12	Wed.,11/19 Fri., 11/21	**6.2** Time-independent Perturbation, degenerate **6.3** Fine Structure of Hydrogen	**Daily 12.W** **Daily 12.F**
13	Mon., 11/24 Tues. 11/25	**6.4-.5** Zeeman & Hyper-fine	**Daily 13.M** **Weekly 13**

Daily: Kyle Jacob Spencer Gigja Anton Jessica Sean Antwain Jonathan Casey Jeremy Mark Connor Brad

Equipment

- Griffith's text

Check dailies

Announcements

Daily 12.W Wednesday 11/19 Griffiths **6.2** Time-independent Perturbations, degenerate

These next few days, we'll be looking at ways to get approximate solutions to problems that are too difficult to exactly solve.

6. Time-Independent Perturbation Theory

When the Hamiltonian is slightly perturbed, we posited that it would be useful to express the new energies and the new wavefunctions in terms of a power series expansion from those appropriate to the un-perturbed Hamiltonian. That is to say,

For

$\hat{H} = \hat{H}^o + \hat{H}'$

we'd have $\psi_n = \psi_n^o + \psi_n^{(1)} + \psi_n^{(2)} + \psi_n^{(3)} + \ldots$ with $E_n = E_n^o + E_n^{(1)} + E_n^{(2)} + E_n^{(3)} + \ldots$

From there, we followed a logical path to arrive at the expression for the first-order correction to the energy,

$$\left\langle \psi_n^o \middle| \hat{H}' \psi_n^o \right\rangle = E_n^{(1)},$$

The first order correction to the wavefunction,

$$\psi_n^{(1)} = \sum_{m \neq n} \frac{\left\langle \psi_m^o \middle| \hat{H}' \psi_n^o \right\rangle}{E_n^o - E_m^o} \psi_m^{(o)}$$

And the second-order correction to the energy,

$$\sum_{m \neq n} \frac{\left| \left\langle \psi_m^o \middle| \hat{H}' \psi_n^o \right\rangle \right|^2}{E_n^o - E_m^o} = E_n^{(2)}$$

Both of the latter involve identifying

$$\psi_n^{(1)} = \sum_{m \neq n} c_{n,m} \psi_m^{(o)} \text{ where } c_{n,m} = \left\langle \psi_m^{(o)} \middle| \psi_n^{(1)} \right\rangle = \frac{\left\langle \psi_m^o \middle| \hat{H}' \psi_n^o \right\rangle}{E_n^o - E_m^o}$$

Clearly, this factor has trouble if one of the states you're using to build this correction term, one of the ψ_m^o's, happens to have the same uncorrected energy, E_m^o, as the uncorrected state you're looking to tweak, ψ_n^o with E_n^o. That is, if $E_n^o - E_m^o = 0$.

To understand how things go wrong, we can look one step back in the derivation. That comes from the 1st-order correction equation which leads to

$$\left(E_n^o - E_m^o\right)\left\langle \psi_m^o \middle| \psi_n^{(1)} \right\rangle = \left\langle \psi_m^o \middle| \hat{H}' \psi_n^o \right\rangle$$

So, if the energies happen to be the same, then we're multiplying $\left\langle \psi_m^o \middle| \psi_n^{(1)} \right\rangle$ by zero, so we have no way of recovering the value of $c_{n,m} = \left\langle \psi_m^o \middle| \psi_n^{(1)} \right\rangle$.

How *do* we find its value?

6.2 Degenerate Perturbation Theory

That's what Degenerate Perturbation Theory addresses. Or rather, it gives us a way to sidestep the question – create linear combinations of the degenerate states, combinations which have distinct energies, and use *those* in your description of the unperturbed system. Surprisingly, though the issue is apparent in the 1st-order wavefunction and 2nd-order energy corrections, this change even effects the 1st-order energy correction.

Qualitatively, that can be understood because the different linear combinations of the degenerate wavefunctions would 'ovelap' with the perturbation differently, and so one combo might *gain* energy due to the perturbation while another might *lose* energy due to it. For example, consider how sine and cosine are two different linear combinations of e^{+ikx} and e^{-ikx}, but one would be severely affected by a delta spike at the origin and the other wouldn't.

6.2.1 Two-Fold Degeneracy

Suppose we have just two (orthogonal) states that have the same energy under the influence of the original Hamiltonian, both ψ_{na}^o and ψ_{nb}^o have energy E_n^o when subject to $\hat{H}^o$. Of course, that means that we can build linear combinations of them which necessarily have the same energy,

$$\psi_n^o = \alpha \psi_{na}^o + \beta \psi_{nb}^o$$

Can we go over what griffiths means by lifting in the paragraph below 6.18, I don't really follow what he is talking about there?" Jessica

We'll be interested in two combinations that are orthogonal to each other, so they're still good members of the ortho-normal basis set, and that have different first order energy corrections, so when the perturbation is applied, the degeneracy "lifts" in the same sense that a

mist lifts or a veil is lifted –the degeneracy is removed and so you can see distinctly (in the energy spectrum) that there actually are distinct states., i.e. two states that had the same energy with the original Hamiltonian now have distinct energies, and so we can, for example, distinguish their presence in the system's spectrum. These will be our "good" degenerate wavefunctions.

An example is having Helium in the ground state, with the two electrons with the same energy- so degenerate spin states. Apply a magnetic field, and now one has spin that's aligned and one has spin that's anti-aligned, and there's an energy difference.

1. *Conceptual*: If the states in the unperturbed case are not the good states, what are their energies in the perturbed system? How do they relate to the energies of the "good" states?

They must then be constructable from a linear combination of the good states, and so their energies must be a linear combination of the good states' energies – thus energies somewhere between / less extreme than those extreme ones.

Now, let's start down the whole perturbation path with this.
The argument about creating a polynomial in λ that we'd started with will still hold, leading to the same expressions for 0th, 1st, and 2nd-order corrections:

$$\hat{H}^o\psi_n^o = E_n^o\psi_n^o$$
$$\hat{H}^o\psi_n^{(1)} + \hat{H}'\psi_n^o = E_n^o\psi_n^{(1)} + E_n^{(1)}\psi_n^o$$
$$\hat{H}^o\psi_n^{(2)} + \hat{H}'\psi_n^{(1)} = E_n^o\psi_n^{(2)} + E_n^{(1)}\psi_n^{(1)} + E_n^{(2)}\psi_n^o$$
...

Of course, the 0th-order relation tells us nothing we didn't already know.

1st-order energy correction

To get useful information out of the 1st-order relation we'd previously taken its inner product with the 0th-order wavefunction, but now we've got a choice – how about just taking the product ψ_{na}^o or ψ_{nb}^o?

That gets us

$$\left\langle \psi_{na}^o \middle| \hat{H}^o \middle| \psi_n^{(1)} \right\rangle + \left\langle \psi_{na}^o \middle| \hat{H}' \middle| \psi_n^o \right\rangle = E_n^o \left\langle \psi_{na}^o \middle\| \psi_n^{(1)} \right\rangle + E_n^{(1)} \left\langle \psi_{na}^o \middle\| \psi_n^o \right\rangle$$

Where, as before, we use the hermetian-ness of the Hamiltonian to rewrite the first term,

$$\left\langle \hat{H}^o\psi_{na}^o \middle\| \psi_n^{(1)} \right\rangle + \left\langle \psi_{na}^o \middle| \hat{H}' \middle| \psi_n^o \right\rangle = E_n^o \left\langle \psi_{na}^o \middle\| \psi_n^{(1)} \right\rangle + E_n^{(1)} \left\langle \psi_{na}^o \middle\| \psi_n^o \right\rangle$$

Which allows us to replace it with the eigen energy,

$$E_n^o \left\langle \psi_{na}^o \middle\| \psi_n^{(1)} \right\rangle + \left\langle \psi_{na}^o \middle| \hat{H}' \middle| \psi_n^o \right\rangle = E_n^o \left\langle \psi_{na}^o \middle\| \psi_n^{(1)} \right\rangle + E_n^{(1)} \left\langle \psi_{na}^o \middle\| \psi_n^o \right\rangle$$

and so, cancel the first term on each side, leaving

$$\left\langle \psi_{na}^o \middle| \hat{H}' \middle| \psi_n^o \right\rangle = E_n^{(1)} \left\langle \psi_{na}^o \middle\| \psi_n^o \right\rangle$$

So far, just the same as before, but here's the new part, substituting in $\psi_n^o = \alpha\psi_{na}^o + \beta\psi_{nb}^o$,

$$\left\langle \psi_{na}^o \middle| \hat{H}' \middle| \alpha\psi_{na}^o + \beta\psi_{nb}^o \right\rangle = E_n^{(1)} \left\langle \psi_{na}^o \middle\| \alpha\psi_{na}^o + \beta\psi_{nb}^o \right\rangle = \alpha E_n^{(1)} \left\langle \psi_{na}^o \middle\| \psi_{na}^o \right\rangle + \beta E_n^{(1)} \left\langle \psi_{na}^o \middle\| \psi_{nb}^o \right\rangle$$
$$\alpha \left\langle \psi_{na}^o \middle| \hat{H}' \middle| \psi_{na}^o \right\rangle + \beta \left\langle \psi_{na}^o \middle| \hat{H}' \middle| \psi_{nb}^o \right\rangle = \alpha E_n^{(1)}$$

Where the last step on the right hand side used the ortho-normality of the unperturbed basis set (these two wavefunctions may have the same energy, but they're still orthogonal to each other!)

> **Note:** if β is 0, i.e., the wavefunction wasn't really degenerate at all, then we just recover our result from section 2.1.

Obviously, had we taken the inner product with ψ_{nb}^o instead, we would have gotten

$$\alpha \left\langle \psi_{nb}^o \middle| \hat{H}' \middle| \psi_{na}^o \right\rangle + \beta \left\langle \psi_{nb}^o \middle| \hat{H}' \middle| \psi_{nb}^o \right\rangle = \beta E_n^{(1)}$$

Where $\left\langle \psi_{nb}^o \middle| \hat{H}' \middle| \psi_{na}^o \right\rangle = \left\langle \psi_{na}^o \middle| \hat{H}' \middle| \psi_{nb}^o \right\rangle^*$ as usual.

Naming these inner products $W_{ba} \equiv \left\langle \psi_{nb}^o \middle| \hat{H}' \middle| \psi_{na}^o \right\rangle$, $W_{aa} \equiv \left\langle \psi_{na}^o \middle| \hat{H}' \middle| \psi_{na}^o \right\rangle$, $W_{bb} \equiv \left\langle \psi_{nb}^o \middle| \hat{H}' \middle| \psi_{nb}^o \right\rangle$

The two algebraic relations can be phrased efficiently as a single matrix relation,

$$\begin{aligned} \alpha W_{aa} + \beta W_{ab} &= \alpha E_n^{(1)} \\ \alpha W_{ba} + \beta W_{bb} &= \beta E_n^{(1)} \end{aligned} \quad \text{or} \quad \begin{pmatrix} W_{aa} & W_{ab} \\ W_{ba} & W_{bb} \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = E_n^{(1)} \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

(here, I'm jumping to the more general phrasing of section 6.2.2)

Then the 'characteristic equation' generated by

$$\det \left| \begin{pmatrix} W_{aa} & W_{ab} \\ W_{ba} & W_{bb} \end{pmatrix} - E_n^{(1)} \mathbf{I} \right| = 0$$

or

$$\det \left| \begin{pmatrix} W_{aa} - E_n^{(1)} & W_{ab} \\ W_{ba} & W_{bb} - E_n^{(1)} \end{pmatrix} \right| = 0$$

Is

$$\left(W_{aa} - E_n^{(1)}\right)\left(W_{bb} - E_n^{(1)}\right) - W_{ab}W_{ba} = 0$$

Or, using that $W_{ba} = W_{ab}^*$ and multiplying out the terms,

$$\left(E_n^{(1)}\right)^2 - \left(W_{bb} + W_{aa}\right)E_n^{(1)} + W_{bb}W_{aa} - \left|W_{ab}\right|^2 = 0$$

Which is solved by the quadratic equation

$$E_n^{(1)} = \frac{1}{2}\left[W_{bb} + W_{aa} \pm \sqrt{\left(W_{bb} + W_{aa}\right)^2 - 4\left(W_{bb}W_{aa} - \left|W_{ab}\right|^2\right)} \right]$$

$$E_n^{(1)} = \frac{1}{2}\left[W_{bb} + W_{aa} \pm \sqrt{\left(W_{aa}\right)^2 + \left(W_{bb}\right)^2 + 2W_{bb}W_{aa} - 4W_{bb}W_{aa} + 4\left|W_{ab}\right|^2} \right]$$

$$E_n^{(1)} = \frac{1}{2}\left[W_{bb} + W_{aa} \pm \sqrt{\left(W_{aa}\right)^2 + \left(W_{bb}\right)^2 - 2W_{bb}W_{aa} + 4\left|W_{ab}\right|^2} \right]$$

$$E_{n\pm}^{(1)} = \frac{1}{2}\left[W_{bb} + W_{aa} \pm \sqrt{\left(W_{aa} - W_{bb}\right)^2 + 4\left|W_{ab}\right|^2} \right]$$

1. *Starting Weekly HW*: Griffiths problem 6.9

Exercise: returning to this, no let's consider the

$$\hat{H} = \hat{H}^o + \hat{H}' = \begin{pmatrix} V_o & 0 & 0 \\ 0 & -2V_o & 0 \\ 0 & 0 & V_o \end{pmatrix} + \begin{pmatrix} -\varepsilon & \varepsilon & 0 \\ \varepsilon & -\varepsilon & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

What are the energies of the 'good' eigenvectors for the two degenerate ones?

$$\left|\psi_1^o\right\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad \left|\psi_2^o\right\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \text{ and } \quad \left|\psi_3^o\right\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Find W_{11}, W_{33}, and W_{13}

$$W_{11} = \left\langle \psi_1^{(o)} \middle| \hat{H}' \middle| \psi_1^{(o)} \right\rangle = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} -\varepsilon & \varepsilon & 0 \\ \varepsilon & -\varepsilon & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = -\varepsilon$$

Etc.

Put together to get the two energies

$$E_{n\pm}^{(1)} = \frac{1}{2}\left[W_{bb} + W_{aa} \pm \sqrt{(W_{aa} - W_{bb})^2 + 4|W_{ab}|^2} \right]$$

You get that $W_{13} = W_{33} = 0$ – the perturbation doesn't affect the third state, but it does affect the first, so the degeneracy already naturally separates out. This is the case Griffiths mentions with one of the coefficients being 0, so you already have the good states.

Now try

$$\hat{H} = \hat{H}^o + \hat{H}' = \begin{pmatrix} V_o & 0 & 0 \\ 0 & -2V_o & 0 \\ 0 & 0 & V_o \end{pmatrix} + \begin{pmatrix} -\varepsilon & 0 & \varepsilon \\ 0 & -\varepsilon & 0 \\ \varepsilon & 0 & 0 \end{pmatrix}$$

Now you'll get that there *is* a W_{13}.

2. *Starting Weekly HW*: Griffiths problem 6.7 parts b and c only. Hint: for part c, use equation 6.22.

So, apparently there are two possible 1st-order corrections to the energy corresponding to two possible ways of combining ψ_{na}^o and ψ_{nb}^o.

$$\psi^o_{n\pm} = \alpha_\pm \psi^o_{na} + \beta_\pm \psi^o_{nb}$$

1. *Conceptual*: In a two-fold degenerate system, generally the perturbation will break the degeneracy. One state will go to a higher energy and an orthogonal state will go to a lower energy. But, these states may not be the same states used in the unperturbed case. Note that there are an infinite number of wavefunctions with the same energy: $\psi = \alpha\psi_a + \beta\psi_b$, where a and b are the 2 wavefunctions in the unperturbed case. If these happed to be the states with the most extreme new energies, what are α and β?

In fact, returning to the two equations,

$\alpha W_{aa} + \beta W_{ab} = \alpha E^{(1)}_n$ rearranges to $\alpha_\pm \dfrac{E^{(1)}_{n_\pm} - W_{aa}}{W_{ab}} = \beta_\pm = \alpha_\pm \dfrac{\left[W_{aa} - W_{bb} \mp \sqrt{(W_{aa} - W_{bb})^2 + 4|W_{ab}|^2}\right]}{2W_{ab}}$

Exercise Okay, return to the example we were just working on and find the two "good" eigen vectors.

Notice that these two combinations are orthogonal. That's something Griffiths told us would be the case at the outset, but didn't actually prove (left to Problem 6.6 a) or use.

$$\left\langle \psi^o_{n\pm} \middle| \psi^o_{n\mp} \right\rangle = \alpha_\pm \alpha_\mp + \beta_\pm \beta_\mp$$

$$= \alpha_\pm \alpha_\mp \left(1 + \frac{\left[W_{aa} - W_{bb} + \sqrt{(W_{aa} - W_{bb})^2 + 4|W_{ab}|^2}\right]}{2W_{ab}} \frac{\left[W_{aa} - W_{bb} - \sqrt{(W_{aa} - W_{bb})^2 + 4|W_{ab}|^2}\right]}{2W_{ab}} \right)$$

$$= \alpha_\pm \alpha_\mp \left(1 + \frac{\left[(W_{aa} - W_{bb})^2 - (W_{aa} - W_{bb})^2 - 4|W_{ab}|^2\right]}{4(W_{ab})^2} \right) = 0$$

So, as Griffiths had predicted at the beginning of the section, the two linear combinations of degenerate states are orthogonal to each other.

However, notice that this doesn't really solve the whole 1st-order wavefunction correction term's problem!

Or does it?

1st-order Wavefunction correction

The problem that we'd identified was that we couldn't use

$$\left(E^o_n - E^o_m\right)\left\langle \psi^o_m \middle| \psi^{(1)}_n \right\rangle = \left\langle \psi^o_m \middle| \hat{H}' \psi^o_n \right\rangle \quad \text{and} \quad c_{n,m} = \left\langle \psi^o_m \middle| \psi^{(1)}_n \right\rangle$$

To determine the $c_{n,m}$ term in

$$\psi^{(1)}_n = \sum_{m \neq n} c_{n,m} \psi^{(o)}_m$$

Contributed by the state that was degenerate with the one we were interested in since the energies being equal killed off the equation. However, if we have reason to believe that $c_{n,m} = \langle \psi_m^o | \psi_n^{(1)} \rangle$ actually *is* zero for this term, then we don't have to worry. And it looks awfully likely that that's the case for these special linear combinations. At least,

$$\langle \psi_{n+}^o | \hat{H}' \psi_{n-}^o \rangle = \langle \alpha_+ \psi_a^o + \beta_+ \psi_b^o | \hat{H}' | \alpha_- \psi_a^o + \beta_- \psi_b^o \rangle =$$
$$\alpha_+^* \alpha_- \langle \psi_a^o | \hat{H}' | \psi_a^o \rangle + \beta_+^* \beta_- \langle \psi_b^o | \hat{H}' | \psi_b^o \rangle + \alpha_+^* \beta_- \langle \psi_a^o | \hat{H}' | \psi_b^o \rangle + \beta_+^* \alpha_- \langle \psi_b^o | \hat{H}' | \psi_a^o \rangle$$
$$\alpha_+^* \alpha_- W_{aa} + \beta_+^* \beta_- W_{bb} + \alpha_+^* \beta_- W_{ab} + \beta_+^* \alpha_- W_{ba}$$

But we've got an expression for $\beta_\pm$ in terms of $\alpha_\pm$, and substituting that in, and doing a little algebra (problem 6.6 b if you're interested), you'll find that

$$\langle \psi_{n+}^o | \hat{H}' \psi_{n-}^o \rangle = 0$$

This suggests that, rather than blowing up, $c_{n_+,n_-} = \langle \psi_{n+}^o | \psi_{n-}^{(1)} \rangle = \frac{\langle \psi_{n+}^o | \hat{H}' \psi_{n-}^o \rangle}{(E_{n+}^o - E_{n-}^o)} = 0$, or at least is finite, and so can be skipped over with only minimal harm to our approximation.

So, when you're after the 1st-order correction to one of the degenerate states, first rephrase in terms of these "good" linear combinations that are still orthogonal and still have the same energy, but smoothly evolve into states with extreme, non-degnerate energies, then you can sum as before:

$$\psi_{n\pm}^{(1)} = \sum_{m \neq n\pm \text{ or } n\mp} \frac{\langle \psi_m^o | \hat{H}' \psi_{n\pm}^o \rangle}{E_{n\pm}^o - E_m^o} \psi_m^{(o)}$$

Can we go over the theorem about hermitian operators commuting?" Casey P.

Griffith's Theorem

Griffiths asserts that (and I'll use my notation here) if ψ_a^o and ψ_b^o (degenerate eigen functions of $\hat{H}^o$) are also eigenfunctions of Hermitian operator $\hat{\mathbf{A}}$ with distinct eigenvalue $\hat{\mathbf{A}}\psi_a = a\psi_a$ and $\hat{\mathbf{A}}\psi_b = b\psi_b$, and $[\hat{\mathbf{A}}, \hat{H}'] = 0$, (Griffiths says it commutes with $\hat{H} = \hat{H}^o + \hat{H}'$, but doesn't use that fact), then ψ_a^o and ψ_b^o are the 'good' eigenfunctions, i.e. W_{nb}.

Proof:

$$[\hat{\mathbf{A}}, \hat{H}'] = 0$$
$$\langle \psi_a^o | [\hat{\mathbf{A}}, \hat{H}'] \psi_b^o \rangle = \langle \psi_a^o | \hat{\mathbf{A}} \hat{H}' | \psi_b^o \rangle - \langle \psi_a^o | \hat{H}' \hat{\mathbf{A}} | \psi_b^o \rangle = \langle \hat{\mathbf{A}} \psi_a^o | \hat{H}' | \psi_b^o \rangle - \langle \psi_a^o | \hat{H}' | \hat{\mathbf{A}} \psi_b^o \rangle = a \langle \psi_a^o | \hat{H}' | \psi_b^o \rangle - b \langle \psi_a^o | \hat{H}' | \psi_b^o \rangle$$
$$(a - b) W_{ab} = 0$$

But that 0 can't be because a*=b; by supposition, they *don't* equal each other. So we're left with the conclusion that

$$W_{ab} = 0$$

But what good is this theorem? It gives you a quick-ish way to check a reasonable guess at the solution, without having to do as much hard work of deriving it.

Also can we do an example where we use this theorem? Jessica

Here's an example, In problem 6.7, you'll be dealing with waves on a ring. The wavefunctions are

$\psi_n(x) = \frac{1}{\sqrt{L}} e^{2\pi i n x/L}$ for n = 0, ±1,2,…

Clearly there's an energy degeneracy for n = ±|n|.

When we perturb this system with a dimple at x=0, $H'(x) = -V_o e^{-x^2/a^2}$, and we want to know how the energies change. First step, what are the two linear combinations that make the 'good', states?

They must have the form $\psi_{n\pm}(x) = \frac{1}{\sqrt{L}}\left(\alpha_\pm e^{i2\pi|n|x/L} + \beta_\pm e^{-i2\pi|n|x/L}\right)$

And they must be orthogonal to each other.

Here's my guess: I know two linear combinations of such terms which are orthogonal to each other: sine and cosine. I guess

$$\psi_{n-}(x) = \frac{\alpha}{\sqrt{L}}\sin(2\pi|n|x/L) \text{ and } \psi_{n+}(x) = \frac{\alpha}{\sqrt{L}}\cos(2\pi|n|x/L)$$

Am I right? Well, can I dream up an operation for which these are eigenfunctions and which commutes with H'? If so, I'm home free.

In this case, the operation is the parity operator: $\hat{P}f(x) = f(-x) = pf(x)$

since $\hat{P}\psi_{n-}(x) = \frac{\alpha}{\sqrt{L}}\sin(-2\pi|n|x/L) = -\frac{\alpha}{\sqrt{L}}\sin(-2\pi|n|x/L) = -\psi_{n-}(x)$

while $\hat{P}\psi_{n+}(x) = \frac{\alpha}{\sqrt{L}}\cos(-2\pi|n|x/L) = \frac{\alpha}{\sqrt{L}}\cos(2\pi|n|x/L) = \psi_{n+}(x)$

so the distinct eigenvalues are +1 and -1
and this has no effect on H', since it depends on x^2, so they commute.
Done.

6.2.2 Higher Order Degeneracy

So, the way we set up the problem for doubly-degenerate clearly generalizes; if we have more than two states with the same energies under the original Hamiltonian, we can solve for the multiple 'good' eigenvectors and their first-order energy corrections by solving the matrix problem

$$\begin{pmatrix} W_{aa} & W_{ab} & W_{ac} & \dots \\ W_{ba} & W_{bb} & W_{bc} & \dots \\ W_{ca} & W_{cb} & W_{cc} & \dots \\ \dots & \dots & \dots & \dots \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \\ \chi \\ \dots \end{pmatrix} = E_n^{(1)} \begin{pmatrix} \alpha \\ \beta \\ \chi \\ \dots \end{pmatrix}$$ where this is a Hermitian matrix since $W_{ab} = W_{ba}^{*}$

'Could we go over Example 6.2 or a similar example?"

Spencer Post a response

Admin

Example 6.2 is a nice, simple-geometric one: particle in a 3-D cubic box, with added potential in the back-left quarter of the box. Before adding that perturbation, the 1st excited state was 3-ways degenerate: you could have extra bumps along x, along y, or along z.
We can predict what would happen in this case: put the extra potential in one quadrant, and a combination that has a lot of concentration there will increase in energy; a combination that has little concentration there will decrease in energy.

Let's walk through his work.