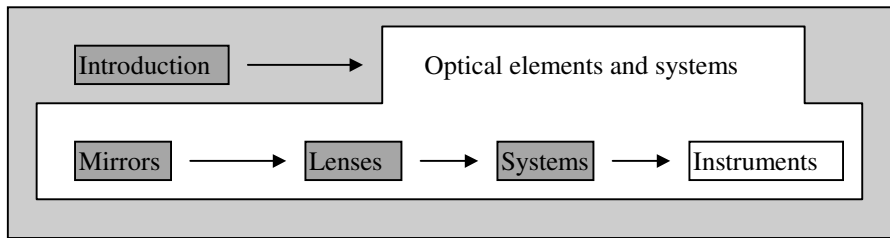# Optical Instruments



## O5.1 Introduction

In this section, we will investigate the operation of three common optical instruments. We will investigate the simple magnifier, the microscope, and the telescope. These systems will require a slightly different approach from the previous chapters because the lens of our eye is part of each of these systems.

O5.2 THE SIMPLE MAGNIFIER. We will investigate the common everyday magnifier. We will examine the behavior of the simple converging lens in conjunction with the human eye. We will begin with a discussion of the properties and behavior of the human eye. Also, in this section we will introduce the concept of angular magnification as an alternative to the standard magnification of the earlier sections. We need to characterize the behavior in terms of angular magnification to accurately answer the question of how big objects look.

O5.3 SIMPLE MICROSCOPES. This is the next most complicated optical system for enlarging small objects. This system is made up of three lenses, two are found in the microscope and one is found in the eye of the observer.

O5.4 REFRACTING TELESCOPES. The behavior of this instrument also requires us to include the properties of the eye if we are to correctly understand its operation. Again there are two lenses in the instrument and a third lens in the observers eye. What makes this optical instrument different from the microscope is the specific lens arrangement.

## O5.2 The simple magnifier.

The simple magnifier is composed of a single converging lens that acts, in conjunction with the human eye, to enlarge an object. The analysis of optical systems involving magnifiers is not straightforward because the lens in the eye is not of a fixed focal length. The situation is also complicated because the size of the image created on the retina of the eye is less important than the interpretation of the retinal image by the brain.
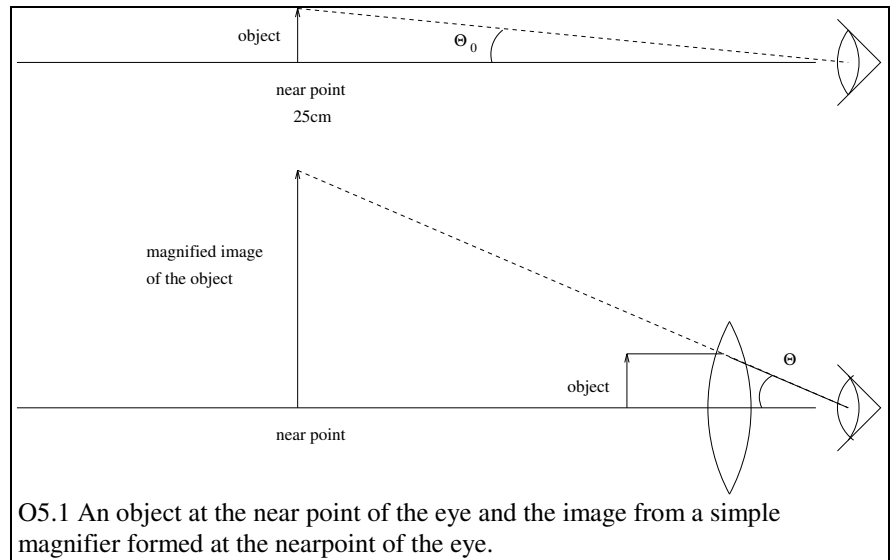
Before we investigate the magnifier system, we need to review some features of the human eye. First of all, the human eye has a variable focal length. There are muscles that can thin or thicken the lens for distance and close-up viewing. These muscles relax for distance viewing and tighten for close-up focusing. (When these muscles lose their ability to relax because of excessive close

work, a person becomes nearsighted.)  The least amount of eye strain results when these muscles are relaxed.

Next, the eye can not focus arbitrarily close to one's face.  For most people the closest distance at which one can focus on an object without excessive strain is taken to be 25 cm.  Finally, the brains interpretation of the size of an object is partially based on how large an object appears in relation to other objects.  Thus if one holds a hand in front of the sun, the hand appears bigger than the sun.  Clearly the hand in not actually bigger than the sun but it definitely appears bigger than the sun.  These illusions combine to make an exact analysis of systems difficult.

Because the size, $h$', of the image formed on the retina is really not relevant for understanding systems which include the eye, we will define a new quantity called **angular magnification** as a measure of the behavior of the system.  The angular magnification is defined as "the ratio of the angle subtended by the image when using the magnifier to the angle subtended by the object when viewed from 25 cm by the "unaided eye" or $m = \theta/\theta_0$ .

We will make one further approximation in our study of the simple magnifier.  We will assume that the simple magnifier lens that we are using is right against the lens of our eye.   This is not actually true, but is not an unreasonable



O5.1 An object at the near point of the eye and the image from a simple magnifier formed at the nearpoint of the eye.

approximation.  With this approximation our understanding can proceed.  Simply place the lens (and your eye) so that the object you are trying to view results in a virtual image at -25 cm.  The lens of the eye then sees this virtual image. Because this virtual image is at the near point of the eye it appears as large as it can be. Figure O5.1 shows some typical rays with and without the magnifying lens. Mathematically, the object distance that gives an image at the near point can be found with

$$\frac{1}{S} + \frac{1}{-25cm} = \frac{1}{f} \Rightarrow S = \frac{25f}{25+f}$$

where $f$ is the focal length in centimeters and

$$M = -\left(\frac{-25\ cm}{S}\right).$$

The angular magnification is now found in short order since

$$\theta_0 \approx \frac{h}{25\ cm} \text{ and } \theta \approx \left(h \cdot \frac{25\ cm}{S}\right)\Big/25\ cm = \frac{h}{S}$$

giving

$$m = 1 + \frac{25\ cm}{f}.$$

This configuration results in the largest magnification but it is very tiring for the muscles of the eye. To focus at the near point the eye muscles must pull quite hard and this results in a great deal of eye strain. It is much more comfortable to relax the eye and view images located infinitely far away. This configuration reduces the magnification slightly but is much less stressful for long term viewing. Another way to state this condition is that the light rays entering the eye must all be parallel. This will occur if the object is placed very near the focal point of the lens so that $S=f+\varepsilon$ (where $\varepsilon$ is some very small distance). If this is the case,

$$\frac{1}{f+\varepsilon} + \frac{1}{S'} = \frac{1}{f} \Rightarrow S' = \frac{f(f+\varepsilon)}{\varepsilon}$$

and the magnification is

$$M = -S'/S = -\frac{\left(f(f+\varepsilon)/\varepsilon\right)}{(f+\varepsilon)} = -\left(f/\varepsilon\right).$$

Then

$$\theta = \frac{\left(hf/\varepsilon\right)}{\left[f(f+\varepsilon)/\varepsilon\right]} = h/(f+\varepsilon) \approx h/f \text{ and } m = \frac{25\ cm}{f}.$$

For most lenses used as simple magnifiers the difference between the two cases is minor but noticeable if one knows to look for it. If we try to get large magnifications from a single lens the image will be distorted due to aberrations associated with real lenses. If we want higher magnifications, then we need to use a microscope that is described in the next section.

---

Exercise O5.2.1: What is the maximum angular magnification of a magnifying glass of focal length 11 cm. What is the angular magnification for relaxed eye viewing.

---

## O5.3 Simple microscopes.

The next optical instrument that aids our natural vision is the simple microscope. This instrument is a simple version of the common laboratory instrument. Commercial microscopes are more complicated than the optical system
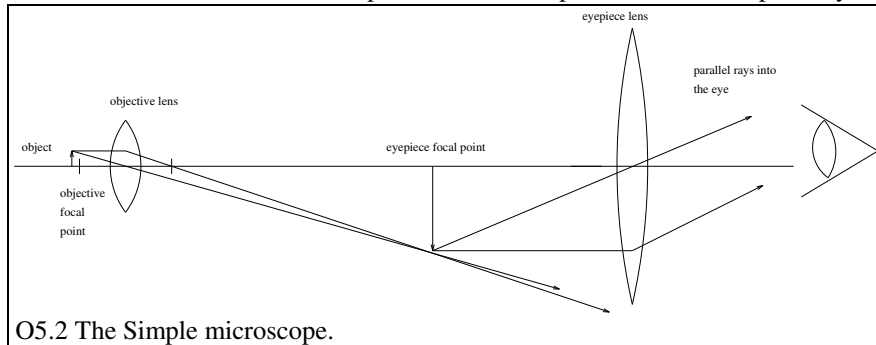


O5.2 The Simple microscope.

we will investigate in order to improve the quality of the image and the total magnification. However, the basic principles of the simple microscope and the more advanced systems are the same.

Just as for the simple magnifier, we could consider a situation for maximum magnification and a situation that is comfortable for extended viewing.

We will only deal with the case that results in minimum eye strain which means that the light rays leaving the microscope and entering the eye must all be parallel.

Figure O5.2 shows the lenses and rays that form the simple microscope. To understand the workings of this device, we will work backwards through the device. Thus, if all the light rays leaving the eyepiece are to be parallel, the object for the eyepiece lens must be located at the focal point of the eyepiece. The eyepiece lens typically has a focal length of a few centimeters. The other lens in the microscope, the objective, has a very short focal length and is placed so that when the small object under study is located just outside the focal point of this lens, a real image is formed at the focal point of the eyepiece. The overall magnification is determined by the magnification of both lenses and also the separation of the two lenses (tube length). The overall magnification is given by the product of the standard magnification for the objective lens times the angular magnification due to the eyepiece. Because the eye is involved we are more concerned with the angular magnification, which tells us how big the image appears, rather than the actual size of the image formed on the retina of the eye.

The overall angular magnification is the product of the magnification due to the objective and the angular magnification of the eyepiece. The magnification due to the objective has the standard form that we saw in Chapter O3 and is

$$M = -(T + f_o)/f_o$$

where T is called the tube length and is the distance between the objective focal point and the eyepiece focal point. Since T is so much bigger than $f_o$ this can be approximated as $-T/f_o$. Since the eyepiece acts as a simple magnifier acting on the intermediate image, it is given by $m = (25cm)/f_e$. Thus the total angular magnification is

$$M \cdot m = -(T/f_o) \cdot (25cm/f_e).$$

---

Exercise O5.3.1: A microscope has a tube length of 20 cm and an eyepiece with a 1 cm focal length. What must be the focal length of the objective lens to get a magnification of -200?

---

## O5.4 Refracting telescopes.

The simplest refracting telescope is another simple optical instrument. The telescope is typically used to view objects located far away unlike the microscope



O5.3 The simple refracting telescope.
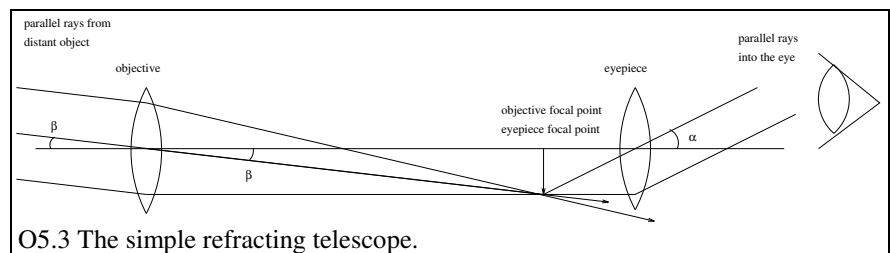
that is used to view small nearby objects. It is composed of two lenses but, as with the microscope, the properties of the eye must be included in order to understand the instrument. In some ways, the telescope is easier to understand than is the microscope. A schematic diagram of the simple refracting telescope is shown in Figure O5.3. The rays from the distant objects under consideration are parallel as they enter the objective lens. Because the rays are parallel as they approach the objective, an image of the distant object is formed at the focal point of the objective lens. As with the microscope the easiest viewing occurs when the rays entering the

eye are parallel and the eye muscles are relaxed. In order to produce such parallel rays for viewing, the eyepiece must be situated so that its focal point is coincident with the focal point of the objective and thus the image formed by the objective lens.

With the objective and eyepiece positioned as described above we can find the angle formed by rays coming from the object and the angle formed by the rays exiting from the eyepiece. Strictly speaking, the angle $\alpha$ is not correct because the angle is measured from the front of the telescope rather than from the viewer's eye. For telescopes where the object being viewed is extremely far away in comparison to the size of the telescope, this very slight difference is not significant. The rays entering the eye do so with an angle $\beta$ formed by rays emanating from the image formed between the two lenses. From Figure O5.3 we can see that

$$\tan(\beta) = h / f_o \quad \text{and} \quad \tan(\alpha) = h / f_e.$$

Because the angles are really quite small (the figure exaggerates the angles a great deal) the tangent of the angle can be replaced with the angle. Thus, the angular magnification becomes

$$m = -\left(f_o / f_e\right).$$

Because of various aberrations, this type of telescope is limited in its ultimate magnification. For really large magnifications, different types of telescopes have been designed. Most often these, like the Hubble space telescope, are based on mirrors.

---

Exercise O5.4.1: A telescope has an angular magnification of -100 and is 1.00 m long. What are the focal lengths of the lenses?

---

## O5.5 HOMEWORK PROBLEMS

1. A simple magnifier is used to assist a forensic scientist at a crime scene. The focal length of the magnifier is 8 cm. What is the maximum angular magnification one can achieve with this lens? What is the angular magnification one can realize if one does not want to suffer from excessive eyestrain?
2. A simple microscope can be constructed from two lenses. A reasonable amount of magnification for such a microscope would have a magnitude or 100. Assume that the tube length will be 18 cm and find values for the eyepiece and the objective lenses. Clearly there are many combinations that will accomplish this but one should give some thought to appropriate values for each lens.
3. What is the magnification and size of a refracting telescope that is equipped with an eyepiece with a focal length of 15 cm and an objective lens with a focal length of 75 cm?

---

# ANSWERS TO EXERCISES

E5.2.1 The maximum magnification is $m = 1 + \dfrac{25\ cm}{11\ cm} = 3.27$ and the relaxed eye angular magnification is $m = \dfrac{25\ cm}{11\ cm} = 2.27$.

E5.3.1 The focal length of the objective lens can be found by solving $-200 = -(20 \ cm/f_o) \cdot (25 \ cm/1 \ cm)$. This gives an objective focal length of 2.5 cm.

E5.4.1 The solution requires us to solve two equations in two unknowns. The first is $-100 = -(f_o/f_e)$ and the other is $f_o + f_e = 1.00 \ m$. This results in $f_e$=9.9 mm and $f_o$=99 cm.