

For Monday 9/24, read Griffiths section 2.5.1 and turn in by 9:30 am:

1. Conceptual: Compare Griffith's definition of a bound state with Q7.1.
2. Conceptual: Compare Griffith's definition of tunneling with Q11.3.
3. Easy Math: Let  $y = \sqrt{a}[x + (b/2a)]$ . Write out the equivalent for  $(ax^2 + bx)$  in terms of  $y$  instead of  $x$ . What is the advantage of this change of variables?
4. Real math: The Gaussian wave packet. A free particle has the initial wave function  $\Psi(x,0) = Ae^{-ax^2}$ , where  $A$  and  $a$  are constants ( $a$  is real and positive).

- a. Normalize  $\Psi(x,0)$ .
- b. Find  $\Psi(x,t)$ .

- c. Find  $|\Psi(x,t)|^2$ . Express your answer in terms of the quantity  $w \equiv \sqrt{\frac{a}{1 + (2\hbar at / m)^2}}$ .

Sketch  $|\Psi|^2$  (as a function of  $x$ ) at  $t=0$ , and again for some very large  $t$ . Qualitatively, what happens to  $|\Psi|^2$  as time goes on?

- d. Find  $\langle x \rangle$ ,  $\langle p \rangle$ ,  $\langle x^2 \rangle$ ,  $\langle p^2 \rangle$ ,  $\sigma_x$ , and  $\sigma_p$ .
- e. Does the uncertainty principle hold? At what time  $t$  does the system come closest to the uncertainty limit?

"For realz" weekly homework due 9:30 am on Wednesday 9/26 is math problems from 9/19, 9/21, and 9/24.

For Wednesday 9/26, read Griffiths' section 2.5 and Q11 and turn in by 9:30 am:

1. Conceptual: Are possible energy levels quantized only for bound states? Why or why not?
2. Conceptual: State the rules from Q11.4 in terms of mathematical equations. Can you match the rules to equations in Griffiths? If you can, give equation numbers.
3. Math: Evaluate the following integrals:

- a.  $\int_{-3}^{+1} (x^3 - 4x^2 + 3x - 2)\delta(x+1)dx$

- b.  $\int_0^{\infty} [\cos(2x) + 5]\delta(x - \pi)dx$

- c.  $\int_{-3}^{+1} e^{(|x|+6)}\delta(x-4)dx$

4. Math: Consider the double delta-function potential  $V(x) = -\alpha[\delta(x+a) + \delta(x-a)]$ , where  $\alpha$  and  $a$  are positive constants.
  - a. Sketch this potential.
  - b. Write the schrodinger equation in each of the three regions.
  - c. What is the solution to each of these differential equations?
  - d. What are the boundary conditions?
  - e. Does problem 2.1(c) apply here? Does it help?
  - f. Write the possible solutions for  $\psi(x)$ .
  - g. How many bound states are there?

For Friday 9/28, read Griffiths' section 2.6 and Q11.2 turn in by 9:30 am:

5. Conceptual: What physical properties determine the number of bound states in a finite well?
6. Conceptual: How do we determine the number of scattering states?
7. Fill in: Derive equations 2.167 and 2.168. Show all steps. Hints:
  - a. First use 2.165 and 2.166 to write C and D in term of F (without A or B).
  - b. Solve for  $Ae^{-ika} + Be^{ika}$  and  $Ae^{-ika} - Be^{ika}$  in terms of  $\sin(2la)$  and  $\cos(2la)$  using the double angle formulas in trig.
  - c. Add the equations you get in the previous section to get rid of B and then solve for F in terms of A (eq. 2.168).
  - d. Now subtract the equations to get rid of A and solve for B in terms of F (eq. 2.167).
8. Math: Consider the "step" potential:  $V(x) = \begin{cases} 0, & \text{if } x \leq 0 \\ V_0, & \text{if } x > 0 \end{cases}$  and  $V(x) = \begin{cases} V_0, & \text{if } x \leq 0 \\ 0, & \text{if } x > 0 \end{cases}$ 
  - a. Sketch both functions. (This is due with conceptual questions.)
  - b. Show that the reflection coefficients for the two cases are equal when  $E > V_0$ .
    - i. Start by doing the first case. How many regions are you using? What are your boundary conditions?
    - ii. Write the schrodinger equation in each region and solve. Solutions should be exponentials, not sines and cosines. You should be using k from equation 2.130 and l similar to (but not exactly) equation 2.148. Careful when defining these in terms of E and  $V_0$ .
    - iii. Label your diagrams in part (h) with coefficients like in figure 2.15. Assume wave enters from left. What goes to zero?
    - iv. Apply boundary conditions and calculate the reflection coefficient. (Hint: write as something over  $(k^2 - l^2)^2$ .) Finally, plug back in for k and l to get reflection in terms of E and  $V_0$  and simplify. You should get something ugly over  $V_0^2$ .
    - v. Repeat above for other case.
  - c. What happens when  $E < V_0$ ? Show calculation and comment on solution. Do only for the first case.