Rephrasing Faraday's Law

As physics educators, we must often find the balance between simplicity and accuracy. Particularly in introductory courses, it can be a struggle to give students the level of understanding for which they're ready without misrepresent reality – often this means giving them simple models, but parenthetically noting limitations. Of course, it's in these introductory courses that our students begin to construct the conceptual framework which they'll flesh out over a physics curriculum – so a misrepresentation at this early stage can seed difficulties and stubborn misconceptions that persist or even strengthen through subsequent courses, especially since many upper level texts focus more on *techniques* and would not directly challenge mistaken *concepts*. In the worst cases, misunderstandings remain with our students past graduation, and are even passed on to their own students.

One important case is the common interpretation of Faraday's Law as showing that a time varying magnetic field *produces* a curled electric field. This paper demonstrates that this is a widely presented interpretation, argues that it is impossible to deduce causality from Faraday's Law, and provides the actual cause of both the curled electric and time varying magnetic fields – a time varying current density. Being one of the fundamental laws of Electricity and Magnetism, its misinterpretation undermines the foundations for a student's understanding of the whole subject of Electricity and Magnetism. Because the subject is conceptually challenging, even mystifying for introductory students, it is particularly important that we avoid seeding and reinforcing this misunderstanding.

In calculus-based introductory and advanced texts, Faraday's Law tends to be presented in one of two forms

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}, \qquad \text{Equation 1}$$

or

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}. \qquad \text{Equation 2}$$

Here, $\Phi_B = \int \vec{B} \cdot d\vec{A}$ and is the magnetic flux, with *B* being the magnetic field and *A* the area it pierces; *E* is the electric field, and *s* is the path encircling *A* and over which *E* is evaluated. The two equations are equivalent through Stokes' Theorem. Unfortunately, there is a disturbing, and disturbingly common, mistaken translation of Equations 1 and 2 into English – specifically, that the time varying magnetic field causes the curled electric field. According to Halliday, Resnick, and Walker's introductory text, Equation 1 "says simply that a changing magnetic field induces an electric field." It goes on to say "Induced electric fields are produced not by static charges but by a changing magnetic flux."[1] Young and Freedman's text goes so far as to comment that "this may be a little jarring; we are accustomed to

[1] D. Halliday, R. Resnick, and J. Walker, Fundamentals of Physics, 6th ed. (Wiley, New York, 2001), p.722.

thinking about electric fields as being caused by electric charges, and now we are saying that a changing magnetic field somehow acts as a source of electric field."[2] Similar statements are to be found in many of the texts that share or have shared the introductory physics market over the years.[3] Commendably, a few texts say that the electric field is "associated with"[4] or "accompanied by"[5] the changing magnetic field; however, without actually indicating what does cause the electric field, these texts allow students to reach the same mistaken conclusion, that the time varying magnetic field causes the electric field. One might hope that this mistake would be corrected by intermediate or advanced texts, but instead it tends to be corroborated by misleading statements. Purcell's intermediate-level text replaces "produces" with "determines", but the subtle difference is likely lost on students.[6] The advanced undergraduate text by Griffith's avoids owning any interpretation itself, and instead attributes one to Faraday: "Faraday had an ingenious inspiration: A Changing magnetic field induces an electric field. It is this 'induced' electric field that […]"[7] Jackson's revered and feared graduate-level text does the same: "Faraday interpreted the transient current flow as being due to a changing magnetic flux linked by the circuit. The changing flux induces an electric field around the circuit, the line integral of which is called the *electromotive* force, $\mathcal{E}$. The electromotive force causes a current flow, according to Ohm's law."[8] While the statements in these three texts are not necessarily incorrect, and the ones that represent the interpretation as Faraday's may be historically accurate, they are unhelpful since they reinforce the misunderstanding that a student would have already developed when reading an introductory text.

Faraday's Law cannot be used to establish the oft claimed causal relationship between the electric and magnetic fields. To establish causality, it is necessary (but not sufficient) to establish a time lag between the cause and the effect. In the case of two events at different locations, the reason is obvious – it takes time for information to travel from one point to another. As Purcell points out, Faraday's Law is local – it relates the time variation of a magnetic field to the curl of an electric field *at the same point in space.* In such a case, the time lag may vanish, but ambiguity replaces it – it is impossible to establish that the changing magnetic field causes the curled electric field (or vice versa.) While *our knowing* that a time varying magnetic field exists causes *us to know* that a curled electric field exists (and vice versa), causing our knowledge of an event is quite different from causing the event. That is why Jefimenko's text

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[2] H. Young and R. Freedman, Sears and Zemansky's University Physics: with Modern Physics, 12th ed. (Pearson Addison-Wesley, San Francisco, 2007), p 1009.

[3] D. Giancoli, Physics for Scientists & Engineers with Modern Physics, 4th ed. (Pearson Prentice Hall, Upper Saddle River, New Jersey, 2008), p. 773; D. Halliday and R. Resnick, Fundamentals of Physics, (Wiley, New York, 1970), p. 584; R. Knight, Physics for Scientists and Engineers: a Strategic Approach, 2nd ed. (Pearson Addison-Wesley, San Fransico, 2008), p. 1059; H. Ohanian, Physics, 2nd ed. (Norton, New York, 1989) p. 792.;R. Serway and J. Jewett Jr., Physics for Scientists and Engineers, 6th ed. (Brooks/Cole – Thomas Learning, Belmont, CA, 2004), p. 981; R. Wolfson and J. Pasachoff, Physics for Scientists and Engineers, 3rd ed. (Addison-Wesley, San Fransico, 1999), p. 820.

[4] P. Tipler, G. Mosca, Physics for Scientists and Engineers, 5th ed. (Freeman, New York, 2004), p. 900.

[5] E. Hecht, Physics: Calculus, 2nd ed. (Brooks/Cole, Pacific Grave CA, 2000), p. 817.

[6] E. Purcell, Electricity and Magnetism Berkeley Physics Course, Volume 2, 2nd ed. (McGraw Hill, New York, 1985), p. 274.

[7] D. Griffiths, Introduction to Electrodynamics, 3rd ed. (Prentice Hall, Upper Saddle River, New Jersey, 1999), p. 302.

[8] J. D. Jackson, Classical Electrodynamics, 2nd ed. (Wiley, New York, 1975), p. 210.

pointedly says that Faraday's Law communicates "correlation;" still, it does not offer the underlying cause of the correlated phenomena.[9]

The cause of the curled electric field is the same as the cause of the time varying magnetic field, a time varying current density. That they share a common cause is why, in accordance with Equations 2 and 3, the two effects always accompany each other. While the proof would not be accessible to introductory students, fellow instructors may appreciate an outline of it. To determine the cause of the curled electric field, it is convenient to begin with what Griffith's text dubs "the *causal* solutions to Maxwell's equations" (his emphasis.) While these were first presented in Jefimenko's text,[10] because Griffith's text is the more common one, we will use his notation;[11]

$$\vec{E}(r,t) = \frac{1}{4\pi\varepsilon_o}\int\left[\frac{\rho(\vec{r}',t_r)}{r^2}\hat{r} + \frac{\dot{\rho}(\vec{r}',t_r)}{cr}\hat{r} \frac{\vec{J}(\vec{r}',t_r)}{c^2 r}\right]d\tau',$$

$$\vec{B}(r,t) = \frac{\mu_o}{4\pi}\int\left[\frac{\vec{J}(\vec{r}',t_r)}{r^2} + \frac{\dot{\vec{J}}(\vec{r}',t_r)}{cr}\right]\times\hat{r}d\tau' \qquad \text{Equation 4}^{12}$$

Here, the electric field (*E*) and magnetic field (*B*) are evaluated at location *r* and time *t*. They are found by integrating the charge density, ρ and its time deriviative, , as well as the current density, ,and its time derivative, , over the volume of all space, . The densities are evaluated at locations *r'*, and the retarded time, t_r. s, J is, r is,… As any cause must precede its effect, it is essential that the charge and current densities and their time derivatives are evaluated in these equations at the *retarded* times. When Jefimenko introduces them in his text, he says "These equations indicate that sources of a time-dependent electric field are electric charges together with conduction and convection currents, while those of a time-dependent magnetic field are only the conduction and convection currents but not the displacement currents. This means that although a displacement current is associated with a magnetic field, this does not constitute a cause and effect relationship." (p. 516-517) As Jefimenko used them to differentiate between an association and a cause (in the case of the displacement current and the magnetic field), we can, with only a little work, do the same for the curled electric field. The simplest way is to plug the expression for B in into Equation 1. This yields

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[9] O. Jefimenko, Electricity and Magnetism, 2nd ed. (Electret Scientific Company, Star City, West Virginia, 1966), p. 498.

[10] *Ibid*, pp. 515-516.

[11] D. Griffiths, pp. 427-428.

[12] The current densities here encompass both "free" currents and the "bound" currents such as in permanent magnets. Through Griffith's equation 6.13, the "bound" current density is related to the magnetization via $\vec{J}_b = \vec{\nabla}\times\vec{M}$.

$$\vec{\nabla}\times\vec{E}(r,t)=-\frac{\mu_o}{4\pi}\int\left[\frac{\dot{\vec{J}}(\vec{r}',t_r)}{r^2}+\frac{\ddot{\vec{J}}(\vec{r}',t_r)}{cr}\right]\times\hat{r}d\tau' \qquad \text{Equation 5}$$

The right hand side of this equation then conveys the *cause* of the curled electric field –a time varying current density.

Mindful of this solution, and the correlation (not causation) between the curled electric field and the time varying magnetic field, interpretations of Faraday's Law should be rephrased as "This equation says simply that a changing magnetic field is *accompanied by* a curled electric field. (These two share a common cause in time varying current densities.)" This rephrasing should significantly demystify electric and magnetic fields by tying them back to their actual sources, rather than teaching students that the fields have the capacity to create each other. It may not be quite as simple, but it is far more accurate than the interpretations that are commonly presented.