi f C}, \quad X_L = 2\pi f L$$

$$R = \sqrt{Z^2 - \left(2\pi f L - \frac{1}{2\pi f C}\right)^2} = \sqrt{(100\Omega)^2 - \left(2\pi \cdot 50\text{Hz} \cdot 0.50\text{H} - \frac{1}{2\pi \cdot 50\text{Hz} \cdot 1.30 \times 10^{-5} \text{ F}}\right)^2}$$

$$R = 48\Omega$$

- 19) (5 pts.) What is the resonant frequency of the circuit above?

$$f_o = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.50\text{H} \cdot 1.3 \times 10^{-5} \text{ F}}} = 62\text{Hz}$$