

Review:

There won't be an equation sheet – you should know the basic starting points. When it comes down to an integral, I'll give you what you would be able to find in an integral table, but you may need to do some approximating, integration by parts, etc. to get into that final, evaluable form.

For system of arbitrary dimensions, polarization possibilities and momentum-energy relation, range of state occupancies, be able to find:

- Single state partition function  $Z_{state} = \sum_{n=n_{min}}^{n_{max}} e^{-n(\mathbf{e}-\mathbf{m})\mathbf{b}}$
- Average occupancy  $\bar{n} = \sum_{n=n_{min}}^{n_{max}} n \frac{e^{-n(\mathbf{e}-\mathbf{m})\mathbf{b}}}{Z_{state}} = -\frac{1}{Z_{state}} \frac{\partial Z_{state}}{\partial((\mathbf{e}-\mathbf{m})\mathbf{b})}$
- Average energy  $\bar{\mathbf{e}} = \sum_{n=n_{min}}^{n_{max}} n \mathbf{e} \frac{e^{-n(\mathbf{e}-\mathbf{m})\mathbf{b}}}{Z_{state}} = \sum_{n=n_{min}}^{n_{max}} \mathbf{e} \bar{n}$
- Density of States  $g(\mathbf{e}) = \frac{ds_{w/e}}{d\mathbf{e}}$
- Debye / Fermi Energy & Temp (if it's not 0, as for photons)
  - $N = \int_0^{\epsilon_F} g(\mathbf{e}) d\mathbf{e}$
  - Looking back at the definition of the density of states, the integral is simply the number of states with energy from 0 to  $\epsilon_F$ . For Fermions, the argument for this is that, at  $T = 0$ , all states with energy less than  $\epsilon_F$  are occupied by one particle, and none above  $\epsilon_F$  are occupied, so the number of states = the number of particles =  $N$ .
  - $n_p \cdot N = \int_0^{\epsilon_D} g(\mathbf{e}) d\mathbf{e}$  where  $n_p$  is the number of polarizations.
  - The integral is then the number of states with energy from 0 to  $\epsilon_D$ , where  $\epsilon_D$  is the maximum energy possible for a phonon. That there are  $n_p N$  states follows from the constraint that the separation of atoms in the solid places on the minimum supportable phonon wavelength (thus maximum phonon energy,  $\epsilon_D$ ).
- Number of particles / chemical potential
  - $N = \int_0^{\mathbf{e}_{max}} g(\mathbf{e}) \bar{n}_s d\mathbf{e}$  For phonons  $\mathbf{e}_{max} = \mathbf{e}_D$ , otherwise,  $\mathbf{e}_{max} = \infty$ .
- If  $N$  is not fixed, as for photons and phonons, then there is no chemical potential. If  $N$  is fixed, as for a system of massive particles, then  $\mathbf{m}$  is non-zero, and this relation can be used to solve for  $\mathbf{m}$ .

- Energy  $U = \int_0^{e_{\max}} \epsilon g(\epsilon) \bar{n}_s d\epsilon$ , again  $e_{\max} = e_D$  for Phonons and  $e_{\max} = \infty$  otherwise.
- Specific Heat  $C_V = \left( \frac{\partial U}{\partial T} \right)_V$
- Entropy  $S(T) = \int_0^T \frac{C_V(T)}{T} dT$

Be able to evaluate these things in the Low temperature limit

- Phonons: one can neglect the constrained upper limit on energy since there won't be enough energy to get there, might as well set it equal to infinity. For everybody else, the upper limit is infinity anyway.
- If it's a system with a chemical potential, you can massage the math until you've an integral in which the integrand is negligible unless  $\frac{\epsilon - \mu}{kT} \ll 1$ , so you can set your integration limits to  $\pm \infty$  with impunity (since the integrand is 0 out there anyway), and you can expand  $\epsilon$  around  $\mu$ . Indeed, this is how you solve for  $\mu$  from the  $N = \dots$  integral.

Be able to evaluate these things in the High temperature limit.

- You should generally get the classical results – the equipartition theorem results.

Be able to make rough sketches of these things if they're simple, or by extrapolating between the two Temperature limits.