Names: KEY **Group Problem 0**

You have been given a job working on a team that is examining telescope pictures for asteroids which might collide with the Earth. In your orientation, your team was told how important these observations might be. Current theories indicate that dinosaurs and many other organisms became extinct when the Earth was struck by a large asteroid. 65 million years ago dust from an asteroid impact was lofted into the upper atmosphere all around the globe, where it lingered for at lest several months and blocked the sunlight reaching the Earth's surface. On the dark and cold Earth that temporarily resulted, many forms of life became extinct. It has been suggested that such an asteroid collisions likely to happen again, perhaps causing the extinction of the current dominant life form on Earth, namely us. As you scan space for this danger, how large an asteroid should you be watching for if the dangerous asteroid size is roughly the same as the one that wiped out the dinosaurs? Available evidence suggests that about 20% of that asteroid's mass ended up as dust spread uniformly over Earth after eventually settling out of the upper atmosphere. About 0.020g/cm^2 of dust, which is chemically different than the Earth's rock covered the Earth's surface. Typical asteroids have a density of about 2.0 g/cm^3.

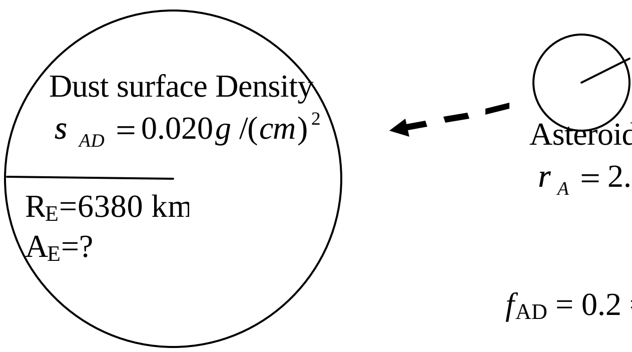
Possibly Useful Information:

Earth's Radius = 6380 km	Volume of a sphere = $\frac{4}{3}\pi R^3$
1 km = 1×10^5 cm	Surface area of a sphere = $4\pi R^2$
Volume density = mass/volume	Surface density = mass/area

Focus & Describe the Physics

(6 pts) **Picture / Diagram & Definitions**

(establish a coordinate system and represent the relevant quantities: symbols and values)



R_A=? (radius)
M_A = ? (mass)
V_A = ? (volume)

Asteroid volume Density
$r_A = 2.0 g/(cm)^3$

$f_{AD} = 0.2 = \frac{M_{AD}}{M_A}$ fraction of asteroid mass dusting Earth's surface

(1 pt) **Question:** What "size", say radius, an asteroid has enough mass so 20% can cover the Earth's surface at 0.020 g/cm^2?

(1 pt) **Target:** R_A asteroid radius (note: from the vague statement of the problem, one could target Volume, or Mass instead)

(1 pt) **Approach:** Geometry Assume that the Earth and asteroid are spheres

(3 pts) **Quantitative Relations**

1) $r \equiv \frac{M}{V} \Rightarrow r_A = \frac{M_A}{V_A}$ 2) $V_{sphere} \equiv \frac{4}{3}pr^3 \Rightarrow V_A = \frac{4}{3}pR_A^3$ 3) $A_{spere} = 4pr^2 \Rightarrow A_E = 4pR_E^2$

4) $s \equiv \frac{M}{A} \Rightarrow s_{AD} = \frac{M_{AD}}{A_E}$ 5) $f_{AD} = \frac{M_{AD}}{M_A}$

Algebra

(7 pts)

Find R_A — Unknowns

$V_A = \frac{4}{3} p R_A^3$

2) $R_A = \sqrt[3]{\frac{V_A}{\frac{4}{3}p}}$ — V_A

Find V_A

1) $r_A = \frac{M_A}{V_A}$, $V_A = \frac{M_A}{r_A}$ — M_A

Find M_A

5) $f_{AD} = \frac{M_{AD}}{M_A}$, $M_A = \frac{M_{AD}}{f_{AD}}$ — M_{AD}

Find M_{AD}

4) $s_{AD} = \frac{M_{AD}}{A_E}$, $M_{AD} = s_{AD} A_E$ — A_E

Find A_E

3) $A_E = 4pR_E^2$ — none

into 4) $M_{AD} = s_{AD} 4pR_E^2$

into 5) $M_A = \frac{s_{AD} 4pR_E^2}{f_{AD}}$

into 1) $V_A = \frac{s_{AD} 4pR_E^2}{r_A f_{AD}}$

into 2) $R_A = \sqrt[3]{\frac{\frac{s_{AD} 4pR_E^2}{r_A f_{AD}}}{\frac{4}{3}p}} = \sqrt[3]{\frac{3 s_{AD} R_E^2}{r_A f_{AD}}}$

(2 pts) **Unit Check**

$$[length]: \sqrt[3]{\frac{1\frac{\cancel{[mass]}}{\cancel{[length]^2}}\cancel{[length]^2}}{\frac{\cancel{[mass]}}{[length]^3}\cancel{1}}} = \sqrt[3]{\frac{1}{\frac{1}{[length]^3}}} = [length]$$

Evaluate Equation

The dependences look right: increase the size of the Earth, you need a bigger asteroid, i.e., $R_A \uparrow$ when $R_E \uparrow$. Similarly, $R_A \uparrow$ when you need a higher density of dust σ_A, less of the asteroid becomes dust, f_{AD}, or the asteroid is less dense, r_A.

Execute

$$R_A = \sqrt[3]{\frac{3 \cdot 0.020g/(cm)^2 \cdot (6380km)^2}{2.0g/(cm)^3 \cdot 0.20}}$$

$$R_A = 183\sqrt[3]{\frac{\cdot (km)^2}{1/(cm)}}$$

$$R_A = 183\sqrt[3]{\cdot (km)^2 cm}$$

$$R_A = 183\sqrt[3]{\cdot (km)^2 1cm \frac{0.01m}{1cm}\frac{0.001km}{1m}} = 3.94km$$

The asteroid's radius is around 3.94 km

Evaluate Numbers

(1pt) The answer is(n't) reasonable because…

You'd expect it to be bigger than a house but smaller than the moon. This qualifies.

(1/2 pt) Is the answer properly stated? Yes.

(1/2 pt) Is the answer complete? Yes.