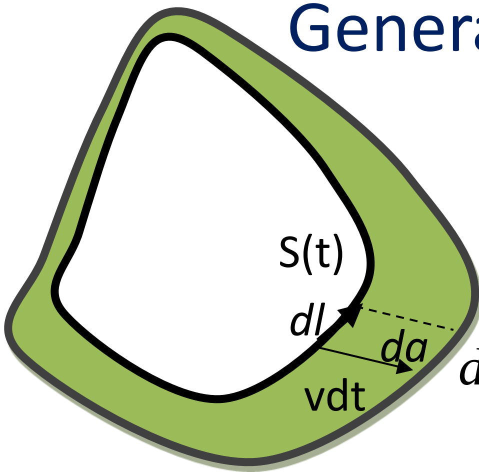
Wed. Fri.,	7.1.3-7.2.2 Emf & Induction 7.2.3-7.2.5 Inductance and Energy of B	
Mon., Tues. Wed.	7.3.1-.3.3 Maxwell's Equations 10.1 - .2.1 Potential Formulation Lunch with UCR Engr – 12:20 – 1:00	HW10

Generalization of Flux Rule



$$d\vec{a} = \vec{v}dt \times d\vec{l}$$

Using vector identity (1)

$$\vec{A}\cdot\left(\vec{B}\times\vec{C}\right)=\left(\vec{A}\times\vec{B}\right)\cdot\vec{C}$$

$$\vec{B}\cdot d\vec{a} = \vec{B}\cdot\left(\vec{v}dt \times d\vec{l}\right) = \left(\vec{B}\times\vec{v}dt\right)\cdot d\vec{l} = -\left(\vec{v}dt \times \vec{B}\right)\cdot d\vec{l}$$

Thus change in magnetic flux through the loop

$$d\Phi_B = -\oint\left(\vec{v}dt \times \vec{B}\right)\cdot d\vec{l}$$

rate of change in magnetic flux through the loop

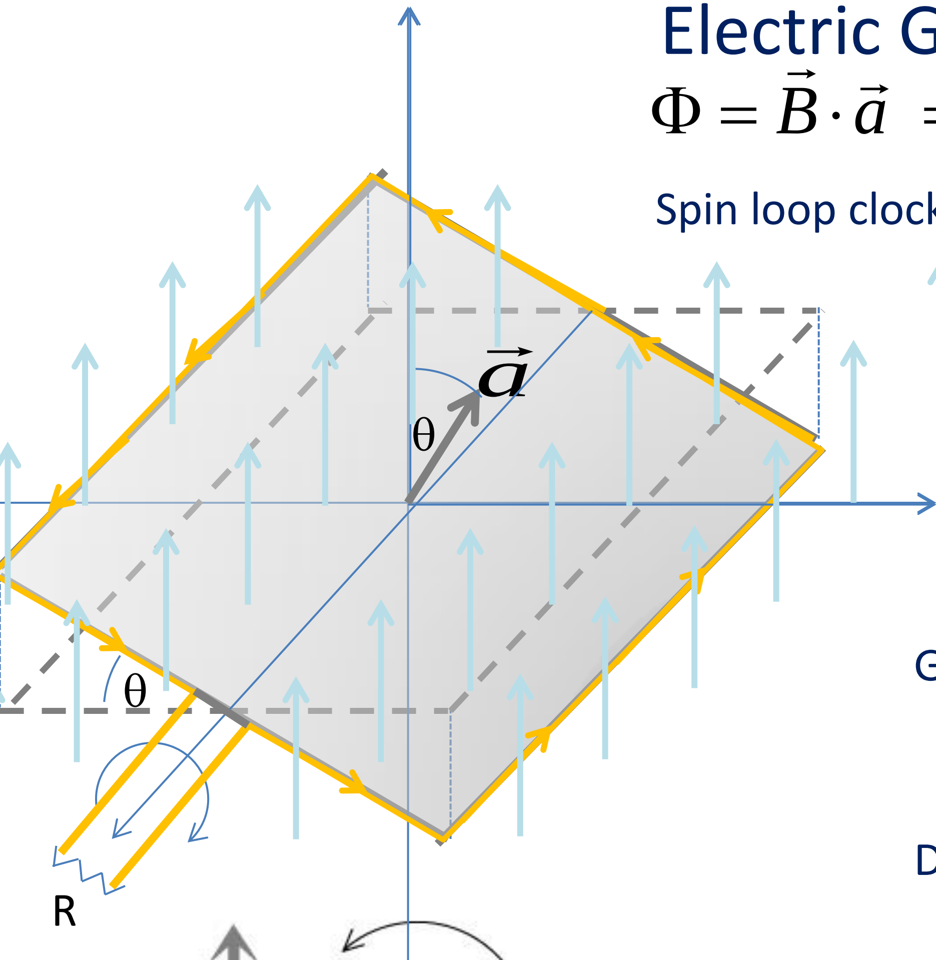
$$\left.\frac{\partial\Phi_B}{\partial t}\right|_B = -\oint\left(\vec{v}\times\vec{B}\right)\cdot d\vec{l} = -\oint\frac{\vec{F}_{mag}}{q}\cdot d\vec{l}$$

$$\left.\frac{\partial\Phi_B}{\partial t}\right|_B = -Emf_{mag}$$

Warning: our derivation used that the changing, *da/dt*, corresponded to moving charge, *vdl*. Not applicable when that's not the case.
– *thar be "paradoxes"*

(We will later extend this reasoning to discuss stationary charges but changing fields)

Electric Generator



$$\Phi = \vec{B}\cdot\vec{a} \;= Ba\cos\theta$$

Spin loop clockwise at ω (perhaps steam turbines make it spin)

$$\frac{d\Phi}{dt} = Ba\frac{d}{dt}(\cos\theta) = -(Ba\sin(\omega t))\omega$$

$$\mathcal{E}mf = -\frac{d\Phi}{dt} = (Ba\sin(\omega t))\omega$$

Generates voltage between terminals

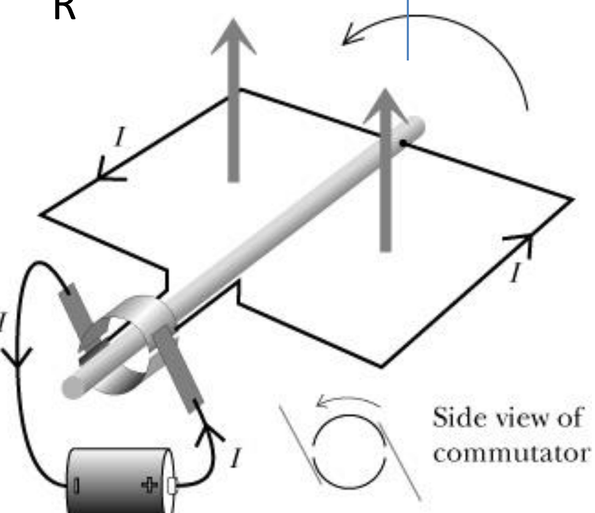
$$\Delta V = \mathcal{E}mf = (Ba\sin(\omega t))\omega$$

Drives a current *I* through a resistive load

$$-IR = \Delta V = (Ba\sin(\omega t))\omega$$

Demo! - “crank generator”

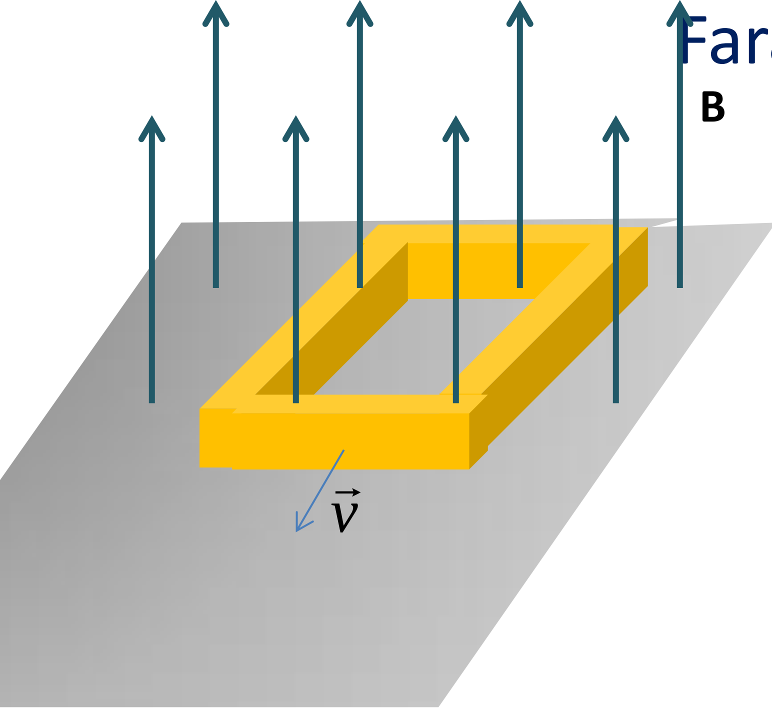
Electric Motor



Same process run in reverse

Demo! - “home-made motor”

Faraday's Law

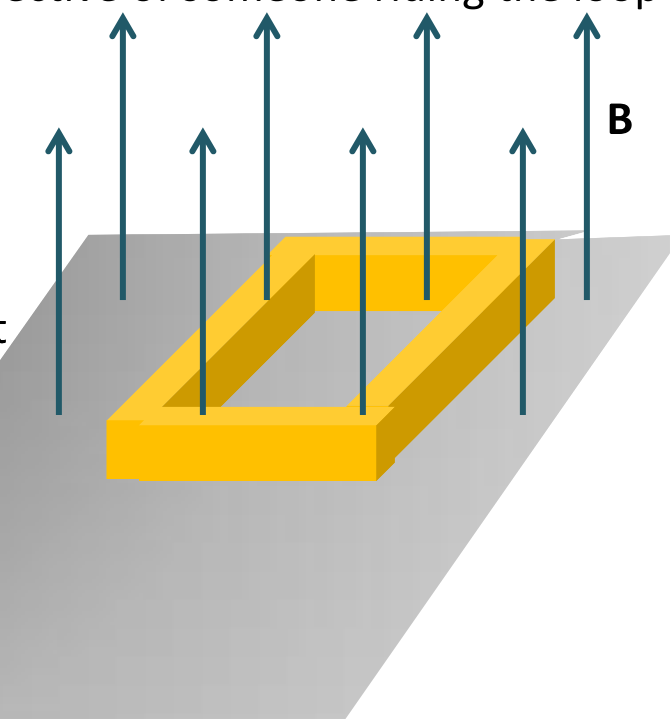


$$\frac{\int \vec{F}_{mag} \cdot d\vec{l}}{q} \equiv \mathcal{E}mf \quad = -\left.\frac{\partial \Phi}{\partial t}\right|_B = -B\frac{da}{dt} = -BvL$$

Which drives charges around the loop, via magnetic force

From the perspective of someone riding the loop

$$\left.\frac{\partial \Phi}{\partial t}\right|_a = \frac{\partial B}{\partial t} a$$

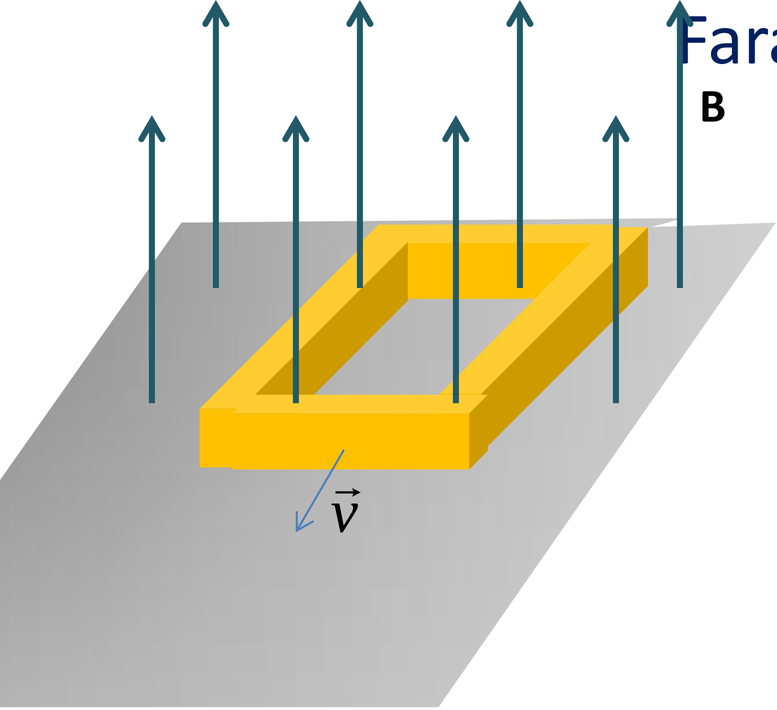


From this perspective too we must see charges move around the loop, there must be a force

But "magnetic" is *defined* as charge force proportional to charge's velocity; from this perspective, there is no *v*, so we can't call it "magnetic", have to call it "electric."

$$\frac{\int \vec{F}_{elect} \cdot d\vec{l}}{q} = \mathcal{E}mf = -\frac{\partial B}{\partial t} a$$

Faraday's Law



$$\frac{\int \vec{F}_{mag} \cdot d\vec{l}}{q} \equiv \mathcal{E}mf \quad = -\left.\frac{\partial \Phi}{\partial t}\right|_B = -B\frac{da}{dt} = -BvL$$

Which drives charges around the loop, via magnetic force

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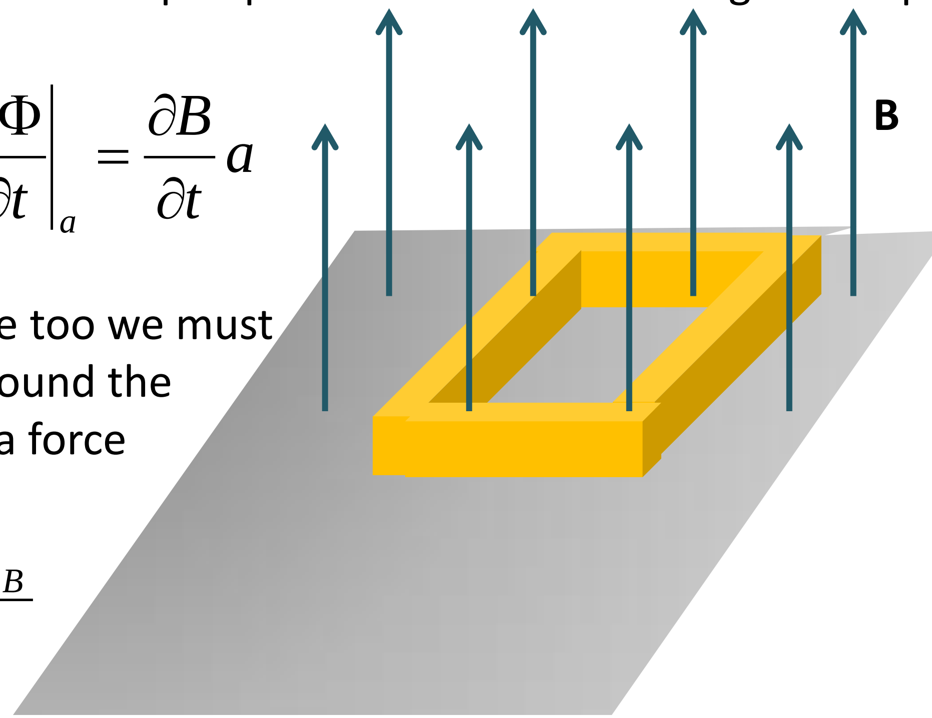
$$\left.\frac{\partial \Phi}{\partial t}\right|_a = \frac{\partial B}{\partial t} a$$



From this perspective too we must see charges move around the loop, there must be a force

In most general case

$$\mathcal{E}mf = -\left(\frac{\partial B}{\partial t} a + B\frac{\partial a}{\partial t}\right) = -\frac{dBa}{dt} = -\frac{d\Phi_B}{dt}$$

Full time derivative

Faraday's Law

$$\frac{\int \vec{F}_{elect} \cdot d\vec{l}}{q} = -\frac{\partial B}{\partial t} a$$

$$\frac{\int q\vec{E} \cdot d\vec{l}}{q} = -\oint \frac{\partial \vec{B}}{\partial t} \cdot d\vec{a}$$

$$\int \vec{E} \cdot d\vec{l} = -\oint \frac{\partial \vec{B}}{\partial t} \cdot d\vec{a}$$

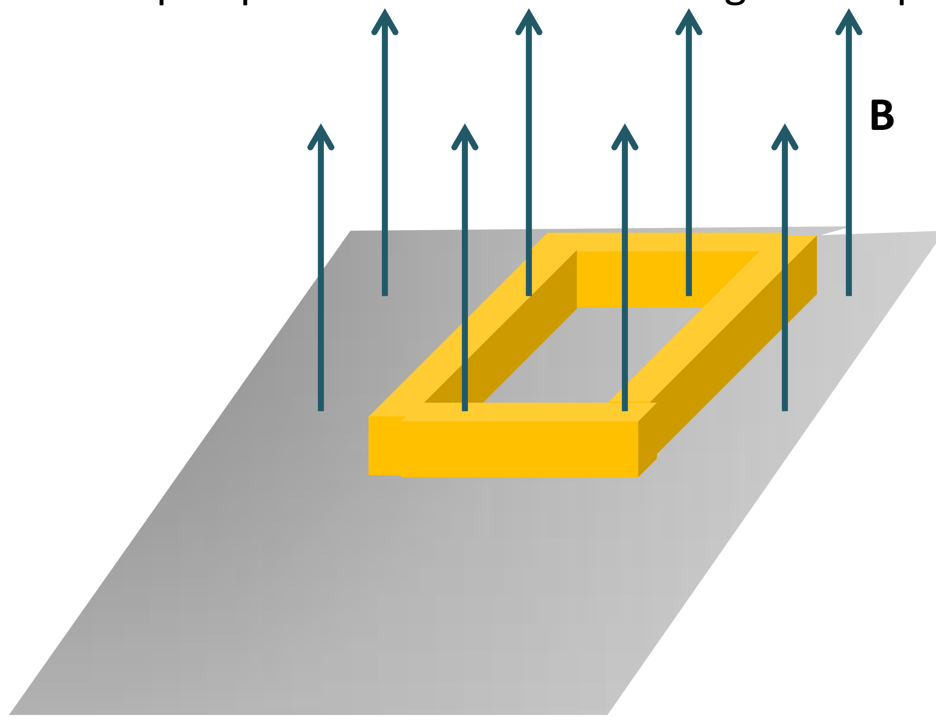
$$\oint \left(\vec{\nabla} \times \vec{E}\right) \cdot d\vec{a} = -\oint \frac{\partial \vec{B}}{\partial t} \cdot d\vec{a}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

circulating electric field is accompanied by time varying magnetic field

Both are produced by time varying current and charge distributions

From the perspective of someone riding the loop



Oh, Induction, let me count the ways…

induced *emf* in the coil 2 on the right

Change the current in coil 1

Move coil 1 (with current through it)

Come up with some more

Induction of the falling magnet

Why does the magnet fall so slowly?

Induction of the falling magnet

Why does the magnet fall so slowly?

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

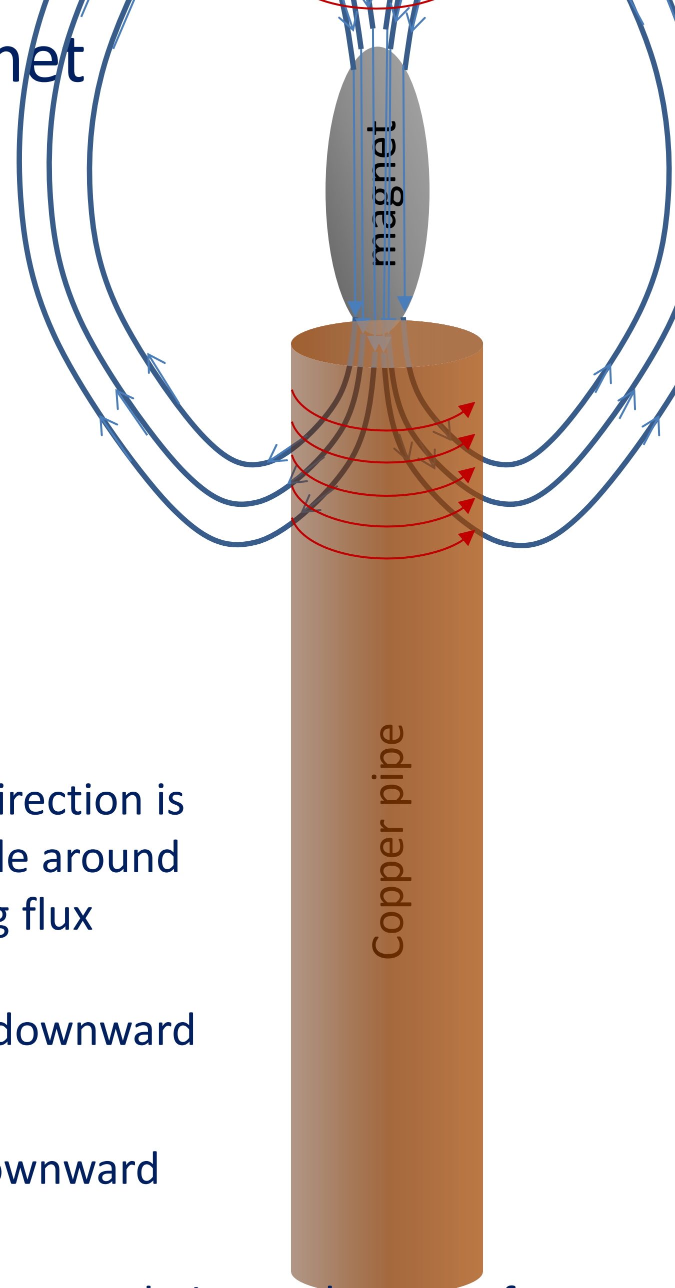
or

$$\mathcal{E}mf = -\frac{d\Phi_B}{dt}$$

Means Emf's direction is by *left* hand rule around area containing flux

To the right as downward flux increases

To the left as downward flux decreases



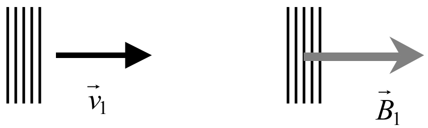
Thus drives charges around pipe and so transfers energy. *These* charges in motion produce field which exerts force on moving charges in magnets.

23_Farady_Magnet.py

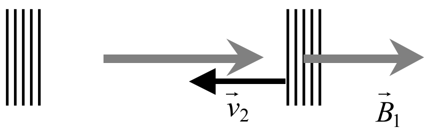
induced *emf* in the coil 2 on the right

Lenz's Law

Change the current in coil 1

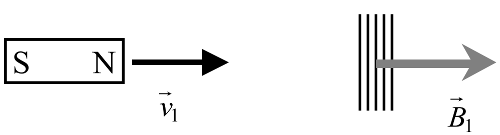


Move coil 1 (with current through it)



Move coil 2 (with current through coil 1)

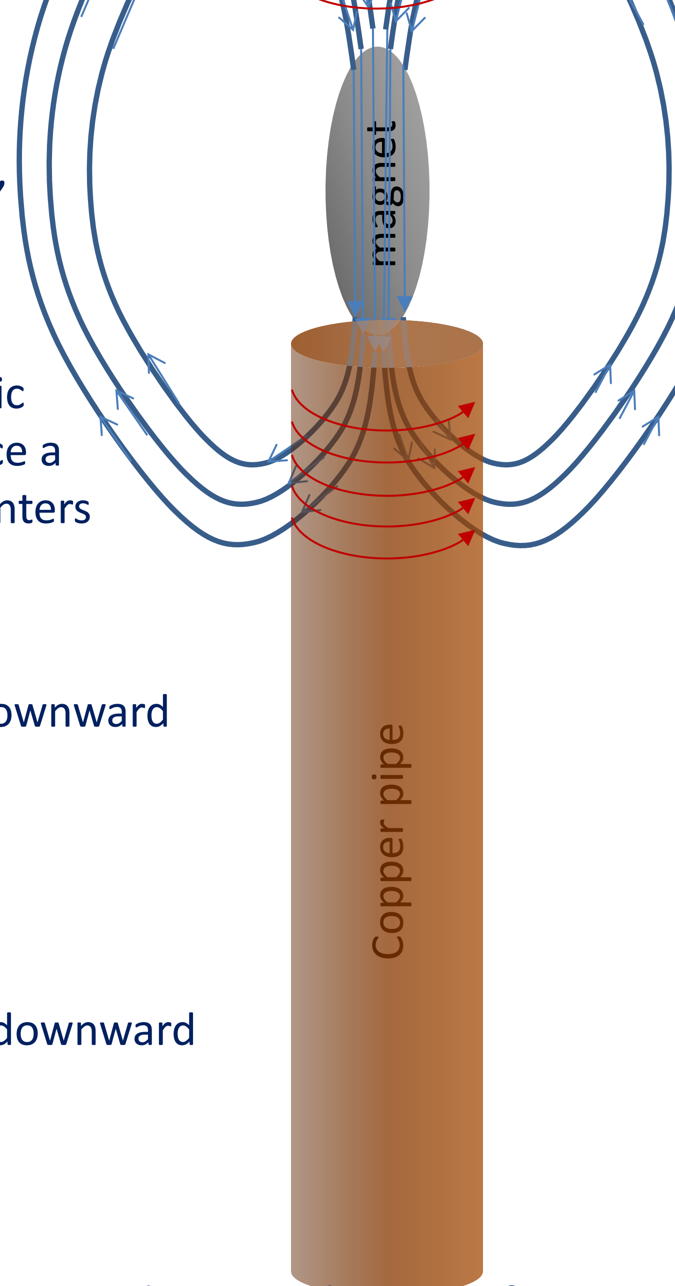
Rotate coil 1 (with current)



Demo !

'nature abhors a change in flux'

"Induced" $\mathcal{Emf}$ (or curled Electric field) drives current that produce a magnetic field which partly counters the change in magnetic flux.



To the left as downward flux decreases

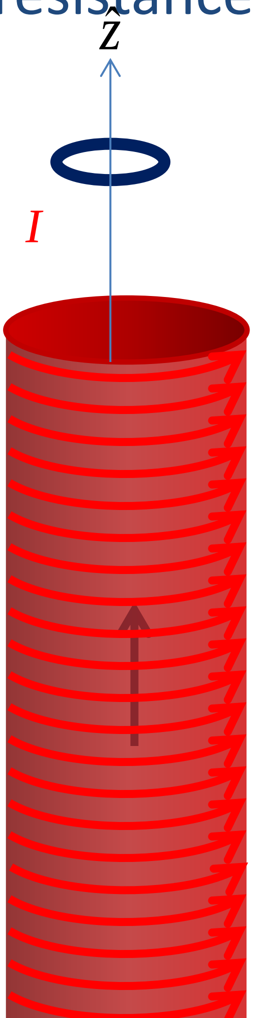
To the right as downward flux increases

Thus drives charges around pipe and so transfers energy. *These* charges in motion produce field which exerts force on moving charges in magnets.

Using Faraday's Law

$$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{or} \quad \mathcal{E}mf = -\frac{d\Phi_B}{dt} \quad \text{or} \quad \oint \vec{E}\cdot d\vec{l} = -\frac{\partial \Phi_B}{\partial t}\bigg|_{area}$$

Example: 'very long' solenoid of radius *a* with sinusoidally varying current such that $\vec{B} = B_o \cos(\omega t)\hat{z}$. A circular loop of radius a/2 and resistance R is inserted. What is the current induced around the loop?



Demo!

Using Faraday's Law

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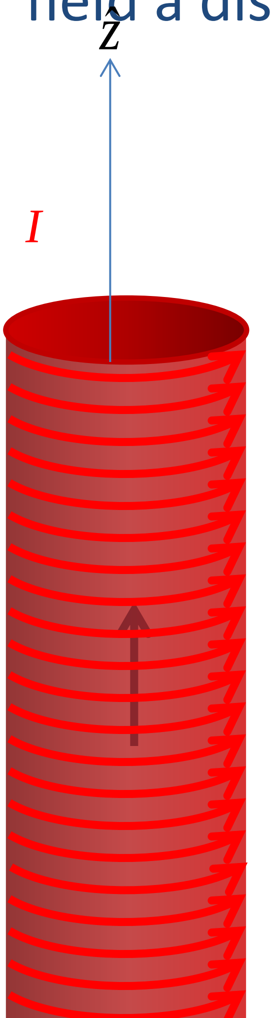
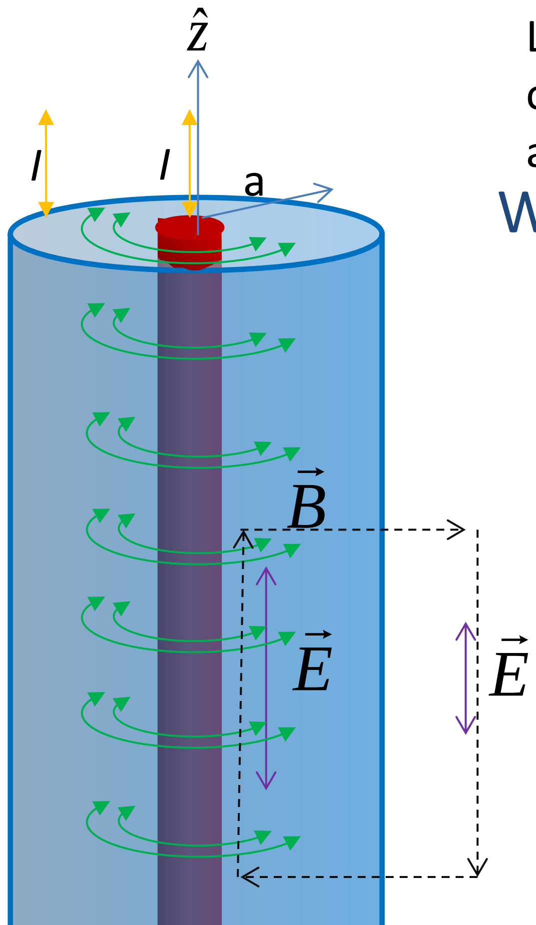
Exercise: 'very long' solenoid, with radius *a* and *n* turns per unit length, carries time varying current, *I*(*t*). What's an expression for the electric field a distance *s* from axis? Recall that inside a solenoid $\vec{B} = \mu_o I n \hat{z}$.



Using Faraday's Law

$$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{or} \quad \mathcal{Emf} = -\frac{d\Phi_B}{dt} \quad \text{or} \quad \oint \vec{E}\cdot d\vec{l} = -\frac{\partial \Phi_B}{\partial t}\bigg|_{area}$$

Example: A slowly varying alternating current, $I(t) = I_0\cos(\omega t)$, flows down a long, straight, thin wire and returns along a coaxial conducting tube of radius *a*.



In what direction must the electric field point?

Lenz' law says in the direction to drive current that would oppose changing flux, so down and up as the current varies up and down. $\hat{z}$

What's the electric field?

$$\oint \vec{E}\cdot d\vec{l} = -\frac{\partial}{\partial t}\int \vec{B}\cdot d\vec{a} \qquad \text{where} \quad \vec{B} = \begin{cases} \dfrac{\mu_0 I}{2\pi s}\hat{\phi} & s < a, \\ 0 & s > a. \end{cases}$$

Calls for an Amperian loop

and $I(t) = I_0\cos(\omega t)$

$$\oint \vec{E}\cdot d\vec{l} = \int_{in} \vec{E}\cdot d\vec{l} + \int_{top} \vec{E}\cdot d\vec{l} + \int_{out} \vec{E}\cdot d\vec{l} + \int_{bottom} \vec{E}\cdot d\vec{l}$$

$$\oint \vec{E}\cdot d\vec{l} = E(s_{in})\cdot\Delta z - E(s_{out})\cdot\Delta z$$

$$\frac{\partial}{\partial t}\int \vec{B}\cdot d\vec{a} = \frac{\partial}{\partial t}\int_{s_{in}}^{a} B\,ds'\Delta z = \frac{\partial}{\partial t}\int_{s_{in}}^{a} \frac{\mu_o I}{2\pi s'}ds'\Delta z = \frac{\partial}{\partial t}\left(\frac{\mu_o I}{2\pi}\ln\left(\frac{a}{s_{in}}\right)\Delta z\right.$$

Using Faraday's Law

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \quad \text{or} \quad \mathcal{E}mf = -\frac{d\Phi_B}{dt} \quad \text{or} \quad \oint\vec{E}\cdot d\vec{l} = -\frac{\partial\Phi_B}{\partial t}\Big|_{area}$$

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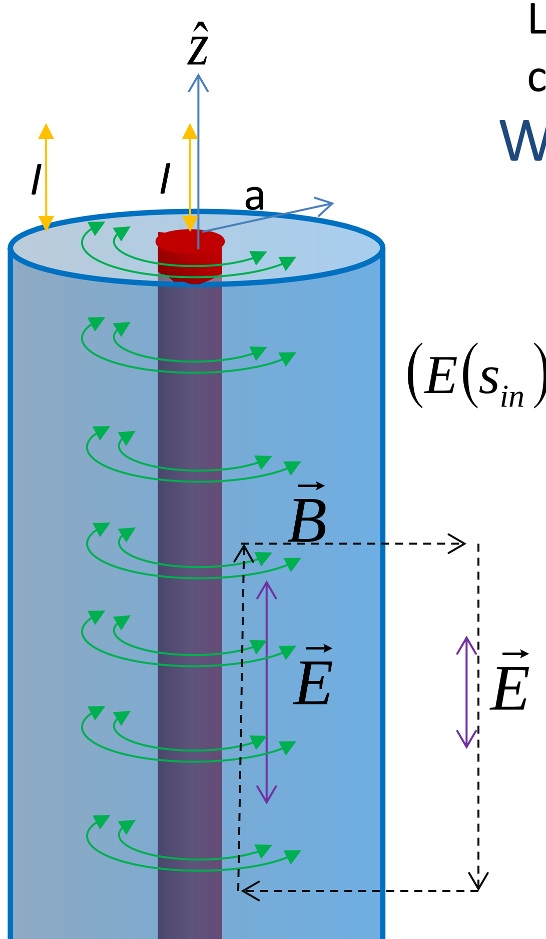

What's the electric field?

$$\oint\vec{E}\cdot d\vec{l} = -\frac{\partial}{\partial t}\int\vec{B}\cdot d\vec{a}$$

$$\left(E(s_{in}) - E(s_{out})\right)\Delta z = -\frac{\partial}{\partial t}\left(\frac{\mu_o I}{2\pi}\ln\left(\frac{a}{s_{in}}\right)\Delta z\right)$$

Right-hand-side is independent of how far out of loop s_{out} is, so E is constant outside. But it should be 0 *quite* far away, so must be 0 everywhere outside.

$$E(s_{in}) = -\frac{\partial}{\partial t}\left(\frac{\mu_o I}{2\pi}\ln\left(\frac{a}{s_{in}}\right)\right) = -\frac{\partial}{\partial t}\left(\frac{\mu_o I_o\cos(\omega t)}{2\pi}\ln\left(\frac{a}{s_{in}}\right)\right)$$

$$E(s_{in}) = \frac{\mu_o I_o \omega\sin(\omega t)}{2\pi}\ln\left(\frac{a}{s_{in}}\right)$$

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