Subgroups

1. Let G be an Abelian group and let H_2 be the subset of G given by

   $H_2 = \{ x \in G \mid x^2 = e \}$.

   (a) For $G = U(20)$, $H_2 = \{$________________________$\}$ (list the elements).

   (b) Prove that H_2 is a subgroup of G.

   Closure: Let $a, b \in H_2$.

   Identity: $e \in H_2$ because:

   Inverses: Let $a \in H_2$.

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2. Let G be a group (not necessarily Abelian), let $a \in G$, and let $C(a)$ be the subset of G given by $C(a) = \{ g \in G \mid ga = ag \}$.

(a) Use the group table for D_4 to list the elements of $C(V)$ for $G = D_4$. Explain how you know that each element you list is in $C(V)$.

$C(V) = \{$__________________________________$\}$

(b) Prove that $C(a)$ is a subgroup of G. You may let $H = C(a)$, if you like.

Closure: Let $x, y \in C(a)$.

Identity: $e \in C(a)$ because:

Inverses: Let $x \in C(a)$.

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