Calculus I: Another Fundamental Theorem of Calculus!

Goal: Find a formula for the derivative of a definite integral with a variable upper limit.

1) Let $G(x) = \int_3^x (t^3 + 2t)\,dt$. To compute $G'(x)$, first use the Fundamental Theorem of Calculus to evaluate $\int_3^x (t^3 + 2t)\,dt$. Your answer will have at least one x in it.

$G(x) = \int_3^x (t^3 + 2t)\,dt =$ $\Big|_{t=3}^{t=x} =$

Second, compute the derivative of $G(x)$: $G'(x) =$

Another way to write $G'(x)$ is as $G'(x) = \dfrac{d}{dx}\left[\int_3^x (t^3 + 2t)\,dt\right] =$

2) Let $G(x) = \int_1^x \sqrt{t}\,dt$. Compute $G'(x)$ using the two-step method of Problem 1.

$G(x) = \int_1^x \sqrt{t}\,dt = \int_1^x t^{1/2}\,dt =$ $\Big|_{t=1}^{t=x} =$

$G'(x) =$

Another way to write $G'(x)$ is as $\dfrac{d}{dx}\left[\int_1^x \sqrt{t}\,dt\right] =$

3) Let $G(x) = \int_a^x f(t)\,dt$, and let $F(t)$ be an antiderivative of $f(t)$. Compute $G'(x)$ using the two-step method of Problem 1.

$G(x) = \int_a^x f(t)\,dt =$ $\Big|_{t=a}^{t=x} =$

$G'(x) =$

Another way to write $G'(x)$ is as $\dfrac{d}{dx}\left[\int_a^x f(t)\,dt\right] =$

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3) (continued) We have just established the formula:

$F(x)=\int_a^x f(t)\,dt$ or $\frac{d}{dx}\left[\int_a^x f(t)\,dt\right]=$

$F'(x)=$

In Problems 4, 5, and 6, use the formula from Problem 3 to write the derivative as quickly as you can. This means write the derivative without showing any work!

4) $G(x)=\int_0^x \sin(t^2)\,dt$

$G'(x)=$

5) $\frac{d}{dx}\left[\int_2^x e^{3t}\,dt\right]=$

6) $F(x)=\int_{-1}^x \cos(t)\,dt$

$F'(x)=$

7) Explain why it makes sense to call $F(x)=\int_a^x f(t)\,dt$ an *antiderivative* of $f(x)$.

8) The rule you wrote in Problem 3 is called the **Second Fundamental Theorem of Calculus.** Explain how the Second Fundamental Theorem of Calculus shows that integration and differentiation are inverse processes.

Begin with f: f

Integrate:

Differentiate:

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9) Let $H(x) = \int_2^{x^3} \cos(t)\,dt$. Compute $H'(x)$ using the two-step method of Problem 1.

$$H(x) = \int_2^{x^3} \cos(t)\,dt = \qquad \Big|_{t=2}^{t=x^3} =$$

$$H'(x) =$$

10) Let $H(x) = \int_1^{\sin(x)} e^{3t}\,dt$. Compute $H'(x)$ using the two-step method of Problem 1.

$$H(x) = \int_1^{\sin(x)} e^{3t}\,dt = \qquad \Big|_{t=1}^{t=\sin(x)} =$$

$$H'(x) =$$

11) Let $H(x) = \int_a^{g(x)} f(t)\,dt$, and let $F(t)$ be an antiderivative of $f(t)$. Compute $H'(x)$ using the two-step method of Problem 1.

$$H(x) = \int_a^{g(x)} f(t)\,dt = \qquad \Big|_{t=a}^{t=g(x)} =$$

$$H'(x) =$$

Another way to write $H'(x)$ is as $\dfrac{d}{dx}\left[\int_a^{g(x)} f(t)\,dt\right] =$

We have just established the formula:

$F(x) = \int_a^{g(x)} f(t)\,dt$ or $\dfrac{d}{dx}\left[\int_a^{g(x)} f(t)\,dt\right] =$

$F'(x) =$

12) Use the formula from Problem 11 to write the derivative as quickly as you can. This means write the derivative without showing any work!

$$F(x) = \int_2^{\cos(x)} \sqrt{1+t^2}\,dt$$

$$F'(x) =$$

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13) If $y = \int_a^{g(x)} f(t)\,dt$, then y is a composite function of x, with $y = \int_a^{z} f(t)\,dt$ and $z = g(x)$. Show how to apply the Chain Rule in the form $\frac{dy}{dx} = \frac{dy}{dz} \cdot \frac{dz}{dx}$ to obtain the rule you wrote in Problem 11.

Source: Inspired by Exploration 35 in *Calculus Explorations,* by Paul A. Foerster, Key Curriculum Press, 1998, page 36