Calculus I: The Most Ubiquitous Initial Value Problems

1) (a) Write a function $f(x)$ for which $f'(x) = f(x)$: $f(x) =$

(b) For your function $f(x)$ from part (a), let $g(x) = 3f(x)$: $g(x) =$

Compute $g'(x)$: $g'(x) =$

Does $g(x)$ have the property that $g'(x) = g(x)$?

Compute $g(0)$: $g(0) =$

(c) Write a function $h(x)$ for which $h'(x) = h(x)$ and $h(0) = 7$: $h(x) =$

Explain why $h(x) = e^x + 6$ is not a solution to this problem.

(d) Write a function $y(x)$ for which $y'(x) = y(x)$ and $y(0) = y_0$: $y(x) =$

2) (a) Write a function $f(x)$ for which $f'(x) = 4f(x)$: $f(x) =$

(b) For your function $f(x)$ from part (a), let $g(x) = 3f(x)$: $g(x) =$

Compute $g'(x)$: $g'(x) =$

Does $g(x)$ have the property that $g'(x) = 4g(x)$?

Compute $g(0)$: $g(0) =$

(c) Write a function $h(x)$ for which $h'(x) = 4h(x)$ and $h(0) = 7$: $h(x) =$

Explain why $h(x) = e^{4x} + 6$ is not a solution to this problem.

(d) Write a function $y(x)$ for which $y'(x) = k\,y(x)$ and $y(0) = y_0$, where k is any constant real number: $y(x) =$

Together, the *differential equation* $y'(x) = k\,y(x)$ and the *initial value* $y(0) = y_0$ from Problem 2, part (d), form an *initial value problem* (*IVP*). A solution to an IVP is a function $y(x)$ that satisfies the given differential equation and has the given initial value. So, for instance, the function $h(x)$ you wrote in Problem 2, part (c), is a solution to the initial value problem that consists of the differential equation $h'(x) = 4h(x)$ and the initial value $h(0) = 7$.

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3) Solve the following initial value problems.

(a) $f'(x) = 4f(x)$, $f(0) = 7$

$f(x) =$

(b) $g'(x) = -3g(x)$, $g(0) = 12$

$g(x) =$

(c) $\dfrac{dy}{dx} = 4y$, $y(0) = 7$

$y(x) =$

(d) $\dfrac{dy}{dt} = 0.5y$, $y(0) = 2.2$

$y(t) =$

(e) $\dfrac{dP}{dt} = 0.02P$, $P(0) = 6$

$P(t) =$

(f) $y' = -0.01y$, $y(0) = 200$

$y =$

(Usually, the independent variable t or x makes the most sense in the solution.)

(g) $\dfrac{d^2y}{dx^2} = 16y$, $y(0) = 7$

$y(x) =$

Hint: Compare with part (c).

(h) $y'' = 25y$, $y(0) = A$

$y(t) =$

Is $y = Ae^{-5t}$ also a solution to the IVP?

Is $y = Ae^{5t} + Be^{-5t}$ a solution to the initial value problem $y'' = 25y$, $y(0) = A + B$?

(i) $y'' = -25y$, $y(0) = A + B$

$y(t) =$

Hints: $\cos(t)$, $\sin(t)$

(j) You already know why $y = e^{0.5t} + 1.2$ is <u>not</u> a solution to the initial value problem in part (d). Explain why $y = 0.25t^2 + 2.2$ is <u>not</u> a solution to the initial value problem in part (d). Explain why $0.25y^2 + 2.2$ is <u>not</u> a solution to the initial value problem in part (d). For the same reasons, $y = -0.005t^2 + 200$ and $-0.005y^2 + 200$ are not solutions to the initial value problem in part (f)!

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